# THE NORMAL DISTRIBUTION

**Q1)** A) (i) $X \sim N(660, 40)$

$Pr(620 < X < 700) \approx 0.683$

(ii) $Pr(580 < X < 740) \approx 0.954$

(iii) $Pr(X > 740) \approx \frac{1-0.954}{2} = 0.023$

(iv) $Pr(X < 540) \approx \frac{1-0.997}{2} \approx 0.0015$

B) $Pr(X < 620) \approx \frac{1-0.683}{2} = 0.1585$

EXP. # OF TIMES $= 0.1585 \times 80$

$\approx 12.68$ TIMES.

**Q2)** A) $z = \frac{x-\mu}{\sigma} = \frac{52-50}{0.8} = 2.5$

B) $z = \frac{x-\mu}{\sigma} = \frac{97-100}{1.8} \approx 1.67$

C) $x = z\sigma + \mu = 0.8 \cdot 2 + 10 = 11.6$

D) $x = z\sigma + \mu = -1.5 \cdot 1.5 + 80 = 77.75$

**Q3)** $z_A = \frac{x-\mu}{\sigma} = \frac{75-70}{4} = 1.25$

$z_B = \frac{x-\mu}{\sigma} = \frac{60-54}{5} = 1.2$

SINCE $z_A > z_B$, SHE PERFORMS BETTER ON TEST A COMPARED WITH B.

**Q4)** A) $Pr(X < 5) = 0.9234.$

B) $Pr(X < 11) = 0.3085$

C) $Pr(X > 20) = 0.2525$

D) $Pr(X > 9) = 0.9522$

E) $Pr(35 < X < 45) = 0.4680$

F) $Pr(7 < X < 10) = 0.6563$

**Q5)** $X \sim N(30, 7)$

$Pr(X > 45 \mid X > 40)$

$= \frac{Pr(X > 45 \text{ AND } X > 40)}{Pr(X > 40)}$

$= \frac{Pr(X > 45)}{Pr(X > 40)}$

$= 0.2098$

**Q6)** $X \sim N(41, 11)$

A) $Pr(X > 50) = 0.2066$

B) $Pr(X < 20) = 0.0281$

EXP. # OF TIMES $= 0.0281 \times 100 = 2.81$

**Q7)** A) $Pr(X < 113) = 0.9332$

B) $Pr(115 < X < 125) = 0.0606$

C) $Pr(115 < X < 125 \mid X > 115)$

$= \frac{Pr(115 < X < 125 \text{ AND } X > 115)}{Pr(X > 115)}$

$= \frac{Pr(115 < X < 125)}{Pr(X > 115)}$

$= 0.9071.$

**Q8)** A) $x = 8.52$

B) $x = 9.94$

C) $Pr(X \geq x) = 0.15 \therefore Pr(X < x) = 0.85 \therefore x = 17.11$

D) $Pr(X \geq x) = 0.58 \therefore Pr(X \leq x) = 0.42 \therefore x = 11.60$

(E) $Pr(100 < X < x) = 0.125$

$\therefore Pr(X < x) = 0.625$

$\therefore x = 101.59$

(F) $Pr(10-x < X < 10+x) = 0.6$

$\therefore Pr(X < 10-x) = 0.2$ $(\mu = 10)$

$\therefore Pr(X < 10+x) = 0.8$

$\therefore 10+x = 13.37$

$\Rightarrow x = 3.37.$

Q9) A) $X \sim N(30, 7)$

$Pr(X < x) = 0.75$

$\therefore x = 34.72$

B) $Pr(30-x < X < 30+x) = 0.8$

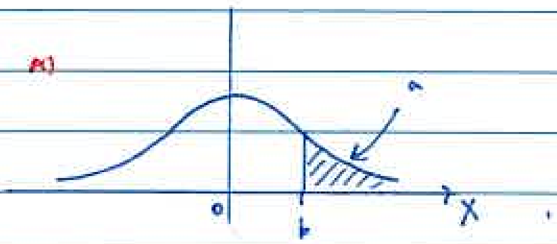
$\therefore Pr(X \leq 30-x) = 0.1$

$\therefore Pr(X < 30+x) = 0.9$

$\therefore x + 30 = 38.97$

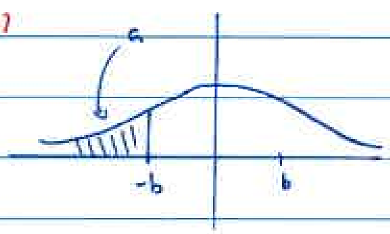
$\Rightarrow x = 8.97.$

Q10) A)




$Pr(X \leq b) = 1-a$

B)



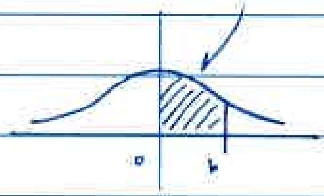

$Pr(X \leq -b) = a$

C) $Pr(X \geq -b) = 1-a$

D) $Pr(-b \leq X \leq b) = \frac{1-2a}{a}$

E) $Pr(0 \leq X \leq b) = 1 - a - \frac{1}{2}$

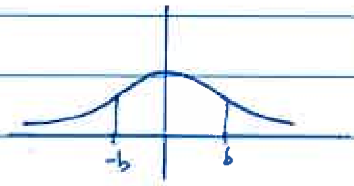
$= \frac{1}{2} - a$



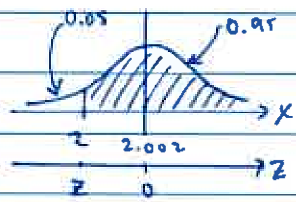

F) $Pr(X \geq b \mid X \geq -b) = \frac{Pr(X \geq b \text{ AND } X \geq -b)}{Pr(X \geq -b)}$

$= \frac{Pr(X \geq b)}{Pr(X \geq -b)}$

$= \frac{a}{1-a}$




Q11) $X \sim N(2.002, \sigma)$




$Pr(Z < z) = 0.05$

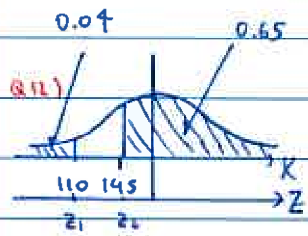
$z = -1.64485$

$z = \frac{x-\mu}{\sigma}$

$\sigma = \frac{x-\mu}{z}$

$= 0.0012$ L

Q12)




$Pr(Z < z_1) = 0.04$

$\therefore z_1 = -1.75069$

$Pr(Z > z_2) = 0.65$

$Pr(Z \leq z_2) = 0.35$

$z_2 = -0.38532$

$z = \frac{x-\mu}{\sigma}$

$-1.75069 = \frac{110-\mu}{\sigma}$ (1)

$-0.38532 = \frac{145-\mu}{\sigma}$ (2)

Solve (1) and (2) to get

$\mu = 155$ g

$\sigma = 26$ g

Q13) A) $X \sim N(175, 6.8)$

$Pr(X > 170) = 0.76892$

Any given man

Let $Y = \#\ > 170$ cm

$Y \sim B(50, 0.76892)$

$Pr(Y > 40) = 0.2507$

B) $X \sim N(175, 6.8)$

$Pr(X < 168) = 0.151643$

Any given man

Let $Y = \#\ < 168$

$Y \sim B(50, 0.151643)$

$Pr(5 \le Y \le 10) = 0.7667$

Q14) (A) $X \sim N(10500, 1000)$

(i) $Pr(X > 11000) = 0.3085$

(ii) $Pr(9000 < X < 10500) = 0.4332$

(iii) $Pr(X > 11000 \mid X > 10000)$

$$= \frac{Pr(X > 11000 \text{ and } X > 10000)}{Pr(X > 10000)}$$

$$= \frac{Pr(X > 11000)}{Pr(X > 10000)}$$

$= 0.4462$

(B) $Pr(X > x) = 0.99.$

$Pr(X < x) = 0.01$

$\Rightarrow x = 8173.64$

$\approx 8200$ hrs.

(C) $Pr(X > 11000) = 0.3085$

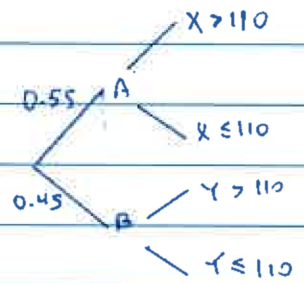
Let $Y = \#$ that last more than 11,000 hrs

$Pr(Y \ge 1) = 1 - Pr(Y = 0)$

$= 1 - (0.3085)^2$

$= 0.9048$

Q16)




$X \sim N(102, 3)$

$Y \sim N(99, 5)$

$Pr(X > 110) = 0.00383$

$Pr(Y > 110) = 0.01390$

$$Pr(A \mid IQ > 110) = \frac{Pr(\text{School A and } X > 110)}{Pr(IQ > 110)}$$

$$= \frac{0.55 \times 0.00383}{0.55 \times 0.00383 + 0.45 \times 0.01390}$$

$= 0.2519.$

Q15) $X \sim N(25, 4.5)$ $X$ = lifetime.

A) $Pr(X \ge x) = 0.95$

$Pr(X \le x) = 0.05$

$\Rightarrow x = 17.5982$ years

B) $Pr(X > 20) = 0.86674.$

C) Let $Y = \#$ working after 20 years. $Y \sim B(10, 0.86674)$

$\therefore Pr(Y \ge 8) = 0.8617.$

Use BinomCDF.

(Q17) $X \sim N(72.3, 5.4)$

A) i) $Pr(\text{NOT FINED})$

$= Pr(X \le 84)$

$= 0.98487$

ii) $Pr(\text{FINED } \$200)$

$= Pr(84 < X < 90)$

$= 0.014607$

(iii) $Pr(\text{FINED } \$800)$

$= Pr(X > 90)$

$= 0.000523$

(B) EXPECTED REVENUE

$= 10{,}000 \times (0.014607 \times \$200$

$+ 0.000523 \times \$800)$

$= \$4331.18$

$\approx \$4300$

(C) INCREASE REVENUE BY FACTOR

OF $\dfrac{\$18000}{\$4331.18}$

$\therefore$ # OF CARS $= \dfrac{18000}{4331.18} \times 10{,}000$

$= 41559.1$

$\approx 42{,}000$ CARS

D) $Pr(\text{EXCESS SPEED})$

$= Pr(X > 80)$

$= 0.07694$

E) $Pr(\text{FINED} \mid \text{SPEEDING})$

$= \dfrac{Pr(\text{FINED AND SPEEDING})}{Pr(\text{SPEEDING})}$

$= \dfrac{Pr(X > 84)}{Pr(X > 80)}$

$= 0.1966.$

Q18) A) (i) $Pr(\text{WORTH} = \$2)$

$= Pr(200 < X < 350)$

$= 0.7500$

(ii) $Pr(\text{WORTH} = \$3)$

$= Pr(X > 350)$

$= 0.1408$

B) EXPECTED TOTAL VALUE

$= 80(2 \times 0.7500 + 3 \times 0.1408)$

$= \$153.79$

$\approx \$154.$

c) $Pr(\text{WORTHLESS}) = Pr(X < 200)$

$= 0.109205$

$Y$ = # WORTHLESS $\quad Y \sim B(4, 0.109205)$

(i) $Pr(Y = 4) = 0.109205^4$

$= 0.0001422$

(ii) $Pr(Y \ge 2) = 0.06156.$

USE BINOMCDF

Q1(a) $X \sim N(\mu_1, \sigma_1)$ $\quad Z = \dfrac{X-\mu_1}{\sigma_1} \therefore X = \sigma_1 Z + \mu_1$

$Y \sim N(\mu_2, \sigma_2)$ $\quad Z = \dfrac{Y-\mu_2}{\sigma_2} \therefore Y = \sigma_2 Z + \mu_2$

$$\Pr(X < a) = \Pr(Y > a)$$

$$\Pr(\sigma_1 Z + \mu_1 < a) = \Pr(\sigma_2 Z + \mu_2 > a)$$

$$\Pr\left(Z < \frac{a-\mu_1}{\sigma_1}\right) = \Pr\left(Z > \frac{a-\mu_2}{\sigma_2}\right)$$

SINCE $Z$ IS SYMMETRIC ABOUT 0

$$\frac{a-\mu_1}{\sigma_1} = -\frac{a-\mu_2}{\sigma_2}$$

$$\sigma_2(a-\mu_1) = \sigma_1(\mu_2 - a)$$

$$\sigma_2 a - \sigma_2\mu_1 = \sigma_1\mu_2 - a\sigma_1$$

$$(\sigma_1+\sigma_2)a = \sigma_1\mu_2 + \sigma_2\mu_1$$

$$\Rightarrow a = \frac{\sigma_1\mu_2 + \sigma_2\mu_1}{\sigma_1+\sigma_2}$$