Statistics

(1) 3G, 5B

(a) $\binom{3}{2} \times \binom{5}{3}$

$= 3 \times \frac{5!}{2!\,3!}$

$= \frac{\cancel{3} \times 120}{2 \times \cancel{3}2}$ $\quad 4\overline{)120}^{30}$

$= 30$

(b) $\binom{3}{3} \times \binom{5}{2}$

$= 1 \times \frac{5!}{2!\,3!}$

$= \frac{5 \times 4 \times 3!}{2 \times 3!}$

$= 10$

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(2) 5O, 4E

(a) $\binom{4}{2} \times \binom{5}{2}$

$= \frac{4!}{2!\,2!} \times 10$

$= \frac{\cancel{4} \times 3 \times 2 \times 1}{\cancel{4}} \times 10$

$= 60$

(b) $\binom{4}{3} \times \binom{5}{1}$

$= 4 \times 5$

$= 20$

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(3) 4G, 3B $\rightarrow$ 4

(a) $\binom{4}{2}\binom{3}{2} = \frac{4!}{2!\,2!} \times 3 = 18$

Total $: \binom{7}{4} = \frac{7!}{4!\,3!} = \frac{7 \times \cancel{6} \times 5 \times \cancel{4!}}{\cancel{4!} \times \cancel{6}}$

$= 35$

Prob $= \frac{18}{35}$

(b) $\binom{4}{3}\binom{3}{1} = 4 \times 3 = 12$

$\therefore$ Prob $= \frac{12}{35}$

(c) $\binom{4}{3}\binom{3}{1} + \binom{4}{4} = 4 \times 3 + 1 = 13$

Prob $= 13/35$

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(4)

(a) H, Y, P, R, B, L $\qquad$ E, O, A

$\binom{3}{2} \times \binom{6}{2} = 3 \times \frac{6!}{2!\,4!} = 3 \times \frac{\overset{3}{\cancel{6}} \times 5}{\cancel{2}} = 45$

Total $= \binom{9}{4} = \frac{9!}{4!\,5!} = \frac{9 \times \overset{2}{\cancel{8}} \times 7 \times \cancel{6}}{\cancel{4} \times \cancel{3} \times \cancel{2}} = 126$

Prob $= \frac{45}{126} = \frac{5}{14}$

(b) $\binom{6}{4} = \frac{6!}{4!\,2!} = \frac{6 \times 5}{2} = 15$

Prob(no vowel) $= \frac{15}{126} = \frac{5}{42}$

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(5) 5R, 5B $\longrightarrow$ 5

(a) $\binom{10}{5} = 252$

no red $= \binom{5}{5} = 1$

Pr (no red) $= \frac{1}{252}$

(b) 1 red $= \binom{5}{1}\binom{5}{4} = 25$

Pr(1 red) $= \frac{25}{252}$

(c) # 2 red $= \binom{5}{2}\binom{5}{3} = 100$

Pr (2 red) $= \frac{100}{252} = \frac{25}{63}$

(d) # 3 red = 100

Pr (3 red) $= \frac{25}{63}$

(e) Pr (4 red) $= \frac{25}{252}$

(f) Pr (5 red) $= \frac{1}{252}$

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(6) 4R, 3B $\longrightarrow$ 4

(a) $Pr(X=2) = {}^nC_x\, p^x (1-p)^{n-x}$

$= \binom{4}{2} \left(\frac{4}{7}\right)^2 \left(\frac{3}{7}\right)^2$

$= \frac{864}{2401}$

(b) $\frac{\binom{4}{2}\binom{3}{2}}{\binom{7}{4}} = \frac{18}{35}$

(7) $\boxed{\begin{matrix}5R\\4B\end{matrix}} \rightarrow 5$ (3 red)

(a) $X = \#$ red out of 5

$X \sim Bi(n=5, p=\frac{5}{9})$

$$Pr(X=3) = \binom{n}{x} p^x (1-p)^{n-x}$$

$$= \binom{5}{3}\left(\frac{5}{9}\right)^3\left(\frac{4}{9}\right)^2$$

$$= \frac{20000}{59049}$$

$$\simeq 0.3387$$

(b) $$\frac{\binom{5}{3}\binom{4}{2}}{\binom{9}{5}} = \frac{60}{126} = \frac{10}{21}$$

$$\simeq 0.476$$

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(8) 120 O 80 A $\longrightarrow$ 4 20

$\frac{120}{200}$

(a) $X = \#$ oranges out of 4

$X \sim Bi(n=4, p=0.6)$

$$Pr(X=2) = \binom{4}{2} 0.6^2 \times 0.4^2$$

$$\simeq 0.346$$

(b) $$Pr(X=2) = \frac{\binom{120}{2}\binom{80}{2}}{\binom{200}{4}} \simeq 0.349$$

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(9) 45B, 55G $\longrightarrow$ 5 3B

(a) $X = \#$ boys out of 5

$X \sim Bi(n=5, p=0.45)$

$$Pr(X=3) = \binom{5}{3} 0.45^3 \times 0.55^2$$

$$\simeq 0.276$$

(b) $$Pr(X=3) = \frac{\binom{45}{3}\binom{55}{2}}{\binom{100}{5}} \simeq 0.280$$

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(10) (a) $p = \frac{6}{16} = \frac{3}{8} = 0.375$

(b) (i) $\hat{p} = \frac{4}{5} = 0.8$

(ii) $\hat{p} = \frac{3}{5} = 0.6$

(iii) $\hat{p} = \frac{2}{5} = 0.4$

(iv) $\hat{p} = \frac{4}{5} = 0.8$

(v) $\hat{p} = \frac{3}{5} = 0.6$

(vi) $\hat{p} = \frac{2}{5} = 0.4$

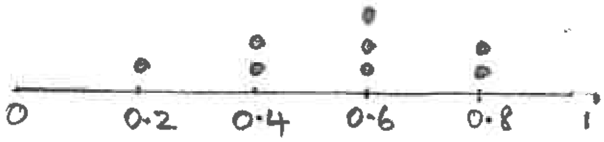
(vii) $\hat{p} = \frac{1}{5} = 0.2$

(viii) $\hat{p} = \frac{3}{5} = 0.6$

(ix) $\hat{p} = \frac{3}{5} = 0.6$

(x) $\hat{p} = \frac{2}{5} = 0.4$

(c)



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(11) $\boxed{5R, 6B} \longrightarrow 5$

$\hat{p}$ = sample prop of red

(a) $$Pr(\hat{p}=0) = \frac{\binom{5}{0}\binom{6}{5}}{\binom{11}{5}} = \frac{1}{77}$$

$$Pr(\hat{p}=\tfrac{1}{5}) = \frac{\binom{5}{1}\binom{6}{4}}{\binom{11}{5}} = \frac{25}{154}$$

$$Pr(\hat{p}=\tfrac{2}{5}) = \frac{\binom{5}{2}\binom{6}{3}}{\binom{11}{5}} = \frac{100}{231}$$

$$Pr(\hat{p}=\tfrac{3}{5}) = \frac{\binom{5}{3}\binom{6}{2}}{\binom{11}{5}} = \frac{25}{77}$$

$$Pr(\hat{p}=\tfrac{4}{5}) = \frac{\binom{5}{4}\binom{6}{1}}{\binom{11}{5}} = \frac{5}{77}$$

$$Pr(\hat{p}=1) = \frac{\binom{5}{5}\binom{6}{0}}{\binom{11}{5}} = \frac{1}{462}$$

$\hat{p}$	0	$\frac{1}{5}$	$\frac{2}{5}$	$\frac{3}{5}$	$\frac{4}{5}$	1
$Pr(\hat{p})$	$\frac{1}{77}$	$\frac{25}{154}$	$\frac{100}{231}$	$\frac{25}{77}$	$\frac{5}{77}$	$\frac{1}{462}$

(b) (i) $$Pr(\hat{p} > 0.5) = \frac{25}{77} + \frac{5}{77} + \frac{1}{462}$$

$$= \frac{181}{462}$$

(ii) $$Pr(\hat{p} < 0.9 \mid \hat{p} > 0.5)$$

$$= \frac{Pr(0.5 < \hat{p} < 0.9)}{Pr(\hat{p} > 0.5)}$$

$$= \frac{30}{77} \div \frac{181}{462}$$

$$= \frac{180}{181}$$

(c) $$E(\hat{p}) = \frac{1}{5} \times \frac{25}{154} + \dots + 1 \times \frac{1}{462}$$

$$= \frac{5}{11}$$

(12) 4B, 6G $\to$ 5

$\hat{p}$ = sample prop boys

(a) $Pr(\hat{p}=0) = \dfrac{\binom{4}{0}\binom{6}{5}}{\binom{10}{5}} = \dfrac{1}{42}$

$Pr(\hat{p}=\frac{1}{5}) = \dfrac{\binom{4}{1}\binom{6}{4}}{\binom{10}{5}} = \dfrac{5}{21}$

$Pr(\hat{p}=\frac{2}{5}) = \dfrac{\binom{4}{2}\binom{6}{3}}{\binom{10}{5}} = \dfrac{10}{21}$

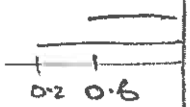
$Pr(\hat{p}=\frac{3}{5}) = \dfrac{\binom{4}{3}\binom{6}{2}}{\binom{10}{5}} = \dfrac{5}{21}$

$Pr(\hat{p}=\frac{4}{5}) = \dfrac{\binom{4}{4}\binom{6}{1}}{\binom{10}{5}} = \dfrac{1}{42}$

$\hat{p}$	0	$\frac{1}{5}$	$\frac{2}{5}$	$\frac{3}{5}$	$\frac{4}{5}$
$Pr(\hat{p})$	$\frac{1}{42}$	$\frac{5}{21}$	$\frac{10}{21}$	$\frac{5}{21}$	$\frac{1}{42}$

(b)(i) $Pr(\hat{p}>0.5) = \frac{5}{21} + \frac{1}{42} = \frac{11}{42}$

(ii) $Pr(\hat{p}>0.6 \mid \hat{p}>0.2)$



$= \dfrac{Pr(\hat{p}>0.6)}{Pr(\hat{p}>0.2)}$

$= \dfrac{\frac{1}{42}}{\frac{31}{42}}$

$= \dfrac{1}{31}$

(c) $E(\hat{p}) = \frac{1}{5}\times\frac{5}{21} + \frac{2}{5}\times\frac{10}{21} + \dots + \frac{4}{5}\times\frac{1}{42}$

$= \frac{2}{5}$

If this experiment were repeated over and over again the long-run average of $\hat{p}$ would approach $\frac{2}{5}$. This is the population proportion of boys in group.

(13)(a) X = # in sample with blue eyes

$X \sim Bi(n=4, p=0.2)$

$Pr(X=x) = \binom{4}{x} 0.2^x \times 0.8^{4-x}$

$\hat{P} = \dfrac{X}{n} = \dfrac{X}{4}$

$Pr\left(\hat{P}=\dfrac{x}{4}\right) = \binom{4}{x} 0.2^x \times 0.8^{4-x}$

$\hat{p}$	0	0.25	0.5	0.75	1
$Pr(\hat{p})$	0.4096	0.4096	0.1536	0.0256	0.001

(b)(i) $Pr(\hat{P}\geqslant 0.75) = 0.0272$

(ii) $Pr(\hat{p}\geqslant 0.25 \mid \hat{p}\leqslant 0.75)$

$= \dfrac{Pr(0.25\leqslant \hat{p}\leqslant 0.75)}{Pr(\hat{p}\leqslant 0.75)}$

$= \dfrac{0.5888}{1-0.0016}$

$= 0.5897$

(c) $E(\hat{p}) = 0.25\times 0.4096 + \dots + 1\times 0.0016$

$= 0.2$

It is the population proportion

(d) $E(\hat{p}) = E\left(\dfrac{X}{n}\right) = \dfrac{1}{n}E(X) = \dfrac{1}{n}\times np =$

$= 0.2$

(14)

(a) The population can be thought of as an infinite set of numbers only containing the numbers 1, 2, 3, 4, 5, 6 all of which occur in equal proportio[n]. So the underlying population proportion is $p=\frac{1}{6}$

(b) $X \sim Bi(n=5, p=\frac{1}{6})$ $\hat{p} = \dfrac{X}{5}$

$\hat{p}$	0	$\frac{1}{5}$	$\frac{2}{5}$	$\frac{3}{5}$	$\frac{4}{5}$	1
$Pr(\hat{p})$	0.4019	0.4019	0.1608	0.0322	0.00322	0.0001286

(c)(i) $Pr(\hat{p}<0.5) = 0.9645$

(c)(ii) $Pr(\hat{p} > 0 \mid \hat{p} < 0.5)$

$$= \frac{Pr(0 < \hat{p} < 0.5)}{Pr(\hat{p} < 0.5)}$$

$$= \frac{0.5626}{0.9645}$$

$$= 0.5833$$

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(15) $\hat{p} \sim N\left(\mu = p, \sigma = \sqrt{\frac{p(1-p)}{n}}\right)$

$p = 0.42$

$\therefore \mu = 0.42$

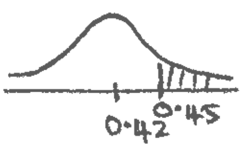
$$\sigma = \sqrt{\frac{0.42 \times 0.58}{1000}}$$

$$\simeq 0.0156$$

$$\hat{p} \sim N(\mu = 0.42, \sigma = 0.0156)$$

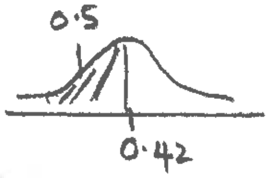
(i) $Pr(\hat{p} > 0.45)$

$\simeq 0.0273$



(ii) $Pr(\hat{p} < 0.42)$

$= 0.5$



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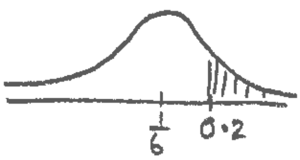
(16) $p = \frac{1}{6} = \mu$

$$\sigma = \sqrt{\frac{\frac{1}{6} \times \frac{5}{6}}{50}} \simeq \frac{\sqrt{10}}{60}$$

$$\hat{p} \sim N\left(\mu = \frac{1}{6}, \sigma = \frac{\sqrt{10}}{60}\right)$$

$Pr(\hat{p} > 0.2)$

$\simeq 0.2635$



$X \sim Bi\left(n = 50, p = \frac{1}{6}\right)$

$Pr(\hat{p} > 0.2) = Pr(X \geqslant 11)$

$\simeq 0.2014$

Results significantly different

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(17)

(i) $n = 10$, $p = 0.5$, $\sigma = \sqrt{\frac{0.5^2}{10}} = \sqrt{\frac{0.25}{10}}$

$\simeq 0.15811$

$$\hat{p} \sim N(\mu = 0.5, \sigma = 0.15811)$$

$Pr(\hat{p} > 0.55) \simeq 0.3759$

(ii) $n = 100$, $p = 0.5$, $\sigma = 0.05$

$$\hat{p} \sim N(\mu = 0.5, \sigma = 0.05)$$

$Pr(\hat{p} > 0.55) = 0.1587$

(iii) $n = 1000$, $p = 0.5$, $\sigma = 0.0158$

$Pr(\hat{p} > 0.55) = 0.000783$

(iv) $n = 10000$, $p = 0.5$, $\sigma = 0.005$

$$\hat{p} \sim N(\mu = 0.5, \sigma = 0.005)$$

$Pr(\hat{p} > 0.55) \simeq 7.77 \times 10^{-24}$

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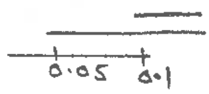
(18)(a) $n = 20$, $p = 0.05$

$\mu = 0.05$

$$\sigma = \sqrt{\frac{0.05 \times 0.95}{20}} \simeq 0.0487$$

$$\hat{p} \sim N(\mu = 0.05, \sigma = 0.0487)$$

$Pr(\hat{p} > 0.1) \simeq 0.1525$



(b) $Pr(\hat{p} > 0.1 \mid \hat{p} > 0.05)$

$$= \frac{Pr(\hat{p} > 0.1)}{Pr(\hat{p} > 0.05)}$$

$$= \frac{0.1525}{0.5}$$

$$\simeq 0.3049$$

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(19) $\hat{p} = \frac{210}{500}$

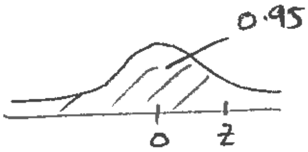
$\simeq 0.42$

(20)

(a) $\hat{p} = \frac{380}{800} \simeq 0.475$

$$\left(\hat{p} - z\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}, \hat{p} + z\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}\right)$$

$$P_r(Z < z) = \frac{\frac{90}{100}+1}{2} = 0.95$$



$z = 1.64485$

$$\sqrt{\frac{0.475\times(1-0.475)}{800}} \simeq 0.0177$$

90% CI

$$(0.475 - 0.029, 0.475 + 0.029)$$

$$\simeq (0.446, 0.504)$$

(b) $P_r(Z<z) = \frac{\frac{95}{100}+1}{2} = 0.975$

$z = 1.95996$

95% CI

$$(.475 - 0.0346, .475 + 0.0346)$$

$$= (0.4404, 0.5096)$$

(c) $P_r(Z<z) = \frac{\frac{99}{100}+1}{2} = 0.995$

$z = 2.5758$

99% CI

$$(.475 - 0.04548, 0.475 + 0.04548)$$

$$= (0.4295, 0.5205)$$

(d) To increase the confidence we must widen the interval.

(21) (a) $\hat{p} = \frac{36}{100} = 0.36$

(b) $\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$

$$= \sqrt{\frac{0.36\times0.64}{100}}$$

$$\simeq 0.048$$

90% CI:

$P_r(Z<z) = \frac{1.9}{2} = 0.95$

$z = 1.64485$

90% CI is

$$(0.36 - 0.0789, 0.36 + 0.0789)$$

$$= (0.281, 0.439)$$

95% CI

$P_r(Z<z) = \frac{1.95}{2} = 0.975$

$z = 1.96$

95% CI is

$$(0.36 - 0.094, 0.36 + 0.094)$$

$$= (0.266, 0.454)$$