TOPIC #2 SOLUTIONS

Q1) BUCKET #1 BUCKET #2

BUCKET #1	BUCKET #2
x	3
$\frac{2}{3}x$	$3+\frac{1}{3}x$

$$\frac{2}{3}x + \frac{1}{3}\left(3+\frac{1}{3}x\right) = \frac{2}{3}\left(3+\frac{1}{3}x\right)$$

$\times 3)$ $$2x + \left(3+\frac{1}{3}x\right) = 2\left(3+\frac{1}{3}x\right)$$

$$2x = \left(3+\frac{1}{3}x\right)$$

$\times 3)$ $$6x = 9 + x$$

$$5x = 9$$

$$x = \frac{9}{5} \text{ LITRES.}$$

Q2) BUCKET #1 BUCKET #2

BUCKET #1	BUCKET #2
x	y
$\frac{1}{2}x$	$y+\frac{1}{2}x$

$$\frac{1}{2}x + \frac{1}{2}\left(y+\frac{1}{2}x\right) = \frac{2}{3}\left(y+\frac{1}{2}x\right)$$

$\times 6)$ $$3x + 2\left(y+\frac{1}{2}x\right) = 4\left(y+\frac{1}{2}x\right)$$

$$3x + 2y + x = 4y + 2x$$

$$4x + 2y = 4y + 2x$$

$$2x = 2y$$

$$\frac{x}{y} = 1$$

Q3 A) $$ax + e = cx + d$$

$$ax - cx = d - e$$

$$x(a-c) = d - e$$

$$x = \frac{d-e}{a-c} \quad (a \neq c)$$

B) $$a(x+b) = c(x+d)$$

$$ax + ab = cx + cd$$

$$ax - cx = cd - ab$$

$$x(a-c) = cd - ab$$

$$x = \frac{cd-ab}{a-c} \quad (a \neq c)$$

C) $$\frac{a-x}{b-x} = 2$$

$$a - x = 2(b-x)$$

$$a - x = 2b - 2x$$

$$a - 2b = x - 2x$$

$$a - 2b = -x$$

$$x = 2b - a$$

D) $$\frac{ax-b}{c-dx} = \frac{e}{f}$$

$$f(ax-b) = e(c-dx)$$

$$fax - fb = ec - dex$$

$$fax + dex = ec + fb$$

$$x(fa+de) = ec + fb$$

$$x = \frac{ec+fb}{fa+de} \quad (fa+de \neq 0)$$

Q4)

$$ax + by = b \quad ①$$

$$bx - ay = a \quad ②$$

$$bax + b^2y = b^2 \quad ① \times b = ③$$

$$bax - a^2y = a^2 \quad ② \times a = ④$$

$③ - ④$ $$(b^2+a^2)y = b^2 - a^2$$

$$y = \frac{b^2-a^2}{b^2+a^2}$$

WHEN $y = \frac{b^2-a^2}{b^2+a^2}$ ⟵ SUB INTO ①

$$\cancel{\frac{a(b^2-a^2)}{b^2+a^2} + by = b}$$

$$ax + b\left(\frac{b^2-a^2}{b^2+a^2}\right) = b \quad \cancel{by = b}$$

$$\frac{a}{b}x + \frac{b^2-a^2}{b^2+a^2} = 1$$

$$\frac{ax}{b} = 1 - \frac{b^2-a^2}{b^2+a^2}$$

$$= \frac{b^2+a^2-b^2+a^2}{b^2+a^2}$$

$$= \frac{2a^2}{b^2+a^2}$$

$$\therefore x = \frac{2ab}{a^2+b^2}$$

$$\cancel{b - \frac{a(b^2-a^2)}{b^2+a^2}}$$

$$\cancel{= \frac{b(b^2+a^2) - a(b^2-a^2)}{b^2+a^2}}$$

$$\cancel{= \frac{b^3 - ba^2 - ab^2 + a^3}{b^2+a^2}}$$

$$\cancel{= \frac{b^3 - ab^2 - ba^2 - a^3}{b^2+a^2}}$$

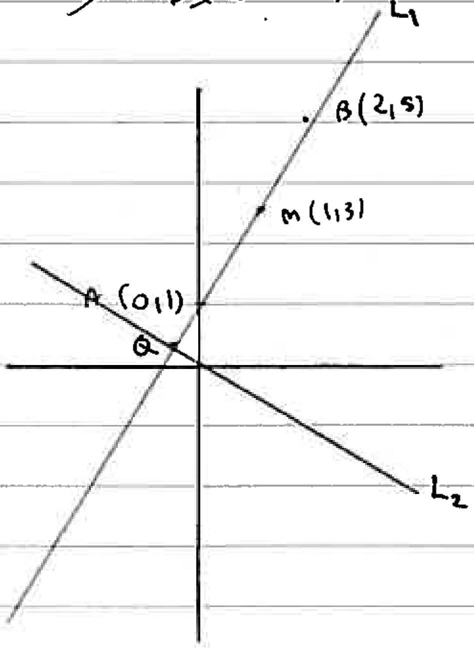
PTO

$$\cancel{by = \frac{b^2(b-a) - a^2(b-a)}{b^2+a^2}}$$

$$\cancel{= \frac{(b-a)(b^2-a^2)}{b^2+a^2}}$$

$$\cancel{\Rightarrow \quad y = \frac{(b-a)(b^2-a^2)}{b(b^2+a^2)}}$$

Q4)



A) $AB = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}$

$= \sqrt{(2-0)^2 + (5-1)^2}$

$= \sqrt{4+16}$

$= \sqrt{20}$

$= 2\sqrt{5}$

B) $y - y_1 = m(x - x_1)$ $\quad m = \frac{5-1}{2-0}$

$y - 1 = 2(x-0)$ $\quad = \frac{4}{2}$

$y - 1 = 2x$ $\quad = 2$

$y = 2x + 1$

C) MIDPOINT: $M = \left(\frac{2+0}{2}, \frac{5+1}{2}\right)$

$= (1, 3)$

GRADIENT: $m = -\frac{1}{2}$

EQUATION: $y - y_1 = m(x - x_1)$

$y - 3 = -\frac{1}{2}(x-1)$

$\times 2)$ $\quad 2y - 6 = -(x-1)$

$2y - 6 = -x + 1$

$x + 2y = 7$

D) EQUATION OF LINE L_2:

$y = -\frac{1}{2}x$

$\underline{L_1 = L_2}$ $\quad -\frac{1}{2}x = 2x + 1$

$\times 2)$ $\quad -x = 4x + 2$

$5x = -2$

$x = -\frac{2}{5}$

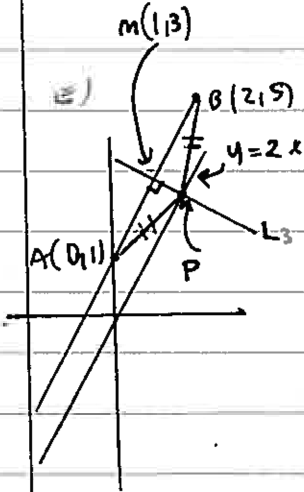
$y = -\frac{1}{2}x$

$= -\frac{1}{2}\left(-\frac{2}{5}\right)$

$= +\frac{1}{5}$

$\therefore Q\left(-\frac{2}{5}, +\frac{1}{5}\right)$

E)



$L_3: \quad x + 2y = 7$

$L_1: \quad y = 2x$

SUB $y = 2x$ INTO L_3

$x + 2(2x) = 7$

$x + 4x = 7$

$5x = 7$

$x = \frac{7}{5}$

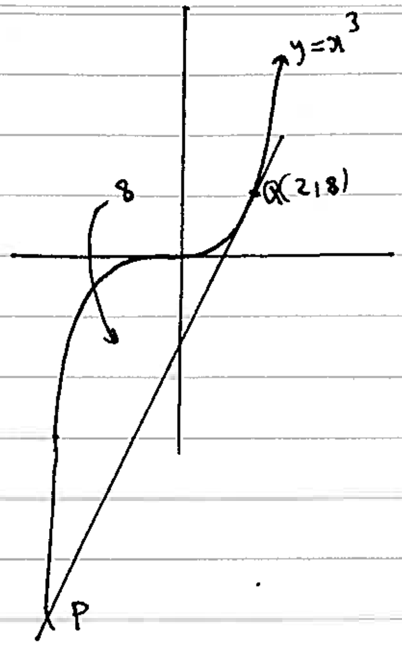
$y = 2x$

$= 2 \times \frac{7}{5}$

$= \frac{14}{5}$

$\therefore P\left(\frac{7}{5}, \frac{14}{5}\right)$

Q6)



$y = x^3 \quad \frac{dy}{dx} = 3x^2$

WHEN $x = 2 \quad \frac{dy}{dx} = 3 \cdot 2^2 = 12$

EQ$^{\underline{N}}$ OF TANGENT : $y - y_1 = m(x - x_1)$

$y - 8 = 12(x-2)$

$y = 12x - 24 + 8$

$= 12x - 16$

THEN, $y = 12x - 16$ ①

$y = x^3$ ②

① = ② $\quad x^3 = 12x - 16$

$x^3 - 12x + 16 = 0$

LET $\quad P(x) = x^3 - 12x + 16$

$P(2) = 8 - 24 + 16 = 0$

$P(x) = (x-2)(x^2 + 2x - 8)$ $\quad -4x \quad -8x$

$= (x-2)(x+4)(x-2)$

$= (x-2)^2(x+4)$

SO IF $\quad (x-2)^2(x+4) = 0$

$\Rightarrow \quad x = 2, \ x = -4$

WHEN $x = -4, \ y = (-4)^3$

$= -64.$

$\therefore \ P(-4, -64)$

TOP $\quad$ BOTTOM

$\therefore$ AREA $= \int_{-4}^{2} x^3 - (12x - 16) \, dx$

$= \int_{-4}^{2} x^3 - 12x + 16 \, dx$

$= \left[\frac{x^4}{4} - \frac{12x^2}{2} + 16x \right]_{-4}^{2}$

$= 108.$

Q7 A) $A + B$

$= \begin{bmatrix} 1 & 0 \\ -2 & 3 \end{bmatrix} + \begin{bmatrix} 2 & 3 \\ -1 & 1 \end{bmatrix}$

$= \begin{bmatrix} 3 & 3 \\ -3 & 4 \end{bmatrix}$

B) $AB = \begin{bmatrix} 1 & 0 \\ -2 & 3 \end{bmatrix}\begin{bmatrix} 2 & 3 \\ -1 & 1 \end{bmatrix}$

$= \begin{bmatrix} 2 & 3 \\ -7 & -3 \end{bmatrix}$

C) $AC = \begin{bmatrix} 1 & 0 \\ -2 & 3 \end{bmatrix}\begin{bmatrix} 2 \\ 1 \end{bmatrix}$

$= \begin{bmatrix} 2 \\ -1 \end{bmatrix}$

D) $2C = 2\begin{bmatrix} 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 4 \\ 2 \end{bmatrix}$

E) $B^{-1} = \frac{1}{2 \cdot 1 - 3(-1)} \begin{bmatrix} 1 & -3 \\ 1 & 2 \end{bmatrix}$

$= \frac{1}{5} \begin{bmatrix} 1 & -3 \\ 1 & 2 \end{bmatrix}$

F) $2A - 3B$

$= 2\begin{bmatrix} 1 & 0 \\ -2 & 3 \end{bmatrix} - 3\begin{bmatrix} 2 & 3 \\ -1 & 1 \end{bmatrix}$

$= \begin{bmatrix} 2 & 0 \\ -4 & 6 \end{bmatrix} - \begin{bmatrix} 6 & 9 \\ -3 & 3 \end{bmatrix}$

$= \begin{bmatrix} -4 & -9 \\ -1 & 3 \end{bmatrix}$

Q8) A) $A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$

$A^2 = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$

$= \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$

$A^4 = A^2 . A^2$

$= \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$

$= \begin{bmatrix} 1 & 4 \\ 0 & 1 \end{bmatrix}$

$A^8 = A^4 . A^4$

$= \begin{bmatrix} 1 & 4 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 4 \\ 0 & 1 \end{bmatrix}$

$= \begin{bmatrix} 1 & 8 \\ 0 & 1 \end{bmatrix}$

$A^9 = A^8 A$

$= \begin{bmatrix} 1 & 8 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$

$= \begin{bmatrix} 1 & 9 \\ 0 & 1 \end{bmatrix}$

b) $B = \begin{bmatrix} 1 & -1 \\ 1 & 0 \end{bmatrix}$

$B^2 = \begin{bmatrix} 1 & -1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} 1 & -1 \\ 1 & 0 \end{bmatrix}$

$= \begin{bmatrix} 0 & -1 \\ 1 & -1 \end{bmatrix}$

$B^4 = B^2 . B^2$

$= \begin{bmatrix} 0 & -1 \\ 1 & -1 \end{bmatrix}\begin{bmatrix} 0 & -1 \\ 1 & -1 \end{bmatrix}$

$= \begin{bmatrix} -1 & 1 \\ -1 & 0 \end{bmatrix}$

$B^8 = B^4 . B^4$

$= \begin{bmatrix} -1 & 1 \\ -1 & 0 \end{bmatrix}\begin{bmatrix} -1 & 1 \\ -1 & 0 \end{bmatrix}$

$= \begin{bmatrix} 0 & -1 \\ 1 & -1 \end{bmatrix}$

$B^{10} = B^8 B^2$

$= \begin{bmatrix} 0 & -1 \\ 1 & -1 \end{bmatrix}\begin{bmatrix} 0 & -1 \\ 1 & -1 \end{bmatrix}$

$= \begin{bmatrix} -1 & 1 \\ -1 & 0 \end{bmatrix}$

Q9) $AX + 2B = 3A$

$AX = 3A - 2B$

$X = A^{-1}(3A - 2B)$

$= \frac{1}{1(-1)-(2)(-1)}\begin{bmatrix} -1 & 1 \\ -2 & 1 \end{bmatrix}\left(\begin{bmatrix} 3 & -3 \\ 6 & -3 \end{bmatrix} - \begin{bmatrix} 0 & -2 \\ 2 & 4 \end{bmatrix}\right)$

$= \frac{1}{-1+2}\begin{bmatrix} -1 & 1 \\ -2 & 1 \end{bmatrix}\begin{bmatrix} 3 & -1 \\ 4 & -7 \end{bmatrix}$

$= \begin{bmatrix} -1 & 1 \\ -2 & 1 \end{bmatrix}\begin{bmatrix} 3 & -1 \\ 4 & -7 \end{bmatrix}$

$= \begin{bmatrix} 1 & -6 \\ -2 & -5 \end{bmatrix}$

Q10) A) $3x - 2y = -1$ $\qquad$ $6x + 4y = -2$

$2y = 3x + 1$ $\qquad$ $4y = -6x - 2$

① $y = \frac{3}{2}x + \frac{1}{2}$ $\qquad$ ② $y = -\frac{3}{2}x - \frac{1}{2}$

$m_1 = 3/2$ $\qquad$ $m_2 = -3/2$

$m_1 \neq m_2$

$\Rightarrow$ LINES INTERSECT

@ ONE UNIQUE POINT

$\Rightarrow$ ONE SOLUTION

① = ② $\quad \frac{3}{2}x + \frac{1}{2} = -\frac{3}{2}x - \frac{1}{2}$

×2) $\quad 3x + 1 = -3x - 1$

$6x = -2$

$x = -\frac{1}{3}$

$\therefore y = \frac{3}{2}x + \frac{1}{2}$

$= \frac{3}{2}(-\frac{1}{3}) + \frac{1}{2}$

$= -\frac{1}{2} + \frac{1}{2}$

$= 0$

$\begin{cases} x = -1/3 \\ y = 0 \end{cases}$

B) $2x - y = -8$ $\qquad$ $-4x + 2y = 3$

$y = 2x + 8$ $\qquad$ $2y = 4x + 3$

$m_1 = 2, c_1 = 8$ $\qquad$ $y = 2x + 3/2$

$m_2 = 2, c_2 = 3/2$

$m_1 = m_2$ AND $c_1 \neq c_2$

$\Rightarrow$ LINES DO NOT INTERSECT

$\Rightarrow$ NO SOLUTION

c) $y - 3x = -1$ $\quad$ $-2y + 6x = 2$

$y = 3x - 1$ $\quad$ $2y = 6x - 2$

$m_1 = 3,\ c_1 = -1$ $\quad$ $y = 3x - 1$

$\begin{cases} m_1 = m_2 \\ c_1 = c_2 \end{cases}$ $\quad$ $m_2 = 3,\ c_2 = -1$

$\Rightarrow$ LINES ARE IDENTICAL

LET $x = t \in \mathbb{R}$

THEN $y = 3x - 1$

$= 3t - 1$

SOLUTION : $\begin{cases} x = t \\ y = 3t - 1 \end{cases}, t \in \mathbb{R}$

Q11 $mx + 3y = 1$ $\quad$ $x + (m+2)y = m$

$3y = -mx + \frac{1}{3}$ $\quad$ $(m+2)y = -x + m$

$y = -\frac{m}{3}x + \frac{1}{3}$ $\quad$ $y = -\frac{1}{m+2}x + \frac{m}{m+2}$

$\therefore$ $c_1 = \frac{1}{3}$ $m_1 = -\frac{m}{3}$ $\quad$ $c_2 = \frac{m}{m+2}$ $m_2 = -\frac{1}{m+2}$

A) (i) ONE SOL^n IFF $m_1 \neq m_2$

$-\frac{m}{3} \neq -\frac{1}{m+2}$

$m(m+2) \neq 3$

$m^2 + 2m - 3 \neq 0$

$(m+3)(m-1) \neq 0$

$m \neq 1, -3$ $\quad$ i.e. $m \in \mathbb{R} \setminus \{1, -3\}$.

CALCULATOR GIVES SOLUTION AS :

$\begin{cases} x = -\frac{2}{m+3} \\ y = \frac{m+1}{m+3} \end{cases}$

(ii) NO SOL^n IFF $m_1 = m_2$ AND $c_1 \neq c_2$

$m_1 = m_2$ AND $c_1 \neq c_2$

$m = 1, -3$ AND $\frac{1}{3} \neq +\frac{m}{m+2}$

$m + 2 \neq +3m$

$2m \neq 2$

$m \neq +1$

$\therefore$ NO SOL^n IFF $m = -3$

(iii) ∞ MANY SOL^n IFF

$m_1 = m_2$ AND $c_1 = c_2$

$m = 1, -3$ AND $m = 1$

$\therefore$ $m = 1$

$\therefore$ ∞ SOL^n IFF $m = 1$.

NOTE : IF $m = 1$ THEN

$y = -\frac{1}{3}x + \frac{1}{3}$

LET $\begin{cases} x = t \in \mathbb{R} \\ y = -\frac{1}{3}t + \frac{1}{3} \end{cases}$ THEN

Q12 A) $x + y + z = 1$ ①

$-x + y + z = 2$ ②

$-x - y + z = 2$ ③

①+② $2y + 2z = 3$ ④

1+③ $2z = 3 \therefore z = 3/2$

Sub $z = 3/2$ into ④

$2y + 2 \cdot \frac{3}{2} = 3$

$2y + 3 = 3$

$2y = 0$

$y = 0$

Sub $y = 0$, $z = 3/2$ into ①

$x + 0 + 3/2 = 1$

$x = 1 - 3/2 = -1/2$

$$\begin{cases} x = -1/2 \\ y = 0 \\ z = 3/2 \end{cases}$$

B) $x + y + z = 1$ ①

$2x - y + 2z = 2$ ②

$x - 2y + z = 3$ ③

NOTE ③ = ① + ②

$\therefore$ $x + y + z = 1$ ①

$2x - y + 2z = 2$ ②

①+② $3x + 3z = 3$

$x + z = 1$

Let $z = t \in \mathbb{R}$

Then $x = 1 - t$

Then $y = 1 - x - z$

$= 1 - (1 - t) - t$

$= 1 - 1 + t - t$

$= 0$

$$\begin{cases} x = 1 - t \\ y = 0 \\ z = t \end{cases}, \quad t \in \mathbb{R}.$$

Q13 $(x, y) \to (x - 2y, 3x - 2y)$

$(3, 4) \to (3 - 2 \cdot 4, 3 \cdot 3 - 2 \cdot 4)$

$= (-5, 1)$

Q14 Unit square has co-ordinates

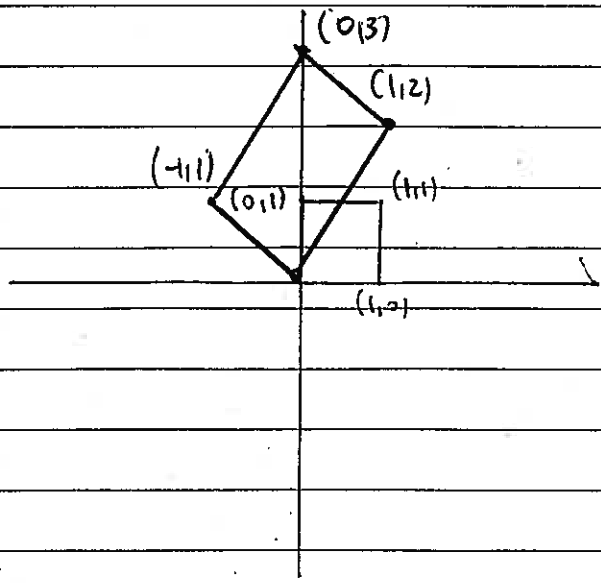
$(0,0)$, $(1,0)$, $(0,1)$, $(1,1)$ So:

$(0,0) \to (0,0)$ Matrix $= \begin{bmatrix} 1 & -1 \\ 2 & 1 \end{bmatrix}$

$(1,0) \to (1,2)$

$(0,1) \to (-1,1)$

$(1,1) \to (1-1, 2+1) = (0,3)$



Q15) $\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & -2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$

$x' = x \qquad x = x'$

$y' = -2y \qquad y = -\frac{1}{2}y'$

$y = x^2 + 1$

$-\frac{1}{2}y' = (x')^2 + 1$

IGNORE DASHES, OBTAIN:

$-\frac{1}{2}y = x^2 + 1$

$y = -2(x^2+1)$

$y = -2x^2 - 2$ ← IMAGE

Q16) $\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$

$\left.\begin{array}{ll} x' = y & y = x' \\ y' = x & x = y' \end{array}\right\}$ NOTE: SWITCHES x WITH y!!

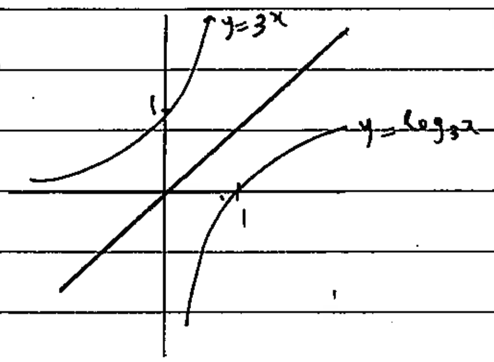
$y = 3^x$

$x' = 3^{y'}$

IGNORE DASHES. OBTAIN:

$x = 3^y$

$y = \log_3 x$



Q17) $\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} -3 \\ 2 \end{bmatrix}$

$x' = x - 3 \qquad x = x' + 3$

$y' = y + 2 \qquad y = y' - 2$

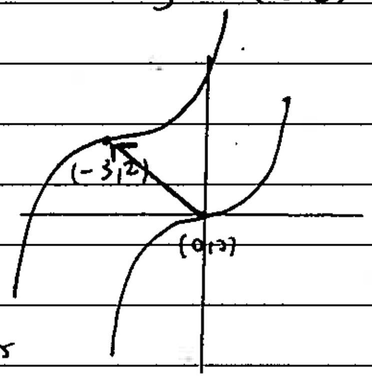
$y = x^3$

$y' - 2 = (x'+3)^3$

IGNORE DASHES. OBTAIN:

$y - 2 = (x+3)^3$

$y = (x+3)^3 + 2$



Q18) $y = 2^x \to y = 2^{-x}$

REFLECTION IN THE y-AXIS

$(x,y) \to (-x,y)$ ← RULE

$\boxed{\text{OR}}$ $\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$

$\boxed{\text{OR}}$ $T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$

Q19) $y = 2^x \to y = -2^x$

REFLECTION IN x-AXIS

$(x,y) \to (x,-y)$ ← RULE

$\boxed{\text{OR}}$ $\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$

$\boxed{\text{OR}}$ $T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$

Q20) $y = \sin x \rightarrow y = 2\sin x$

DILATION BY FACTOR 2

FROM THE x-AXIS

$(x, y) \rightarrow (x, 2y)$ ← RULE

OR $\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$

OR $T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$

Q21) $y = \sin x \rightarrow y = \sin(3x)$

DILATION BY FACTOR $\frac{1}{3}$

FROM y-AXIS

$(x, y) \rightarrow (\frac{1}{3}x, y)$ ← RULE

OR $\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} \frac{1}{3} & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$

OR $T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} 1/3 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$

Q22) $y = x^3 \rightarrow y = (x-1)^3 + 2$

TRANSLATION BY VECTOR $\begin{bmatrix} 1 \\ 2 \end{bmatrix}$

(ONE RIGHT, TWO UP)

$(x, y) \rightarrow (x+1, y+2)$ ← RULE

OR $\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} 1 \\ 2 \end{bmatrix}$

OR $T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} 1 \\ 2 \end{bmatrix}$

Q23) $y = x + \log_2 x \rightarrow y = -x + \log_2(-x)$

REFLECTION IN y-AXIS

$(x, y) \rightarrow (-x, y)$ ← RULE

OR $\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$

OR $T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$

Q24 A) $T_1 : \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$

$T_2 : \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix}$

$\begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$

$= \begin{bmatrix} 2 & 0 \\ 0 & -1 \end{bmatrix}$

B) $\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix}$

$= \begin{bmatrix} 2 & 0 \\ 0 & -1 \end{bmatrix}$

C) MATRICES ARE THE SAME

∴ ORDER DOES NOT MATTER

IN THIS INSTANCE

Q25 A) $T_1 : \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ $T_2 : \begin{bmatrix} 1 & 0 \\ 0 & 3 \end{bmatrix}$

$\begin{bmatrix} 1 & 0 \\ 0 & 3 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

$= \begin{bmatrix} 0 & 1 \\ 3 & 0 \end{bmatrix}$

B) $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 3 \end{bmatrix}$

$= \begin{bmatrix} 0 & 3 \\ 1 & 0 \end{bmatrix}$

C) FINAL MATRICES ARE DIFFERENT

∴ ORDER DOES MATTER IN THIS INSTANCE

Q26) $T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} 2 & 0 \\ 0 & -1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} -1 \\ 2 \end{bmatrix}$

$(x, y) \rightarrow (2x-1, -y+2)$

A) ① DILATION BY FACTOR 2 FROM THE y-AXIS

② REFLECTION IN x-AXIS

③ TRANSLATE 1 UNIT LEFT, 2 UNITS UP

B) METHOD #1

$y = \sin x$

$\rightarrow y = \sin\left(\frac{x}{2}\right)$

$\rightarrow y = -\sin\left(\frac{x}{2}\right)$

$\rightarrow y = -\sin\left(\frac{x+1}{2}\right) + 2.$

METHOD #2

$x' = 2x-1 \qquad y' = -y+2$

$\Rightarrow x = \frac{x'+1}{2} \qquad y = 2-y'$

$y = \sin x$ BECOMES

$2-y' = \sin\left(\frac{x'+1}{2}\right)$

IGNORE DASHES :

$2-y = \sin\left(\frac{x+1}{2}\right)$

$y = 2-\sin\left(\frac{x+1}{2}\right)$ (SAME ANSWER!)

Q27 $T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} -2 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} 1 \\ -2 \end{bmatrix}$

$(x, y) \rightarrow (-2x+1, y-2)$

A) ① DILATE BY A FACTOR OF 2 FROM THE y-AXIS

② REFLECTION IN y-AXIS

③ TRANSLATE 1 UNIT RIGHT, 2 UNITS DOWN

B) METHOD #1

$y = \sin x$

$\rightarrow y = \sin\left(\frac{x}{2}\right)$

$\rightarrow y = \sin\left(-\frac{x}{2}\right)$

$\rightarrow y = \sin\left(-\frac{(x-1)}{2}\right) - 2$

METHOD #2

$x' = -2x+1 \qquad y' = y-2$

$+2x = 1-x' \qquad y = y'+2$

$x = \frac{1-x'}{2}$

$y = \sin x$ BECOMES

$y'+2 = \sin\left(\frac{1-x'}{2}\right)$

IGNORE DASHES :

$y+2 = \sin\left(\frac{1-x}{2}\right)$

$y = \sin\left(\frac{1-x}{2}\right) - 2$ (SAME ANSWER)

#10

Q28) $y = -2\log(x+1)+3$

① DILATION FACTOR OF 2 FROM X-AXIS

② REFLECT IN X-AXIS

③ TRANSLATE 1 UNIT LEFT, 3 UNITS UP

$$T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} 1 & 0 \\ 0 & -2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} -1 \\ 3 \end{bmatrix}$$

OR $x' = x-1 \qquad y' = -2y+3$

CHECK: $x = x'+1 \qquad y = \frac{3-y'}{2}$

$y = \log_2 x$ BECOMES:

$$\frac{3-y'}{2} = \log_2(x'+1)$$

IGNORE DASHES:

$$\frac{3-y}{2} = \log_2(x+1)$$

$$3-y = 2\log_2(x+1)$$

$$y = 3-2\log_2(x+1)$$

CORRECT ANSWER

Q29) $y = 3\times 2^{-(x-3)}+1$

① DILATION FACTOR OF 3 FROM X-AXIS

② REFLECT IN Y-AXIS

③ TRANSLATE 3 UNITS RIGHT, 1 UNIT UP.

$$T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} -1 & 0 \\ 0 & 3 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} 3 \\ 1 \end{bmatrix}$$

OR $x' = -x+3 \qquad y' = 3y+1$

CHECK: $x = 3-x' \qquad y = \frac{y'-1}{3}$

$y = 2^x$ BECOMES:

$$\frac{y'-1}{3} = 2^{3-x'}$$

IGNORE DASHES:

$$\frac{y-1}{3} = 2^{3-x}$$

$$y-1 = 3\times 2^{3-x}$$

$$y = 3\times 2^{3-x}+1$$

CORRECT ANSWER.

#11

Q30) $y = -\frac{3}{2(x+2)} - 1$

① DILATE BY FACTOR $\frac{1}{2}$ FROM y-AXIS

② DILATE BY FACTOR 3 FROM x-AXIS

③ REFLECT IN x-AXIS

④ TRANSLATE BY 2 UNITS LEFT, 1 UNIT DOWN

$T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} \frac{1}{2} & 0 \\ 0 & -3 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} -2 \\ -1 \end{bmatrix}$

$\boxed{OR}$ $x' = \frac{1}{2}x - 2$ $\quad y' = -3y - 1$

$x = 2(x'+2)$ $\quad y = \frac{-y'-1}{3}$

CHECK: $y = \frac{1}{x}$ BECOMES

$\frac{-y'-1}{3} = \frac{1}{2(x'+2)}$

IGNORE THE DASHES:

$\frac{-y-1}{3} = \frac{1}{2(x+2)}$

$-y-1 = \frac{3}{2x+4}$

$-y = \frac{3}{2x+4} + 1$

$y = -\frac{3}{2x+4} - 1$

CORRECT ANSWER.

~~Q31) $y = -(x+2)^2 - 3$~~

~~① REFLECT IN x-AXIS~~

~~② TRANSLATE 2 UNITS LEFT, 3 UNITS DOWN~~

~~$T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} -2 \\ -3 \end{bmatrix}$~~

~~$\boxed{OR}$ $x' = x - 2$ $\quad y' = -y - 3$~~

~~$x = x' + 2$ $\quad y = -y' - 3$~~

~~CHECK: $y = x$~~

PTO

Q31)

$$y = -(x+2)^2 - 3$$

① REFLECT IN x-AXIS

$$y = -\left(-(x+2)^2 - 3\right)$$
$$= (x+2)^2 + 3$$

② TRANSLATE 2 UNITS RIGHT, 3 UNITS DOWN

$$y = ((x-2)+2)^2 + 3 - 3$$
$$= x^2$$

TRANSFORMATION :

$$T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} 2 \\ -3 \end{bmatrix}$$

OR

$$x' = x+2 \qquad y' = -y-3$$
$$x = x'-2 \qquad y = -y'-3$$

CHECK : $y = -(x+2)^2 - 3$ BECOMES:

$$-y' - 3 = -(x'-2+2)^2 - 3$$

IGNORE DASHES :

$$-y - 3 = -(x-2+2)^2 - 3$$
$$-y = -x^2$$
$$y = x^2$$

REQUIRED ANSWER

Q32

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} 2 \\ 1 \end{bmatrix}$$

$$X' = AX + B$$
$$X' - B = AX$$
$$A^{-1}(X' - B) = X$$
$$X = A^{-1}(X' - B)$$
$$= \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}^{-1}\left(\begin{bmatrix} x' \\ y' \end{bmatrix} - \begin{bmatrix} 2 \\ 1 \end{bmatrix}\right)$$
$$= \frac{1}{1}\begin{bmatrix} 1 & 0 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} x'-2 \\ y'-1 \end{bmatrix}$$
$$= \begin{bmatrix} 1 & 0 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} x'-2 \\ y'-1 \end{bmatrix}$$
$$= \begin{bmatrix} x'-2 \\ -(x'-2) + y' - 1 \end{bmatrix}$$
$$= \begin{bmatrix} x'-2 \\ -x' + y' + 1 \end{bmatrix}$$

$$x = x' - 2 \qquad y = -x' + y' + 1$$

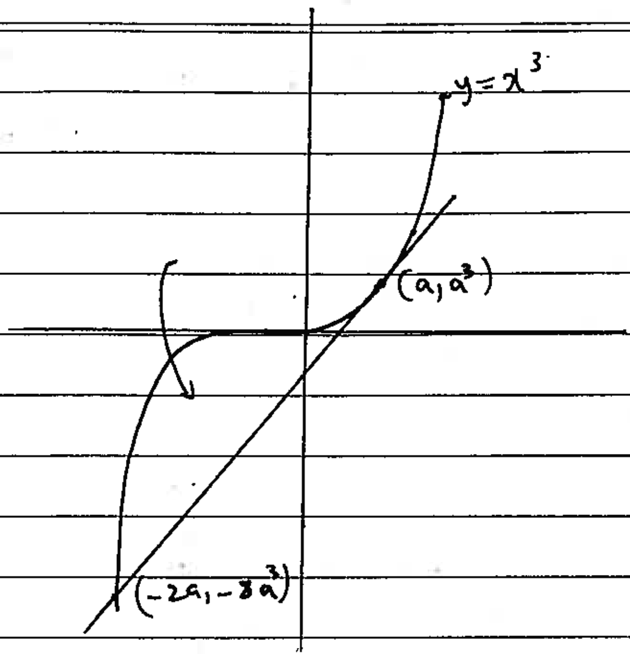
$3x + 2y = 6$ BECOMES :

$$3(x'-2) + 2(-x'+y'+1) = 6$$
$$3x' - 6 - 2x' + 2y' + 2 = 6$$

IGNORE DASHES

$$3x - 6 - 2x + 2y + 2 = 6$$
$$x + 2y - 4 = 6$$
$$x + 2y = 10$$

#13



$y = x^3$ ②

$\frac{dy}{dx} = 3x^2$

WHEN $x = a$ $\frac{dy}{dx} = 3a^2 = m$

$y - y_1 = m(x - x_1)$

$y - a^3 = 3a^2(x-a)$

$y - a^3 = 3a^2x - 3a^3$

$y = 3a^2x - 2a^3$ ①

① = ② $x^3 = 3a^2x - 2a^3$

$P(x) = x^3 - 3a^2x + 2a^3 = 0$

NOTE $P(a) = a^3 - 3a^3 + 2a^3 = 0$

$\Rightarrow P(x) = (x-a)(x^2 + ax - 2a^2)$

$-a^2x$, $-2a^2x$

$= (x-a)(x-a)(x+2a)$

$= (x-a)^2(x+2a) = 0$

$\Rightarrow x = a, -2a$

$A = \int_{-2a}^{a} x^3 - (3a^2x - 2a^3)\, dx$

$= \frac{27a^4}{4}$

SO IF $A = \frac{4}{3}$

THEN $\frac{27a^4}{4} = \frac{4}{3}$

$a^4 = \frac{16}{81}$

$a = \sqrt[4]{\frac{16}{81}}$

$= \frac{2}{3}.$