POLYNOMIALS SOLUTIONS

(Q1) A) $x = \frac{x+12}{x}$

$x^2 = x + 12$

$x^2 - x - 12 = 0$

$(x-4)(x+3) = 0$

$x = -3, 4$

B) $x^2 - 6x = 11$

$x^2 - 6x - 11 = 0$

$(x^2 - 6x + 9) - 9 - 11 = 0$

$(x-3)^2 - 20 = 0$

$(x-3)^2 = 20$

$x - 3 = \pm 2\sqrt{5}$

$x = 3 \pm 2\sqrt{5}$

C) $0.2x^2 + \sqrt{3}x - 5 = 0$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$= \frac{-\sqrt{3} \pm \sqrt{3 - 4(0.2)(-5)}}{0.4}$

$= \frac{-\sqrt{3} \pm \sqrt{3+4}}{0.4}$

$= \frac{-\sqrt{3} \pm \sqrt{7}}{0.4}$

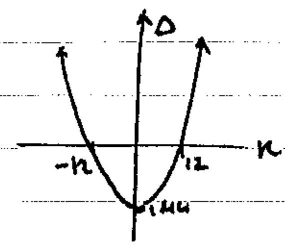
$= \frac{5}{2}(-\sqrt{3} \pm \sqrt{7})$

(Q2) $2x^2 - kx + 18 = 0$

$D = b^2 - 4ac$

$= k^2 - 4(2)(18)$

$= k^2 - 144$



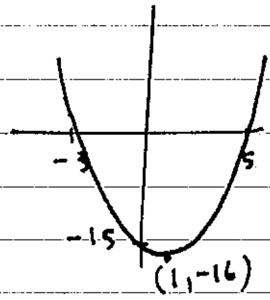
TWO SOL^NS IFF $D > 0$

I.E $K > 12$ OR $K < -12$

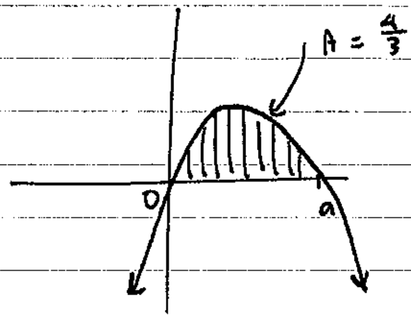
I.E $K \in (-\infty, -12) \cup (12, \infty)$

(Q3) $y = x^2 - 2x - 15$

$= (x-5)(x+3)$



(Q4)



$\int_0^a -x(x-a)\,dx = \int_0^a -x^2 + ax\,dx$

$= \left[-\frac{x^3}{3} + \frac{ax^2}{2}\right]_0^a$

$= -\frac{a^3}{3} + \frac{a^3}{2}$

$= +\frac{a^3}{6}$

$\therefore \frac{a^3}{6} = \frac{4}{3}$

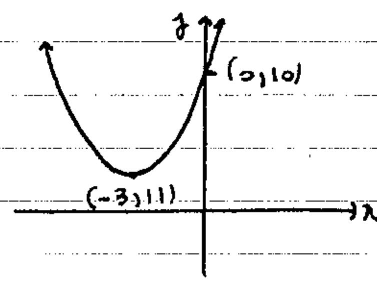
$\Rightarrow a^3 = \frac{4}{3} \times 6 = 8$

$\Rightarrow a = 2$

(Q5) $y = x^2 + 6x + 10$

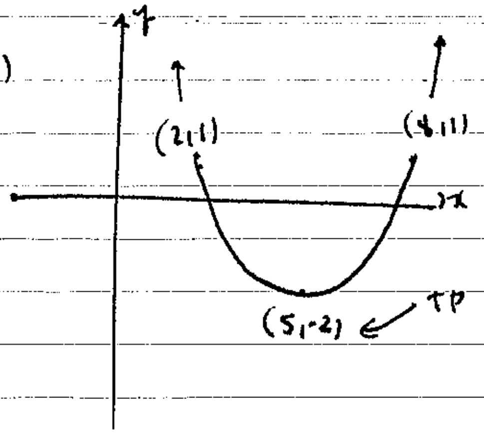
$= (x^2 + 6x + 9) - 9 + 10$

$= (x+3)^2 + 1$



#2

(Q6)



$y = c(x-5)^2 - 2$

WHEN $x = 2, y = 1$

$\therefore 1 = c(2-5)^2 - 2$

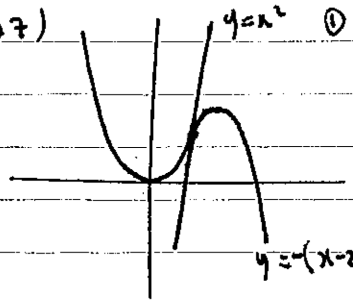
$1 = 9c - 2$

$9c = 3$

$c = \frac{1}{3}$

$\begin{cases} c = 1/3 \\ h = 5 \\ k = -2 \end{cases}$

(Q7)



① = ② $x^2 = -(x-2)^2 + k$

$x^2 = -x^2 + 4x - 4 + k$

$2x^2 - 4x + 4 - k = 0$

$\Delta = b^2 - 4ac$

$= 16 - 4(2)(4-k)$

$= 16 - 8(4-k)$

$= 16 - 32 + 8k$

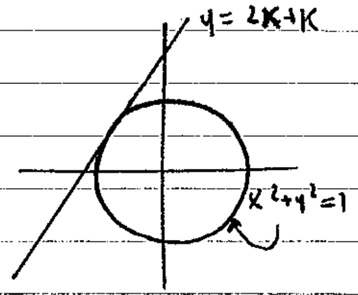
$= 8k - 16$

ONE SOL^{n} IFF $\Delta = 0$

$\therefore 8k - 16 = 0$

$\underline{k = 2}$

(Q8)



$x^2 + y^2 = 1$

$x^2 + (2x+k)^2 = 1$

$x^2 + 4x^2 + 4kx + k^2 - 1 = 0$

$5x^2 + (4k)x + k^2 - 1 = 0$

ONE SOL^{n} IFF $\Delta = 0$

$\therefore \Delta = b^2 - 4ac$

$= (4k)^2 - 4(5)(k^2-1)$

$= 16k^2 - 20k^2 + 20$

$= -4k^2 + 20$

$= 20 - 4k^2$

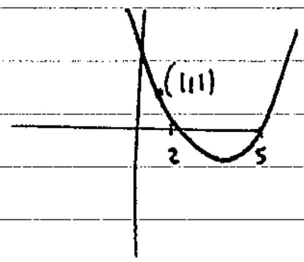
$20 - 4k^2 = 0$ (ONE SOL^{n})

$4k^2 = 20$

$k^2 = 5$

$\underline{k = \pm\sqrt{5}}$

(Q9) (a)



$y = c(x-2)(x-5)$

WHEN $x = 1, y = 1$

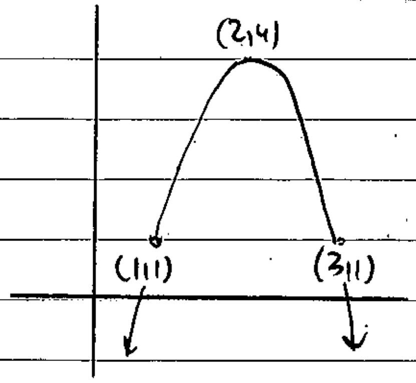
$\therefore 1 = c(1-2)(1-5)$

$1 = 4c$

$c = \frac{1}{4}$

$\therefore y = \frac{1}{4}(x-2)(x-5)$

(B)



$y = c(x-2)^2 + 4$

WHEN $x=1,\ y=1$

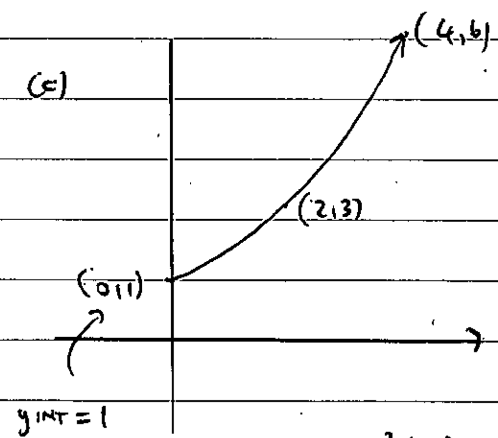
$1 = c(1-2)^2 + 4$

$1 = c + 4$

$c = -3$

$\therefore\ y = -3(x-2)^2 + 4$

(c)



$y = ax^2 + bx + 1$

$x=2, y=3)$ $\quad 3 = 4a + 2b + 1$

$2 = 4a + 2b$

$1 = 2a + b \quad ① \quad b = 1 - 2a$

$x=4, y=6)$ $\quad 6 = 16a + 4b + 1$

$5 = 16a + 4b \quad ②$

SUB ① INTO ②

$5 = 16a + 4(1 - 2a)$

$5 = 16a + 4 - 8a$

$1 = 8a \Rightarrow a = \frac{1}{8}$

SUB $a = \frac{1}{8}$ INTO ①,

$b = 1 - 2a$

$= 1 - \frac{1}{4}$

$= \frac{3}{4}$

$\therefore\ y = \frac{1}{8}x^2 + \frac{3}{4}x + 1.$

(10) $f(x) = 2x^3 - 18x^2 + 54x - 58$

$f(x) = a(x+h)^3 + k$

$= a(x^3 + 3x^2h + 3xh^2 + h^3) + k$

$= ax^3 + (3ah)x^2 + (3h^2a)x + (ah^3 + k)$

EQUATING COEFFICIENTS :

x^3: $\quad a = 2$

x^2: $\quad -18 = 3ah$

$-18 = 3 \times 2 \times h$

$-18 = 6h \Rightarrow h = -3$

x: $\quad 54 = 3h^2a$

$54 = 3 \times 9 \times 2$ ✓ ok

CONSTANT : $\quad -58 = ah^3 + k$

TERM $\quad -58 = 2 \times (-3)^3 + k$

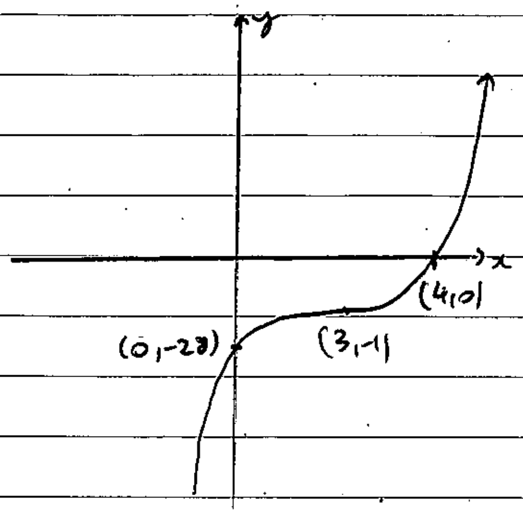
$-58 = -54 + k$

$k = -4.$

$\underline{a = 2,\ h = -3,\ k = -4}$

#4

(11) (A) $y = (x-3)^3 - 1$



WHEN $x=0, y=-28$

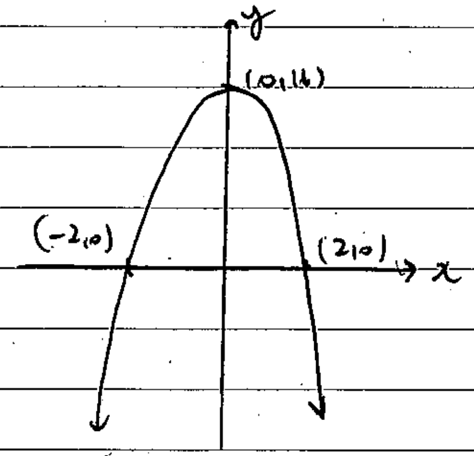
WHEN $y=0$ $\quad 0 = (x-3)^3 - 1$

$(x-3)^3 = 1$

$x-3 = 1$

$x = 4$

(B)

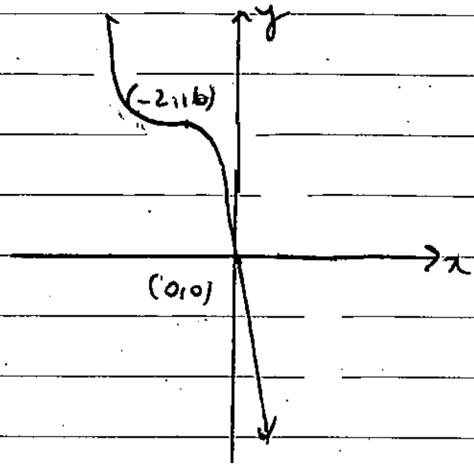


WHEN $y=0$, $\quad 0 = 16 - x^4$

$x^4 = 16$

$x = \pm 2$

(C) $y = -\frac{1}{2}(x+2)^5 + 16$



WHEN $y=0$ $\quad 0 = -\frac{1}{2}(x+2)^5 + 16$

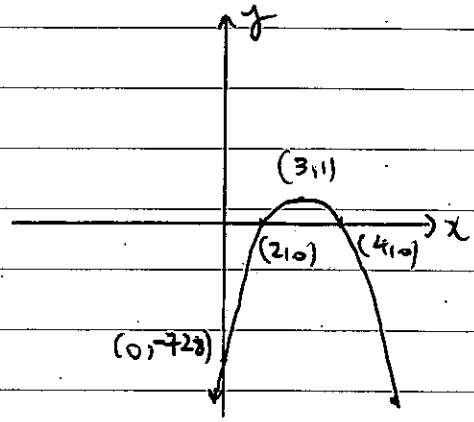
$\frac{1}{2}(x+2)^5 = 16$

$(x+2)^5 = 32$

$x+2 = \sqrt[5]{32} = 2$

$x = 0$

(D) $y = -(x-3)^6 + 1$



WHEN $x=0$, $y = -(-3)^6 + 1 = -728$

$y=0$, $\quad 0 = -(x-3)^6 + 1$

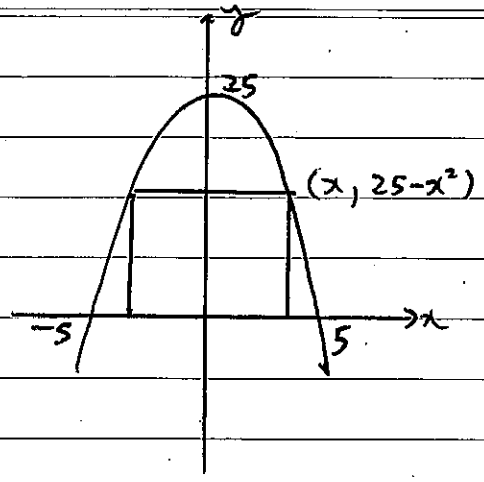
$(x-3)^6 = 1$

$x-3 = \pm 1$

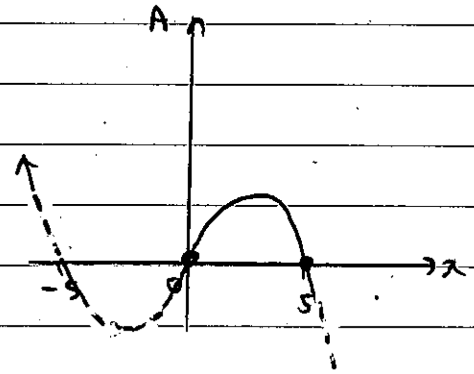
$x = 3 \pm 1$

$= 4, 2$

(Q12)



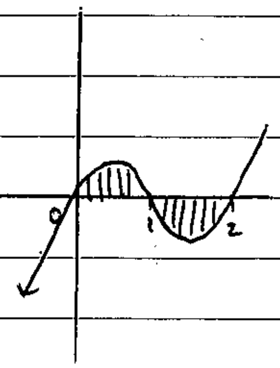
$$A = 2x(25-x^2)$$
$$= 2x(5-x)(5+x)$$



MAXIMISE $A = 50x - 2x^3$

$$\frac{dA}{dx} = 50 - 6x^2 = 0$$
$$\Rightarrow \quad 6x^2 = 50$$
$$\Rightarrow \quad x^2 = \frac{50}{6} = \frac{25}{2}$$
$$x = \frac{5}{\sqrt{2}} \quad (x > 0)$$
$$\Rightarrow \quad A = 50 \cdot \frac{5}{\sqrt{2}} - 2\left(\frac{5}{\sqrt{2}}\right)^3$$
$$= \frac{125}{\sqrt{2}}$$

(Q13)

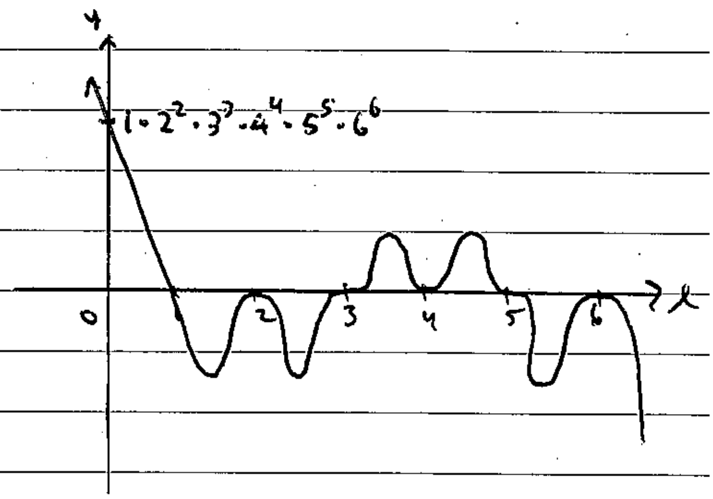


GRAPH IS SYMMETRIC ABOUT THE POINT $(1,0)$

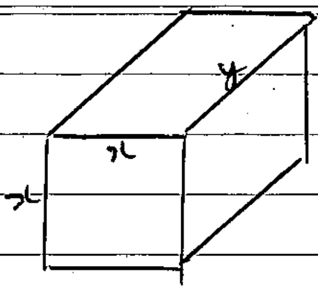
$$\therefore \ A = 2\int_0^1 x(x-1)(x-2)\,dx$$
$$= 2\int_0^1 (x^2-x)(x-2)\,dx$$
$$= 2\int_0^1 x^3 - 3x^2 + 2x\,dx$$
$$= 2\left[\frac{x^4}{4} - x^3 + x^2\right]_0^1$$
$$= 2\left[\left(\frac{1}{4} - 1 + 1\right) - (0)\right]$$
$$= \frac{1}{2}$$

$- \quad + \quad - \quad + \quad - \quad +$

(Q14) $y = (1-x)(2-x)^2(3-x)^3(4-x)^4(5-x)^5(6-x)^6$



(Q15)



$V = x^2 y$

$1 = x^2 y$

$y = \frac{1}{x^2}$

$A = 2x^2 + 4xy$

$= 2x^2 + 4x \cdot \frac{1}{x^2}$

$= 2x^2 + \frac{4}{x}$

$= 2x^2 + 4x^{-1}$

$\frac{dA}{dx} = 4x - 4x^{-2} = 0$

$\Rightarrow 4x^3 - 4 = 0$

$\Rightarrow x = 1$

$\Rightarrow A = 2x^2 + \frac{4}{x}$

$= 2 + 4$

$= 6$ (IT'S A CUBE!)

(Q16)

$$
\begin{array}{r}
2x^2 + 3x + 5 \\
x-2 \overline{\smash{)}\, 2x^3 - x^2 - x + 2} \\
\underline{2x^3 - 4x^2} \\
3x^2 - x + 2 \\
\underline{3x^2 - 6x} \\
5x + 2 \\
\underline{5x - 10} \\
12
\end{array}
$$

$2x^3 - x^2 - x + 2 = (x-2)\underbrace{(2x^2 + 3x + 5)}_{Q(x)} + \underbrace{12}_{R}$

(Q17)

$$
\begin{array}{r}
x^2 + x - 6 \\
x-1 \overline{\smash{)}\, x^3 + 0x^2 - 7x + 6} \\
\underline{x^3 - x^2} \\
x^2 - 7x + 6 \\
\underline{x^2 - x} \\
-6x + 6 \\
\underline{-6x + 6} \\
0
\end{array}
$$

$x^3 - 7x + 6 = (x-1)(x^2 + x - 6)$

$= \underline{(x-1)(x+3)(x-2)}$

(Q18) (A) $P(-3) = (-3)^3 - (-3)^2 - (-3) - 3$

$= -27 - 9 + \cancel{3} \cancel{-3}$

$= -36$

$\therefore R = -36$

(B) $P(3) = 3^3 + 3^2 + 3 + 3$

$= 27 + 9 + 6$

$= 42$

$\therefore R = 42$

(Q19) $P(2) = 1$ $\qquad$ $P(-1) = 2$

$8 + 2a - b = 1$ $\qquad$ $-1 - a - b = 2$

$b = 2a + 7$ $\qquad$ $a + b = -3$

SUB.

$a + 2a + 7 = -3$

$3a = -10$

$a = -\frac{10}{3}$

$\therefore b = 2 \times \left(-\frac{10}{3}\right) + 7$

$= -\frac{20}{3} + \frac{21}{3}$

$= \frac{1}{3}$

$\left\{ \begin{array}{l} a = -\frac{10}{3} \\ b = \frac{1}{3} \end{array} \right.$

(Q20) (A) $R = p(1/2)$

$= (1/2)^3 - (1/2)^2 + 1/2 + 1$

$= 1/8 + 1/4 + 1/2 + 1$

$= 1/8 - \frac{2}{8} + \frac{4}{8} + \frac{8}{8}$

$= \frac{11}{8}$

(B) $R = p(-1/2)$

$= -(-1/2)^3 + (-1/2)^2 - (-1/2)$

$= \frac{1}{8} + \frac{1}{4} + \frac{1}{2}$

$= \frac{1}{8} + \frac{2}{8} + \frac{4}{8}$

$= \frac{7}{8}$

(Q21) $p(x) = x^3 + 2x^2 - 5x - 6$

$p(-1) = (-1)^3 + 2(-1)^2 - 5(-1) - 6$

$= -1 + 2 + 5 - 6$

$= 0$

$\therefore$ $x+1$ IS A FACTOR

$+x$

$(x+1)(x^2 + x - 6)$

$-6x$

$= (x+1)(x+3)(x-2)$

(Q23) $x-2,\ x-3,\ x-4$ ARE FACTORS

$\therefore$ $p(x) = c(x-2)(x-3)(x-4)$

$36 = c(1-2)(1-3)(1-4)$

$36 = c(-1)(-2)(-3)$

$36 = -6c$

$c = -6$

$p(x) = -6(x-2)(x-3)(x-4)$

(Q22) $p(x) = 2x^3 + 5x^2 - 4x - 3$

$p(-1/2) = 2(-1/2)^3 + 5(-1/2)^2 - 4(-1/2) - 3$

$= -1/4 + 5/4 + 2 - 3$

$= 1 + 2 - 3$

$= 0$

$\Rightarrow$ $2x+1$ IS A FACTOR

$2x$

$p(x) = (2x+1)(x^2 + 2x - 3)$

$-6x$

$= (2x+1)(x+3)(x-1)$

(Q24) $p(2) = f(2) - 2 = 0$

$p(3) = f(3) - 3 = 0$

$p(4) = f(4) - 4 = 0$

$\therefore$ $x-2, x-3, x-4$ ARE FACTORS

$\therefore$ $p(x) = c(x-2)(x-3)(x-4)$

$\therefore$ $p(x) = f(x) - x$

$f(5) = 6$

$\Rightarrow$ $p(5) = f(5) - 5$

$c(5-2)(5-3)(5-4) = 6 - 5$

$c(3)(2)(1) = 1$

$c = \frac{1}{6}$

$f(x) = p(x) + x$

$= \frac{1}{6}(x-2)(x-3)(x-4) + x$

$f(1) = \frac{1}{6}(1-2)(1-3)(1-4) + 1$

$= \frac{1}{6}(-1)(-2)(-3) + 1$

$= -1 + 1$

$= 0$

(Q25) $p(x) = x^3 + ax^2 + bx + 4$

$p(-1) = 0$

$0 = -1 + a - b + 4$

$a - b = -3$ ①

$p(1) = 0$

$0 = 1 + a + b + 4$

$a + b = -5$ ②

① + ② $2a = -8$

$a = -4$

$\therefore b = -5 - a$

$= -5 + 4$

$= -1$

$\therefore p(x) = x^3 - 4x^2 - x + 4$

$= x^2(x-4) - (x-4)$

$= (x-4)(x^2-1)$

$= (x-4)(x-1)(x+1)$

SOL$^{\text{N}}$S ARE $x = \pm 1, 4$

(Q26) TRY $p(1/3) = 3(1/3)^3 - (1/3)^2 + 3(1/3) - 1$

$= \frac{1}{9} - \frac{1}{9} + 1 - 1$

$= 0$

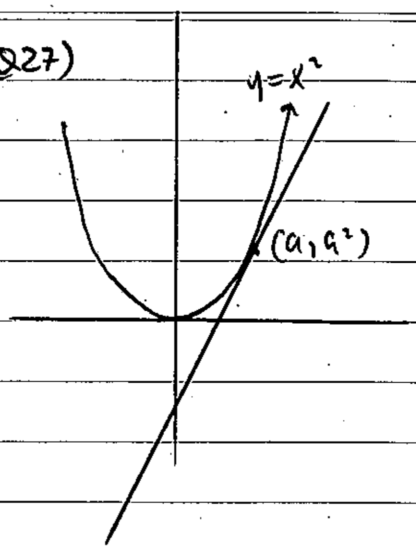
$\Rightarrow$ $3x-1$ IS A FACTOR

$\underbrace{(3x-1)(x^2 + 0x}_{3x} + 1)$

$= (3x-1)(x^2+1)$ ⟵ FULLY FACTORISED

$\therefore$ $x = 1/3$ IS ONLY SOLUTION

(Q27)



$y = x^2 \quad \therefore \frac{dy}{dx} = 2x$

WHEN $x = a \quad m = \frac{dy}{dx} = 2a$

$\therefore y - y_1 = m(x - x_1)$

$y - a^2 = 2a(x - a)$

$y - a^2 = 2ax - 2a^2$

$y = 2ax - a^2$

① $y = x^2$

② $y = 2ax - a^2$

$x^2 = 2ax - a^2$

$x^2 - 2ax + a^2 = 0$

$(x-a)^2 = 0$

$x = a$

ONE SOL$^{\text{N}}$

(Q28) $f(x) = 2x^3 - 9x^2 + 7x + 6$

$f(1) \neq 0$

$f(2) = 2 \cdot 8 - 9 \cdot 4 + 7 \cdot 2 + 6$

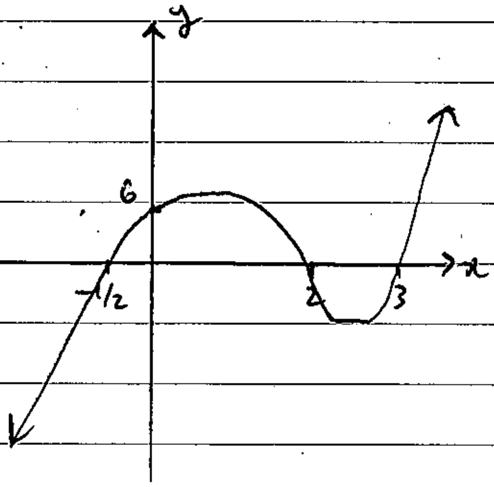
$= 16 - 36 + 14 + 6$

$= 0$

$\therefore x - 2$ IS A FACTOR

$f(x) = (x-2)(2x^2 - 5x - 3)$ (+10x, −3x)

$2x \times 1$, $x \times -3$ $= (x-2)(2x+1)(x-3)$



(Q29) $x^3 + x > 4x^2 - 6$

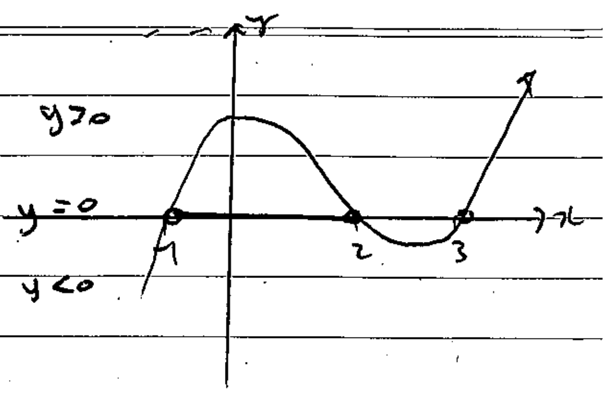
$\Leftrightarrow y = x^3 - 4x^2 + x + 6 > 0$

$p(-1) = -1 - 4 - 1 + 6 = 0$

$\therefore x + 1$ IS A FACTOR

$(x+1)(x^2 - 5x + 6) > 0$ (−5x, 6x)

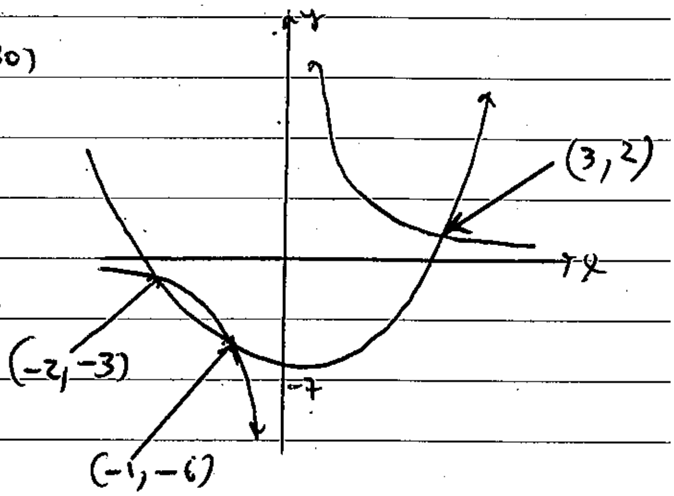
$y = (x+1)(x-3)(x-2) > 0$



$-1 < x < 2$ or $x > 3$

I.E. $x \in (-1, 2) \cup (3, \infty)$

(Q30)



① $y = \frac{6}{x}$

② $y = x^2 - 7$

① = ② $\frac{6}{x} = x^2 - 7$

$6 = x^3 - 7x$

$p(x) = x^3 - 7x - 6 = 0$

$p(-1) = -1 + 7 - 6 = 0$ $\therefore x + 1$ IS A FACTOR

$(x+1)(x^2 - x - 6) = 0$ (−x, −6x)

$(x+1)(x-3)(x+2) = 0$

$x = -1, -2, 3$

$\Rightarrow y = -6, -3, 2$

$(-1, -6), (-2, -3), (3, 2)$

#10

(Q31) $f(x) = x^3 + kx^2 + 6x - 2$

f WILL HAVE AN INVERSE IFF f IS ONE TO ONE

THIS OCCURS PROVIDED f IS STRICTLY INCREASING.

$f'(x) = 3x^2 + (2k)x + 6$

$\Delta = b^2 - 4ac$

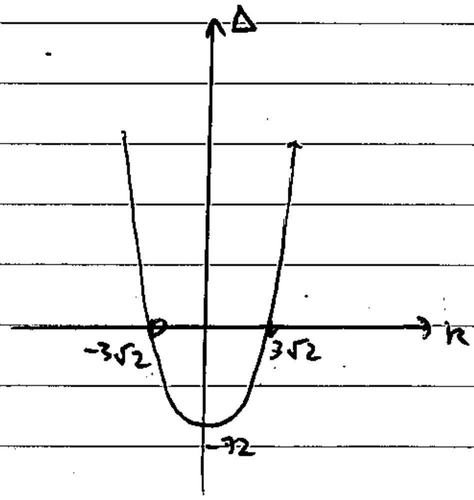
$= 4k^2 - 4(3)(6)$

$= 4k^2 - 72$

$f'(x) > 0$ FOR ALL x IFF

$\Delta < 0$

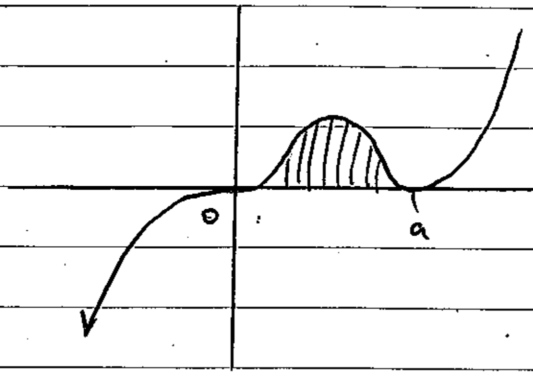
$\Delta = 4k^2 - 72$



$-3\sqrt{2} < k < 3\sqrt{2}$

I.E $k \in (-3\sqrt{2}, 3\sqrt{2})$

(Q32) $a > 0$ $f(x) = x^3(x-a)^2$



$A = \int_0^a x^3(x-a)^2\,dx$

$= \int_0^a x^3(x^2 - 2ax + a^2)\,dx$

$= \int_0^a x^5 - 2ax^4 + a^2x^3\,dx$

$= \left[\frac{x^6}{6} - \frac{2ax^5}{5} + \frac{a^2x^4}{4}\right]_0^a$

$= \frac{a^6}{6} - \frac{2a^6}{5} + \frac{a^6}{4}$

$= \left(\frac{10}{60} - \frac{24}{60} + \frac{15}{60}\right)a^6$

$= \frac{1}{60}a^6$

(Q33) $p(-1) = 0$

$-1 + 2 - a + b = 0$

$1 - a + b = 0$

$a = b + 1$

$\therefore x^3 + 2x^2 + (b+1)x + b = 0$

$\therefore (x+1)(x^2 + x + b) = 0$

REQUIRE ONE SOLN. I.E. $\Delta < 0$

$\Delta = 1^2 - 4(1)(b)$

$= 1 - 4b < 0$

$\Leftrightarrow b > \frac{1}{4}$

I.E. $b > \frac{1}{4}$ AND $a = b + 1$

#11

(Q34) $f(x) = x^3 + kx^2 + 3x - 5$

$f'(x) = 3x^2 + (2k)x + 3$

NO STAT PTS PROVIDED $f'(x) > 0$ NO ZEROES!

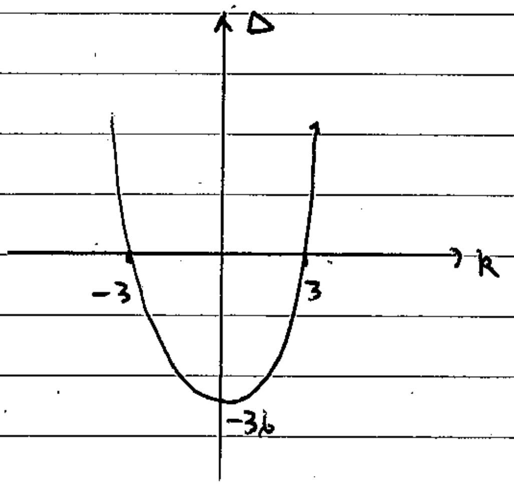
$\Delta = b^2 - 4ac$

$= (2k)^2 - 4(3)(3)$

$= 4k^2 - 36$

WANT $\Delta < 0$

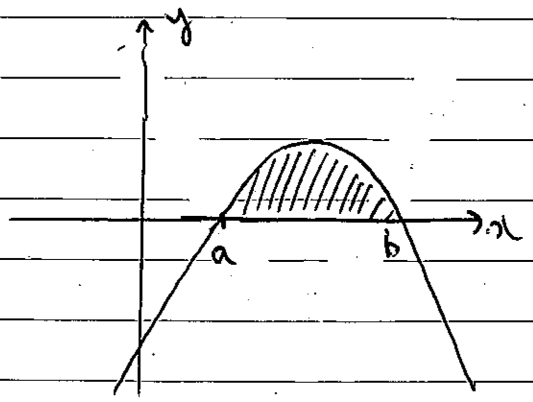
$\Delta = 4k^2 - 36 < 0$



$\underline{-3 < k < 3, \quad k \in (-3, 3)}$

(Q35) (A) $(b-a)^3 = b^3 - 3b^2a + 3ba^2 - a^3$

(B)



$A = \int_a^b -(x-a)(x-b)\, dx$

$= \int_a^b (a-x)(x-b)\, dx$

$= \int_a^b -x^2 + (a+b)x - ab\, dx$

$= \left[-\frac{x^3}{3} + \frac{(a+b)x^2}{2} - abx \right]_a^b$

$= -\frac{b^3}{3} + \frac{(a+b)b^2}{2} - ab^2$

$+ \frac{a^3}{3} - \frac{(a+b)}{2}a^2 + a^2b$

$= -\frac{b^3}{3} + \frac{b^3}{2} + \frac{ab^2}{2} - ab^2$

$+ \frac{a^3}{3} - \frac{a^3}{2} - \frac{ba^2}{2} + a^2b$

$= \frac{b^3}{6} - \frac{ab^2}{2} - \frac{a^3}{6} + \frac{a^2b}{2}$

$= \frac{1}{6}(b^3 - 3ab^2 + 3a^2b - a^3)$

$= \frac{1}{6}(b-a)^3$

$A = \frac{1}{6}$

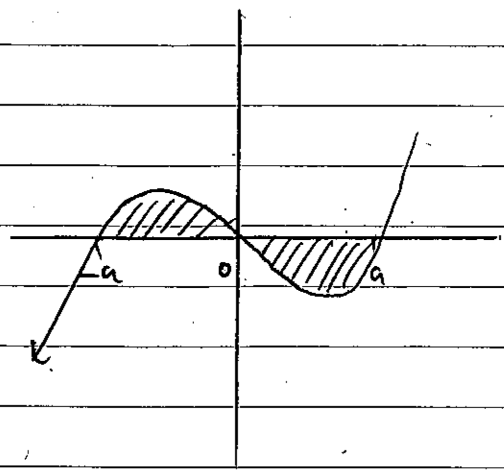
$\Rightarrow \frac{1}{6}(b-a)^3 = \frac{1}{6}$

$(b-a)^3 = 1$

$b - a = 1$

$\underline{b = a + 1}$

(Q36) $f(x) = x(x-a)(x+a)$



THE GRAPH IS SYMMETRIC ABOUT THE ORIGIN.

$$\therefore\ A = -2\int_0^a x(x^2-a^2)\,dx$$
$$= 2\int_0^a x(a^2-x^2)\,dx$$
$$= 2\int_0^a a^2x - x^3\,dx$$
$$= 2\left[\frac{a^2x^2}{2} - \frac{x^4}{4}\right]_0^a$$
$$= 2\left[\left(\frac{a^4}{2} - \frac{a^4}{4}\right) - (0-0)\right]$$
$$= 2\left(\frac{a^4}{4}\right)$$
$$= \frac{a^4}{2}$$

(Q37) $f(x) = x^3 - 6x^2 - 135x$
$$= x(x^2 - 6x - 135)$$
$$= x(x-15)(x+9)$$

WE REQUIRE THE MAX AND MIN OF THIS FUNCTION.

$$f'(x) = 3x^2 - 12x - 135$$

$3x \quad -27$
$x \quad 5$ $= (3x-27)(x+5)$

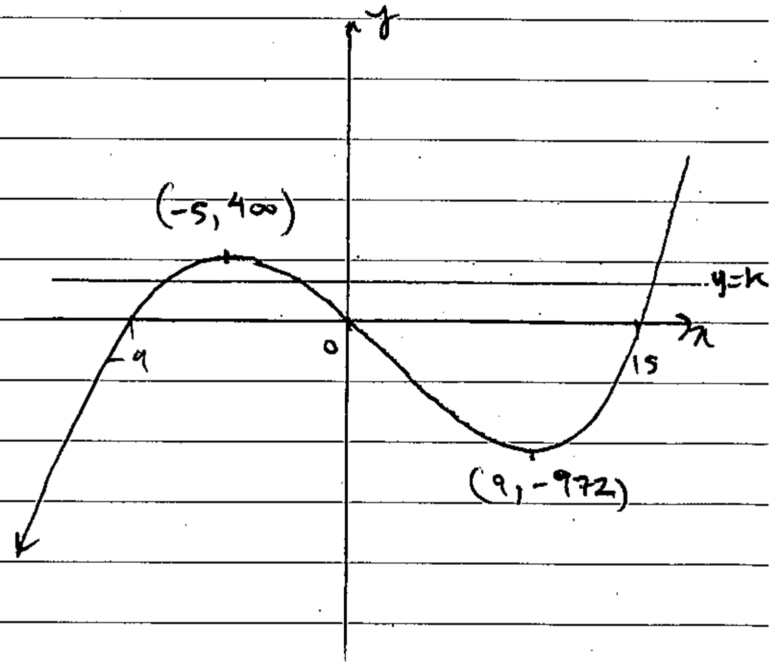
IF $f'(x) = 0$

THEN $(3x-27)(x+5) = 0$

$\Rightarrow x = 9, -5$

$f(9) = -972$

$f(-5) = 400$



$-972 < k < 400$

OR $k \in (-972, 400)$