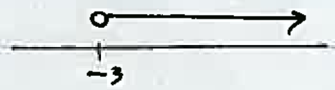
FUNCTIONS AND RELATIONS SOLUTIONS

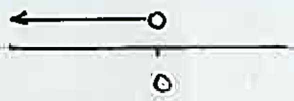
Q1.
A) F D) F G) T
B) T E) F H) F
C) T F) T I) F

Q2.
A) $(-1, 4]$

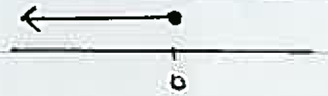
B) $(-3, \infty)$



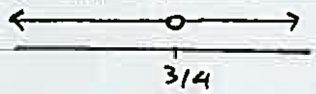
C) $(-\infty, 0)$



D) $(-\infty, 0]$



E) $(-\infty, 3/4) \cup (3/4, \infty)$



F) $(-3, -2]$

Q3.
A) DOM $= \mathbb{R}$
RAN $= \mathbb{R}$

FUNC. ? YES
1-1 ? NO

B) DOM $= [-2, \infty)$
RAN $= [-1, 2] \cup \{4\}$

FUNC ? YES
1-1 ? NO

C) DOM $= \mathbb{R}$
RAN $= \mathbb{R}$

FUNC ? NO
1-1 ? NO

D) DOM $= \mathbb{R}$
RAN $= \{-3\}$

FUNC ? YES
1-1 ? NO

E) DOM $= [-2, 1] \cup (3, \infty)$
RAN $= [-1, \infty)$

FUNC ? YES
1-1 ? YES

F) DOM $=$ RAN $= [-1, 1]$

FUNC ? NO
1-1 ? NO

Q4. a) $f : \mathbb{R} \setminus \{3\} \to \mathbb{R}$,
$f(x) = \frac{1}{x-3}$

b) $f : \mathbb{R}^+ \to \mathbb{R}$,
$f(x) = -3$

Q5. a) REQUIRE $x - 1 \geq 0$

$x \geq 1$

DOM $= [1, \infty)$

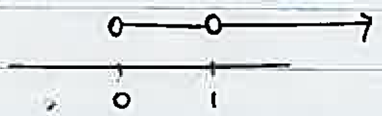
b) $y = \frac{3}{\log_2 x}$

IF $\log_2 x = 0$

$x = 2^0 = 1$

ALSO $\log_2 x$ DEFINED ONLY FOR $x > 0$

$\Rightarrow$ DOM $= \mathbb{R}^+ \setminus \{1\}$



c) IF $x(x+1) = 0$

$\Rightarrow x = 0, -1$

$\Rightarrow$ DOM $= \mathbb{R} \setminus \{0, -1\}$

d) REQUIRE $2x - 4 > 0$

$\Rightarrow x > 2$

DOM $=$ ~~#~~ $(2, \infty)$

e) REQUIRE :

$x + 3 \geq 0$ AND $3 - x \geq 0$

$x \geq -3$ AND $x \leq 3$

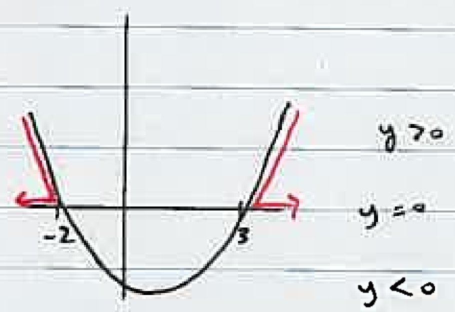
$\Longrightarrow -3 \leq x \leq 3$

DOM $= [-3, 3]$

f) REQUIRE :

$y = x^2 - x - 6 \geq 0$

I.E. $y = (x-3)(x+2) \geq 0$

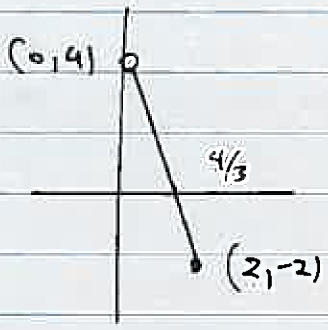


$x \geq 3$ OR $x \leq -2$

I.E. DOM $= (-\infty, -2] \cup [3, \infty)$

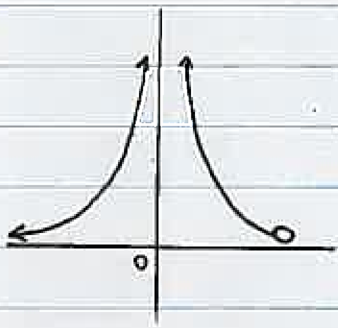
Q6. $f(x) = 4 - 3x \quad 0 < x \leq 2$

A)



DOM $= [0, 2]$ RAN $= [-2, 4)$

B)



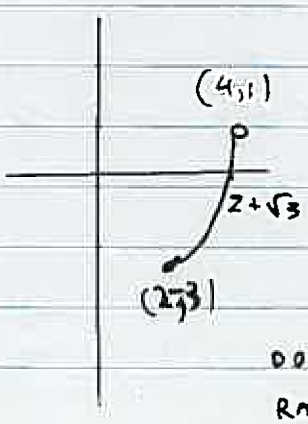
DOM $= (-\infty, 2) \setminus \{0\}$

RAN $= (0, \infty) = \mathbb{R}^+$

C) $f(x) = x^2 - 4x + 1$

$= x^2 - 4x + 4 - 4 + 1$

$= (x-2)^2 - 3$

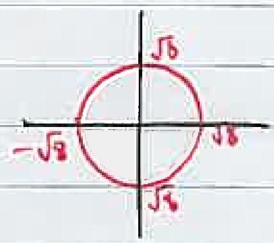


$f(4) = 1$

DOM $= [2, 4)$

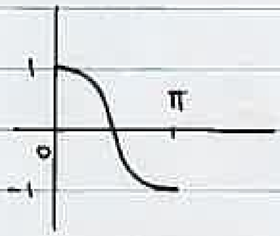
RAN $= [-3, 1)$

D) $x^2 + y^2 = (\sqrt{8})^2$



DOM = RAN $= [-\sqrt{8}, \sqrt{8}]$

E)



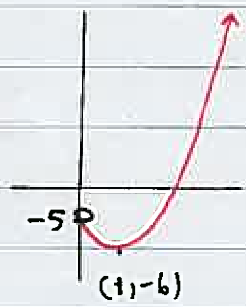
DOM $= [0, \pi]$

RAN $= [-1, 1]$

F) $h(x) = x^2 - 2x - 5$

$= x^2 - 2x + 1 - 1 - 5$

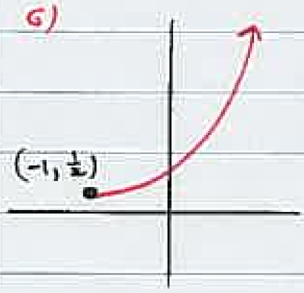
$= (x-1)^2 - 6$



DOM $= \mathbb{R}^+$

RAN $= [-6, \infty)$

G)

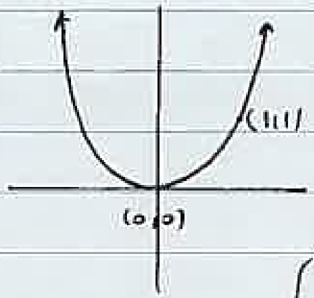


$f(-1) = 2^{-1} = \frac{1}{2}$

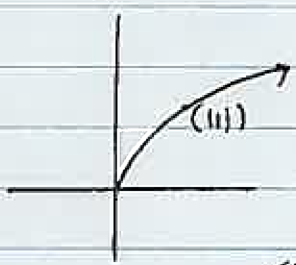
DOM $= [-1, \infty)$

RAN $= [\frac{1}{2}, \infty)$

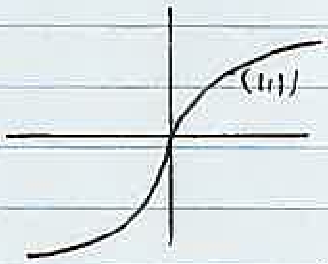
Q7. A) $y = x^{4/3} = \sqrt[3]{x^4}$ EVEN



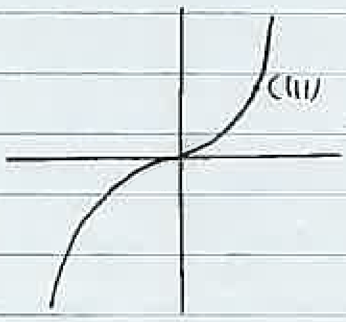
B) $y = x^{3/4} = \sqrt[4]{x^3}$ ONE BRANCH



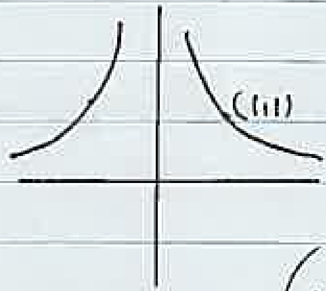
C) $y = x^{\frac{3}{5}} = \sqrt[5]{x^3}$ ODD ODD



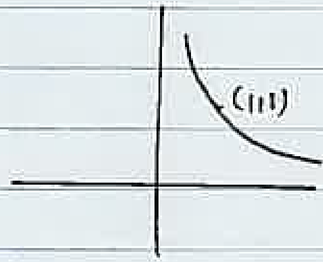
D) $y = x^{5/3} = \sqrt[3]{x^5}$ ODD ODD



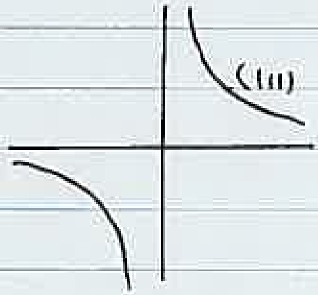
E) $y = x^{-2/3} = \frac{1}{\sqrt[3]{x^2}}$ EVEN



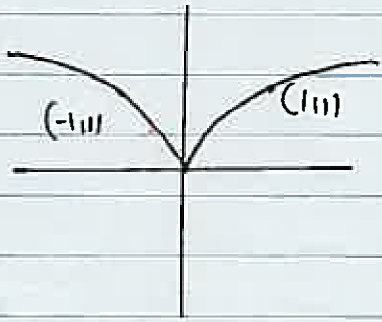
F) $y = x^{-3/2} = \frac{1}{\sqrt[2]{x^3}}$ EVEN ONE BRANCH



G) $y = x^{-5/7} = \frac{1}{\sqrt[7]{x^5}}$ ODD ← ODD



H) $y = x^{6/7} = \sqrt[7]{x^6}$ EVEN



Q8) $n \in \mathbb{N}$ $2n$ IS EVEN

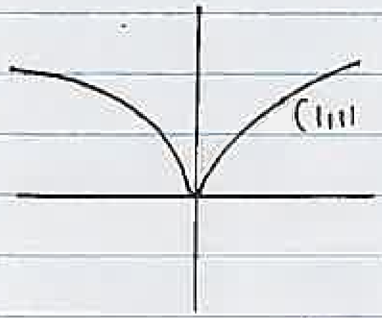
$1+2n$ IS ODD

NS $= \frac{2n}{2n+1} < 1$

$y = x^{\frac{2n}{2n+1}} = \sqrt[2n+1]{x^{2n}}$

ODD

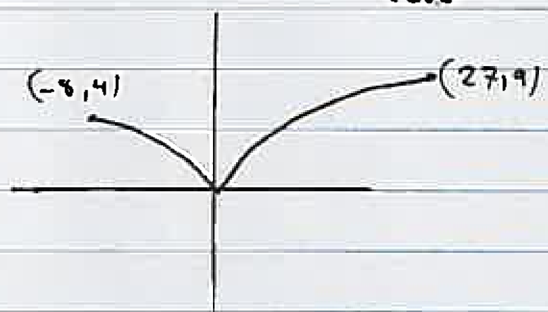
EVEN



Q9) $f(x) = x^{2/3} = \sqrt[3]{x^2}$

ODD

EVEN



$f(-8) = \cancel{\sqrt[3]{(-8)}^2}\ (-8)^{2/3} = (\sqrt[3]{-8})^2 = 4$

$f(27) = (27)^{2/3} = (\sqrt[3]{27})^2 = 9$

Q10) $y = x^3 + 3$

$y = \frac{1}{(x+2)^2}$

$y = 1 + \sqrt{x-2}$

$y = 2(x+2)^2 = \sin(x+2)$

$y = -\frac{1}{\cos x}$

$y = 2^{-x}$

$y = \sqrt{\frac{x}{4} - 4}$

$y = 3\sqrt{3-x}$

$y = \log_2(-x) - 3(-x) = \log_2(-x) + 3x$

$y = -(x^2 + \sin x) = -x^2 - \sin x$

Q11) $y = \log_{10} x$

$\to y = \log_{10}(x-2)$

$\to y = \log_{10}(x-2) + 1$

$\to y = 2(\log_{10}(x-2) + 1)$

Q12) $y = f(x)$

$\to y = f(x) - 2$

$\to y = -(f(x) - 2)$

$\to y = -(f(\frac{x}{3}) - 2)$

$\to y = -(f(\frac{x+1}{3}) - 2)$

$\to y = -(f(\frac{-x+1}{3}) - 2)$

Q13) $y = 3x^2 - 2x$

$\to y = 3(x-2)^2 - 2(x-2)$

$\to y = 3(\frac{x}{2} - 2)^2 - 2(\frac{x}{2} - 2)$

$\to y = -(3(\frac{x}{2} - 2)^2 - 2(\frac{x}{2} - 2))$

$\to y = -(3(\frac{x}{2} - 2)^2 - 2(\frac{x}{2} - 2)) + 5$

CAN BE SIMPLIFIED

Q14) TRANSLATE TO ORIGIN FIRST

TRANSLATE 1 UP

TRANSLATE 1 LEFT

REFLECT IN x-AXIS

DILATE BY FACTOR 2 FROM y-AXIS

TRANSLATE 4 RIGHT

TRANSLATE 1 UP.

Q15) B

Q16) D

Q17) E

Q18) NOTE:
$y = \sqrt{2(x - 3/2)} + 1$ D

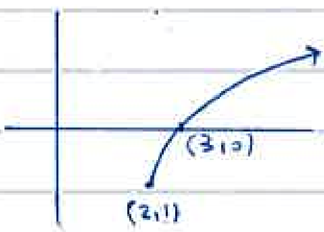
Q19) NOTE:
$y = \sqrt{-(x-3)} + 1$ C

Q20) D

Q21) NOTE: D
$y = -\dfrac{1}{2(x + 3/2)} + 4$

Q22) B (THIS ONE IS HARDER...)

Q23) A) $y = \sqrt{x-2} - 1$



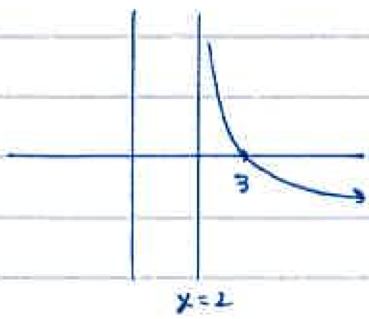
$0 = \sqrt{x-2} - 1$

$\sqrt{x-2} = 1$

$x - 2 = 1$

$x = 3$

B) $y = -\log_2(x-2)$



$0 = -\log_2(x-2)$

$0 = \log_2(x-2)$

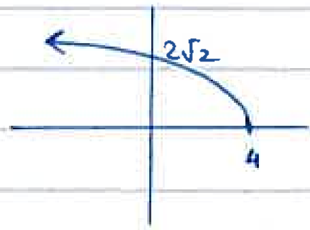
$x - 2 = 2^0$

$x - 2 = 1$

$x = 3$

C) $y = \sqrt{8-2x}$
$= \sqrt{-2(x-4)}$

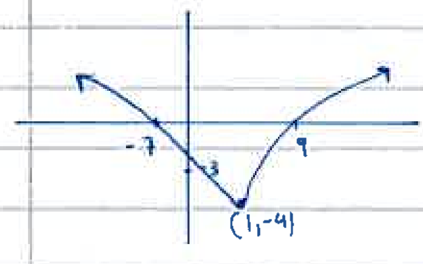
WHEN $x=0$
$y = \sqrt{8} = 2\sqrt{2}$



D) $f(x) = (1-x)^{2/3} - 4$
$= \sqrt[3]{(1-x)^2} - 4$
$= \sqrt[3]{(-(x-1))^2} - 4$

$f(0) = 1 - 4$
$= -3$

NOTE $\frac{2}{3} < 1$



WHEN $y = 0$

$0 = (1-x)^{2/3} - 4$

$\sqrt[3]{(1-x)^2} = 4$

$(1-x)^2 = 4^3 = 64$

$1 - x = \pm\sqrt{64} = \pm 8$

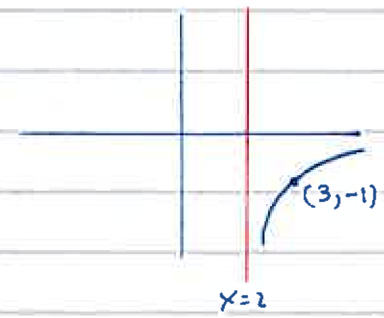
$x = 1 \pm 8 = -7, 9$

REFLECTION

EVEN (ONE BRANCH)

E) $f(x) = -\dfrac{1}{\sqrt[4]{(x-2)^3}}$

$f(3) = -\dfrac{1}{\sqrt{(3-2)^3}}$
$= -1$

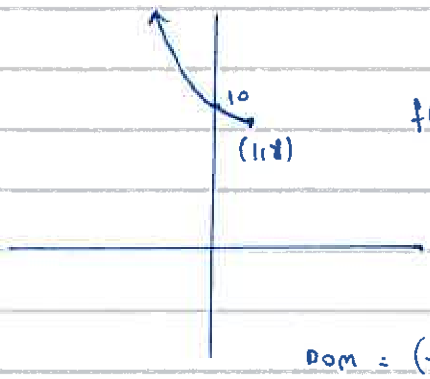


Q24 A) $f(x) = 2(1-x)^{5/4} + 8$ $\quad 5/4 > 1$

$= 2\left(-(x-1)\right)^{5/4} + 8$

$= 2\sqrt[4]{(-(x-1))^5} + 8$

EVEN (ONE BRANCH)



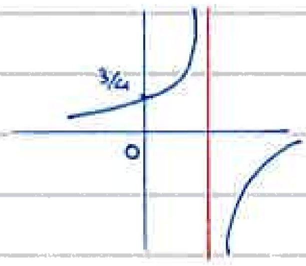
$f(0) = 2(1)^{5/4} + 8$

$= 10$

DOM : $(-\infty, 1]$

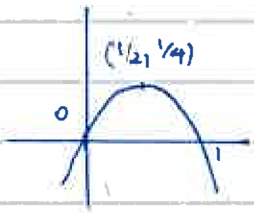
RAN : $[8, \infty)$

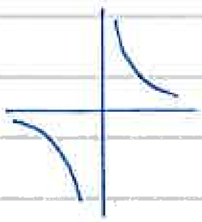
B) $g(x) = \frac{3}{-2(x-2)}$ $\qquad g(0) = \frac{3}{4}$



DOM $= \mathbb{R} \setminus \{2\}$ $\quad$ RAN $= \mathbb{R} \setminus \{0\}$

Q25) $f(x) = x(1-x)$ $\qquad g(x) = \frac{1}{x}$





DOM $= \mathbb{R}$

RAN $= (-\infty, \frac{1}{4}]$ $\qquad$ DOM = RAN $= \mathbb{R} \setminus \{0\}$

A) $\operatorname{ran}(g) = \mathbb{R} \setminus \{0\} \subseteq \mathbb{R} = \operatorname{Dom}(f)$ ✓

$\Rightarrow$ $f \circ g$ IS DEFINED

B) ~~$\operatorname{ran}(f) = (-\infty, \frac{1}{4}] \not\subseteq \operatorname{dom}(g)$~~

$\operatorname{ran}(f) = (-\infty, \frac{1}{4}] \not\subseteq \mathbb{R} \setminus \{0\} = \operatorname{dom}(g)$

$\Rightarrow$ $g \circ f$ IS NOT DEFINED

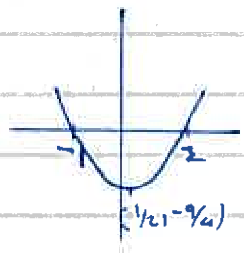
C) $f \circ g(x) = f(g(x))$

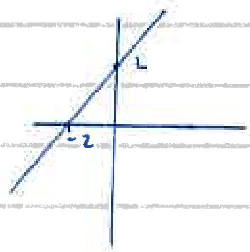
$= f\left(\frac{1}{x}\right)$

$= \frac{1}{x}\left(1 - \frac{1}{x}\right)$

$\operatorname{Dom}(f \circ g) = \operatorname{Dom}(g) = \mathbb{R} \setminus \{0\}$

Q26) $f(x) = (x-2)(x+1)$ $\qquad g(x) = x+2$





Dom $= \mathbb{R}$ $\qquad$ Dom $= \mathbb{R}$

ran $= [-9/4, \infty)$ $\qquad$ RAN $= \mathbb{R}$

A) $\operatorname{ran}(g) = \mathbb{R} \subseteq \operatorname{Dom}(f)$

$\Rightarrow$ $f \circ g$ IS DEFINED ✓

$f \circ g(x) = f(g(x))$

$= f(x+2)$

$= (x+2)^2 - (x+2) - 2$

$= x^2 + 2x + \cancel{4} - x \cancel{-2} \cancel{-2}$

$= x^2 + x$

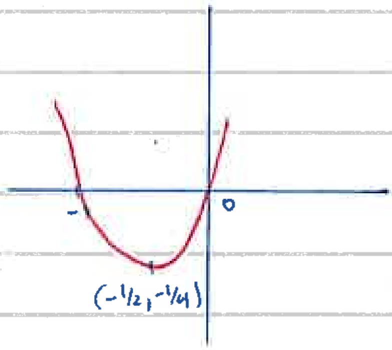
B) OVER PAGE

b) $\operatorname{ran}(f) = [-9/4, \infty) \subseteq \mathbb{R} = \operatorname{dom}(g)$

$\Rightarrow$ $g \circ f$ is DEFINED

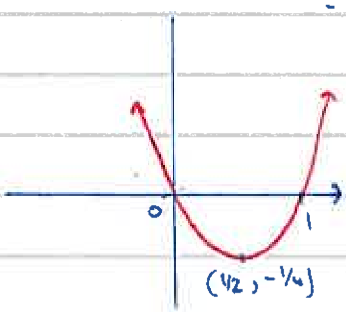
$$g \circ f(x) = g(f(x))$$
$$= g(x^2 - x - 2)$$
$$= x^2 - x - 2 + 2$$
$$= x^2 - x$$

c) $y = f(g(x)) = x^2 + x = x(x+1)$



$\operatorname{DOM}(f \circ g) = \mathbb{R}$ $\quad \operatorname{ran}(f \circ g) = [-1/4, \infty)$

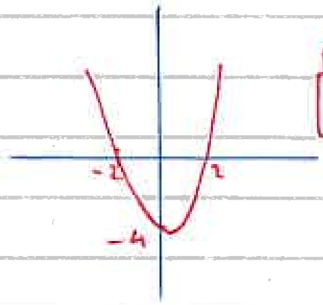
D) CONSIDER $y = (g \circ f)(x) = x^2 - x$
$$= x(x-1)$$



$\operatorname{RAN}(g \circ f) = [-1/4, \infty)$

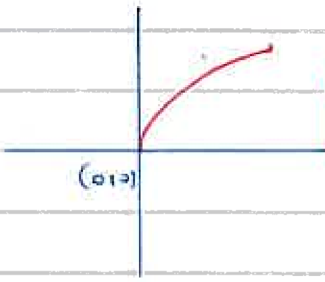
Q27) $g : \mathbb{R} \to \mathbb{R}$ $\quad g(x) = x^2 - 4$

A)



$\begin{cases} \operatorname{DOM} = \mathbb{R} \\ \operatorname{RAN} = [-4, \infty) \end{cases}$

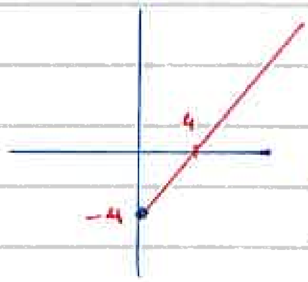
$h : \mathbb{R}^+ \cup \{0\} \to \mathbb{R}$ $\quad h(x) = \sqrt{x}$



$\begin{cases} \operatorname{DOM} = [0, \infty) \\ \operatorname{RAN} = [0, \infty) \end{cases}$

B) $(g \circ h)(x) = g(h(x))$
$$= g(\sqrt{x})$$
$$= (\sqrt{x})^2 - 4$$
$$= x - 4 \qquad (x \geq 0)$$

(C)

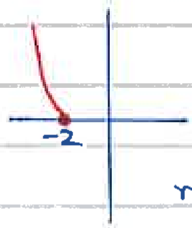


RAN $g \circ h$
$= [-4, \infty)$

(D) $\operatorname{ran}(g) = [-4, \infty) \not\subseteq [0, \infty) = \operatorname{DOM}(h)$

$\Rightarrow$ $h \circ g$ DOES NOT EXIST

(E) IF $k = -2$ $\quad g^* : (-\infty, -2) \to \mathbb{R}$



$g^*(x) = x^2 - 4$

$\operatorname{DOM} g^* = (-\infty, -2)$

$\operatorname{RAN} g^* = [0, \infty)$

$\operatorname{ran}(g^*) = [0, \infty) = \operatorname{DOM}(h)$

Q28) $Dom(f) = \{1,2,3,4\}$

$Ran(f) = \{2,5,7\}$

a) $Dom(g) = \{2,3,4,5,7\}$

$Ran(g) = \{-1,-2,-3,-4,-5\}$

b) f not 1-1 ✗

g is 1-1 ✓

c) $g(f(1)) = g(2) = -1$

$g(f(2)) = g(5) = -4$

$g(f(3)) = g(7) = -5$

$g(f(4)) = g(7) = -5$

$\therefore g \circ f = \{(1,-1),(2,-4),(3,-5),(4,-5)\}$

$ran(g \circ f) = \{-1,-4,-5\}$

d) $ran(g) = \{-1,-2,-3,-4,-5\}$

$\nsubseteq \{2,5,7\}$

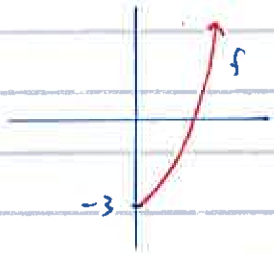
$= dom(f)$

$\therefore f \circ g$ is not defined

Q29) $f(x) = x^{5/4} - 3$ $\quad 5/4 > 1$

$= \sqrt[4]{x^5} - 3$

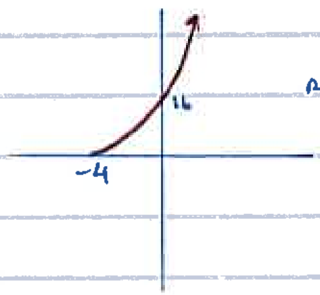
EVEN $\Rightarrow$ ONE BRANCH



$Dom(f) = [0,\infty)$

$Ran(f) = [-3,\infty)$

$g(x) = (x+4)^2 \quad x \geq -4$



$Dom(g) = [-4,\infty)$

$Ran(g) = [0,\infty)$

A) $ran(g) = [0,\infty) \subseteq [0,\infty) = Dom(f)$

$\therefore f \circ g$ is defined

$ran(f) = [-3,\infty) \subseteq [-4,\infty) = Dom(g)$

$\therefore g \circ f$ is defined.

RULES:

$f(g(x)) = f((x+4)^2)$

$= ((x+4)^2)^{5/4} - 3$

$= (x+4)^{5/2} - 3 \quad (x \geq -4)$

$g(f(x)) = g(x^{5/4} - 3)$

$= (x^{5/4} - 3 + 4)^2$

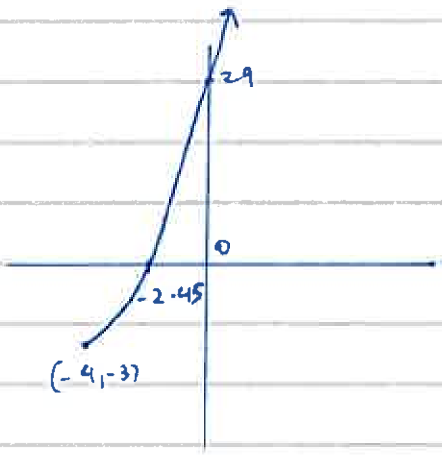
$= (x^{5/4} + 1)^2 \quad (x \geq 0)$

B) OVER PAGE

(b) $y = f(g(x)) = (x+4)^{5/2} - 3$ $\quad x \geq -4$

$= \sqrt[2]{(x+4)^5} - 3$

EVEN $\Rightarrow$ ONE BRANCH $\quad 5/2 > 1$



WHEN $x = 0$ $\quad f(g(0)) = 4^{5/2} - 3$

$= 2^5 - 3$

$= 32 - 3$

$= 29$

WHEN $y = 0$ $\quad 0 = (x+4)^{5/2} - 3$

$3 = (x+4)^{5/2}$

$9 = (x+4)^5$

$x + 4 = \sqrt[5]{9}$

$x = -4 + \sqrt[5]{9}$

≈ -2.45

NOTE: $\operatorname{ran}(f \circ g) = [-3, \infty)$

Q30) A)

$g(h(-1))$

$= g(-\frac{1}{1})$

$= g(-1)$

$= 4 - (-1)^2$

$= 4 - 1$

$= 3$

$g(h(f(5)))$

$= g(h(\sqrt{5-1}))$

$= g(h(2))$

$= g(\frac{1}{2})$

$= 4 - (\frac{1}{2})^2$

$= 4 - \frac{1}{4}$

$= \frac{15}{4}$

B) (i) $g(h(x)) = -5$

LET $h(x) = y$

$g(y) = -5$

$4 - y^2 = -5$

$-y^2 = -9$

$y^2 = 9$

$y = \pm 3$

IF $h(x) = 3$

$\frac{1}{x} = 3$

$x = \frac{1}{3}$

$h(x) = -3$

$h(x) = -3$

$\frac{1}{x} = -3$

$x = -\frac{1}{3}$

$x = \pm \frac{1}{3}$

(ii) OVER PAGE

(ii) $g(h(f(x))) = 3$

LET $h(f(x)) = y$

$g(y) = 3$

$4 - y^2 = 3$

$y^2 = 1$

$y = \pm 1$

IF $h(f(x)) = 1$

LET $f(x) = z$

$h(z) = 1$

$\frac{1}{z} = 1$

$z = 1$

$\Rightarrow f(x) = 1$

$\sqrt{x-1} = 1$

$x - 1 = 1$

$x = 2$

$h(f(x)) = -1$

LET $f(x) = z$

$h(z) = -1$

$\frac{1}{z} = -1$

$z = -1$

$f(x) = -1$

$\sqrt{x-1} = -1$

NOT POSSIBLE

AS $\sqrt{x-1} \geq 0$

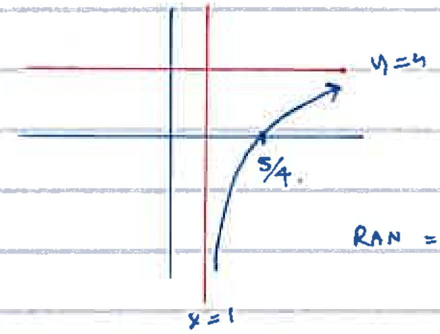
(E) $y = g(h(f(x)))$

$= g(h(\sqrt{x-1}))$

$= g\left(\frac{1}{\sqrt{x-1}}\right)$

$= 4 - \left(\frac{1}{\sqrt{x-1}}\right)^2$

$= 4 - \frac{1}{x-1} \quad (x > 1)$



RAN $= (-\infty, 4)$

WHEN $y = 0$: $0 = 4 - \frac{1}{x-1}$

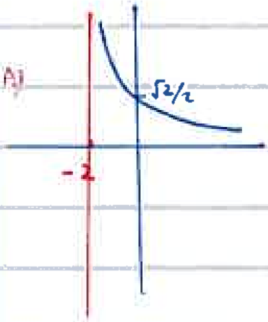
$\frac{1}{x-1} = 4$

$x - 1 = \frac{1}{4}$

$x = 1 + \frac{1}{4} = \frac{5}{4}$

Q31) $f(x) = (x+2)^{-1/2} = \frac{1}{\sqrt{x+2}}$

A)



$D_1 = \text{Dom } f = (-2, \infty)$

RAN $f = (0, \infty)$

$g(x) = -2x^4 + 8$

WHEN $y = 0$

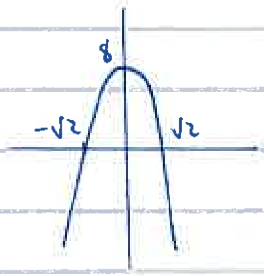
$0 = -2x^4 + 8$

$x^4 = 4$

$x^4 = 2^2$

$x = \pm 2^{2/4} = \pm 2^{1/2}$

$= \pm\sqrt{2}$



$D_2 = \text{Dom } g = \mathbb{R}$

RAN $g = (-\infty, 8]$

B) $g(f(x))$

$= g\left((x+2)^{-1/2}\right)$

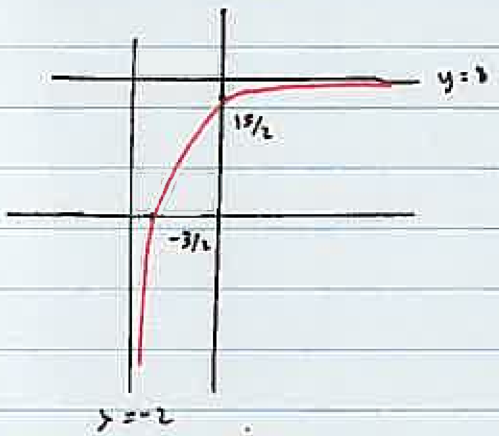
$= -2\left[(x+2)^{-\frac{1}{2}}\right]^4 + 8$

$= -2(x+2)^{-2} + 8$

$= \frac{-2}{(x+2)^2} + 8$

$\text{DOM}(g \circ f) = \text{DOM } f = (-2, \infty)$

C) $y = \frac{-2}{(x+2)^2} + 8 \qquad x > -2$



WHEN $x = 0$ $\quad y = -\frac{2}{2^2} + 8$

$= -\frac{1}{2} + 8$

$= \frac{15}{2}$

$y = 0 \quad 0 = \frac{-2}{(x+2)^2} + 8$

$\frac{2}{(x+2)^2} = 8$

$\frac{1}{4} = (x+2)^2$

$x + 2 = \pm\frac{1}{2}$

$x = -2 \pm \frac{1}{2}$

$= -\frac{3}{2} \quad (x > -2)$

D) $\text{ran}(g \circ f) = (-\infty, 8)$

E) (i) $f \circ g^*$ EXISTS PROVIDED

$\text{ran}(g^*) \subseteq \text{dom}(f)$

$[-2a^4 + 8, 8] \subseteq (-2, \infty)$

I.E WE REQUIRE THAT

$-2a^4 + 8 > -2$

$10 > 2a^4$

$a^4 \leq 5$

$-\sqrt[4]{5} \leq a \leq \sqrt[4]{5}$

$\Rightarrow$ LARGEST POSSIBLE $a = \sqrt[4]{5}$

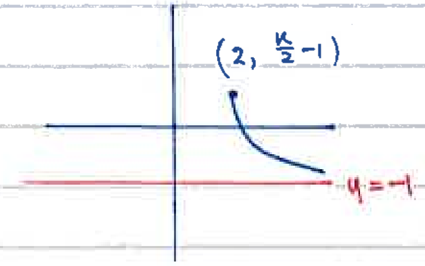
(ii) $\text{dom } g^* = (-\sqrt[4]{5}, \sqrt[4]{5})$

MIN $\quad g^*(0) = 8 \qquad f(8) = \frac{1}{\sqrt{10}} = \frac{\sqrt{10}}{10}$

MAX $\quad g^*(\sqrt[4]{5}) = -2 \qquad \lim_{x \to -2} f(x) = \infty$

$\text{RAN}(f \circ g^*) = \left[\frac{\sqrt{10}}{10}, \infty\right)$

Q32) $g : [2, \infty) \to \mathbb{R}$ $\quad g(x) = \frac{k}{x} - 1$



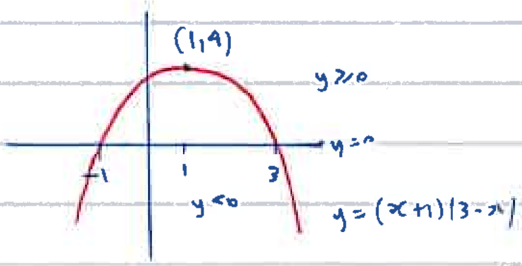
$\text{Dom}(g) = [2, \infty)$

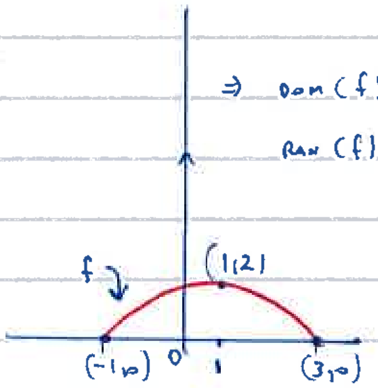
$\text{Ran}(g) = \left(-1, \frac{k}{2} - 1\right]$

$f(x) = \sqrt{(x+1)(3-x)}$

DEFINED PROVIDED $(x+1)(3-x) \geq 0$

LET $y = (x+1)(3-x)$





$\Rightarrow \text{Dom}(f) = [-1, 3]$

$\text{Ran}(f) = [0, 2]$

A) $\underline{\text{dom}(f) = [-1, 3]}$

B) $f \circ g$ EXISTS PROVIDED

$\text{ran}(g) \subseteq \text{dom}(f)$

$\left(-1, \frac{k}{2} - 1\right] \subseteq [-1, 3]$

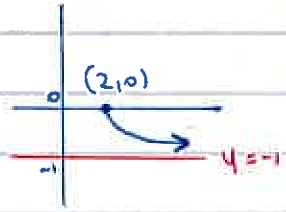
$\frac{k}{2} - 1 \leq 3$

$\Leftrightarrow \frac{k}{2} \leq 4$

$\Leftrightarrow k \leq 8$

SINCE $k > 0$, $\boxed{0 < k \leq 8}$

C) SUPPOSE $k = 2$ $\quad g(x) = \frac{2}{x} - 1$



$\text{ran}(g) = (-1, 0]$

$f(0) = \sqrt{3}$ $\quad \Rightarrow \underline{\text{ran}(f \circ g) = (0, \sqrt{3}]}$

$f(-1) = 0$

D) $g \circ f^*$ IS DEFINED PROVIDED

(i) $\text{ran}(f^*) \subseteq \text{dom}(g)$

$[f(a), 2] \subseteq [2, \infty)$

$\Rightarrow f(a) = 2$

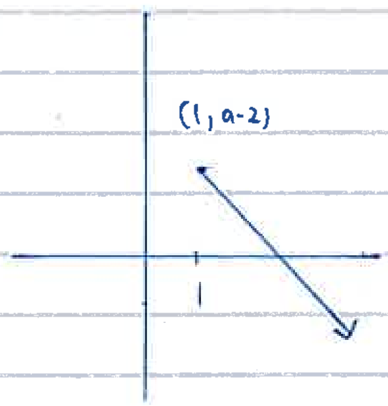
$\Rightarrow a = 1$

(ii) $\therefore g \circ f^*$ ONLY DEFINED @ $x = 1$

$g \circ f^*(1) = g(2) = \frac{2}{2} - 1 = 0$

$\text{ran}(g \circ f^*) = \{0\}$

Q33) $f : [1, \infty) \to \mathbb{R}$ $\quad f(x) = a - 2x$



$\text{dom } f = [1, \infty)$

$\text{ran } f = (-\infty, a-2]$

$g : (-\infty, 0] \to \mathbb{R}$ $\quad g(x) = x^2 + 2a$

$\text{dom } g = (-\infty, 0]$

$\text{ran } g = [2a, \infty)$

$f \circ g$ DEFINED IFF

$\text{ran}(g) \subseteq \text{dom } f$

$[2a, \infty) \subseteq [1, \infty)$

i.e. $2a \geq 1$

$a \geq \frac{1}{2}$

$g \circ f$ DEFINED IFF

$\text{ran}(f) \subseteq \text{dom}(g)$

$(-\infty, a-2] \subseteq (-\infty, 0]$

$a - 2 \leq 0$

$a \leq 2$

OVERALL : $\boxed{\frac{1}{2} \leq a \leq 2}$

Q34) A) $x \leftrightarrow y$

$x = 4y - 8$

$x + 8 = 4y$

$y = \frac{x+8}{4}$

$f^{-1}(x) = \frac{x+8}{4}$

B) $x \leftrightarrow y$

$x = \frac{y+2}{y-1}$

$x(y-1) = y + 2$

$xy - x = y + 2$

$xy - y = 2 + x$

$y(x-1) = 2 + x$

$y = \frac{x+2}{x-1}$

$f^{-1}(x) = \frac{x+2}{x-1}.$

C) $x \leftrightarrow y$ $\quad x = y^{5/3}$

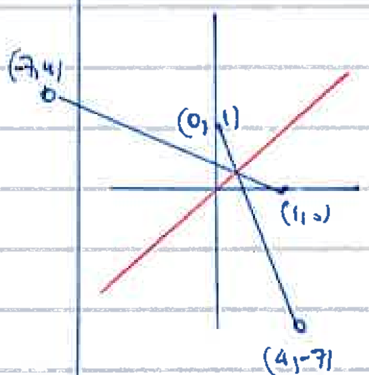
$x^{3/5} = (y^{5/3})^{3/5}$

$x^{3/5} = y^1$

$y = x^{3/5}$

$f^{-1}(x) = x^{3/5}$

Q35 A) $f : [0, 4) \to \mathbb{R}$ $\quad f(x) = 1 - 2x$



$x \leftrightarrow y$ $\quad x = 1 - 2y$

$x - 1 = -2y$

$2y = 1 - x$

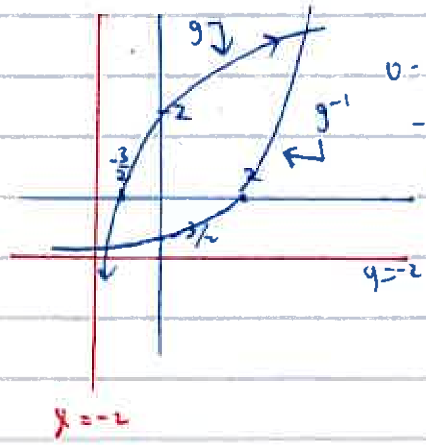
$y = \frac{1-x}{2}$

$f^{-1} : [-7, 1] \to \mathbb{R}$

$f^{-1}(x) = \frac{1-x}{2}$

$\text{RAN } f^{-1} = \text{DOM } f = [0, 4)$

B) $g:[-2,\infty)\to\mathbb{R}$ $\quad g(x)=\log_2(x+2)+1$



$0=\log_2(x+2)+1$

$-1=\log_2(x+2)$

$2^{-1}=x+2$

$\frac{1}{2}=x+2$

$x=-\frac{3}{2}$

$x\Leftrightarrow y$

$x=\log_2(y+2)+1$

$x-1=\log_2(y+2)$

$2^{x-1}=y+2$

$y=-2+2^{x-1}$

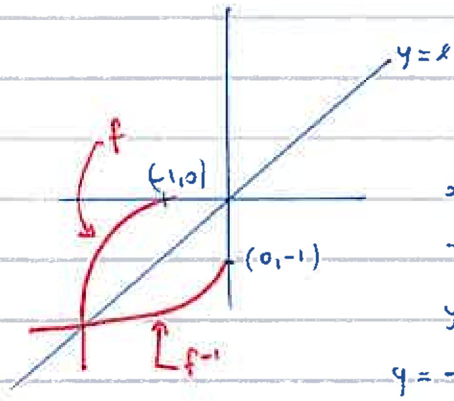
$g^{-1}:\mathbb{R}\to\mathbb{R}$ $\quad g^{-1}(x)=-2+2^{x-1}$

ran $(g^{-1})=[-2,\infty)$

C) $h^{-1}=\{(2,-1),(4,0),(6,1)\}$

ran $h^{-1}=\{-1,0,1\}$

D) $f:(-\infty,-1]\to\mathbb{R}$ $\quad f(x)=-2(x+1)^2$



$x\Leftrightarrow y$

$x=-2(y+1)^2$

$-\frac{x}{2}=(y+1)^2$

$y+1=-\sqrt{-\frac{x}{2}}$ (BOTTOM BRANCH)

$y=-1-\sqrt{-\frac{x}{2}}$

$f^{-1}:(-\infty,0]\to\mathbb{R}$, $\quad f^{-1}(x)=-1-\sqrt{-\frac{x}{2}}$

ran$(f^{-1})=(-\infty,-1]$

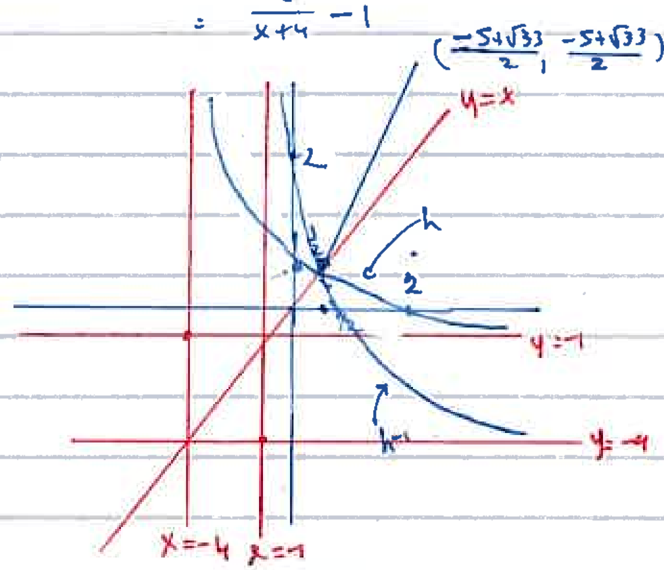
Q36) $h:(-4,\infty)\to\mathbb{R}$

$h(x)=-\frac{x-2}{x+4}$

$=-\frac{(x+4)-4-2}{x+4}$

$=-\left(1-\frac{6}{x+4}\right)$

$=\frac{6}{x+4}-1$



$x\Leftrightarrow y$ $\quad x=\frac{6}{y+4}-1$

$x+1=\frac{6}{y+4}$

$\frac{1}{x+1}=\frac{y+4}{6}$

$\frac{6}{x+1}=y+4$

$y=-4+\frac{6}{x+1}$

(A) $h^{-1}:(-1,\infty)\to\mathbb{R}$ $\quad h^{-1}(x)=-4+\frac{6}{x+1}$

(B) ran $h^{-1}=(-4,\infty)$

(C) SKETCH ABOVE - TO FIND POINT OF INT, LET

$x=h(x)$

$x=-\frac{x-2}{x+4}$

$x^2+4x=-(x-2)$

$x^2+4x=-x+2$

$x^2+5x-2=0$

$x=\frac{-5\pm\sqrt{25+8}}{2}=\frac{-5+\sqrt{33}}{2}$

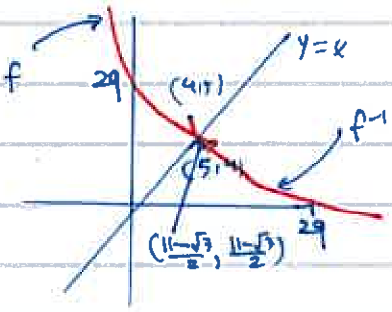
(SINCE $x>0$)

Q37) $f: (-\infty, a] \to \mathbb{R}$

$f(x) = x^2 - 10x + 29$

(A) $= x^2 - 10x + 25 - 25 + 29$

$= (x-5)^2 + 4$



f^{-1} exists iff f is 1-1

so let $\underline{a = 5}$

(B) $x \Leftrightarrow y$

$x = (y-5)^2 + 4$

$x - 4 = (y-5)^2$

$-\sqrt{x-4} = y - 5$ ← BOTTOM BRANCH

$y = 5 - \sqrt{x-4}$

$f^{-1}: [4, \infty) \to \mathbb{R}, \quad f^{-1}(x) = 5 - \sqrt{x-4}$

THESE ONLY INTERSECT ALONG LINE $y = x$. LET $f(x) = x$

$x^2 - 10x + 29 = x$

$x^2 - 11x + 29 = 0$

$x = \dfrac{11 \pm \sqrt{121 - 4(29)}}{2}$

$= \dfrac{11 \pm \sqrt{7}}{2}$

SINCE $x < 5$, $x = \dfrac{11 - \sqrt{7}}{2}$

$y = \dfrac{11 - \sqrt{7}}{2}$

(C) SKETCH ABOVE

Q38) $f: [a, \infty) \to \mathbb{R}$

$f(x) = x^2 - 2ax + 4$

$= (x^2 - 2ax + a^2) - a^2 + 4$

$= (x-a)^2 + 4 - a^2$

if $f^{-1}(4) = 4$ then $f(4) = 4$

$\Rightarrow 16 - 2a(4) + 4 = 4$

$\Rightarrow 20 - 8a = 4$

$\Rightarrow -8a = -16$

$\Rightarrow a = 2$

Q39) $f^{-1}(x) = -1$

$\Leftrightarrow x = f(-1)$

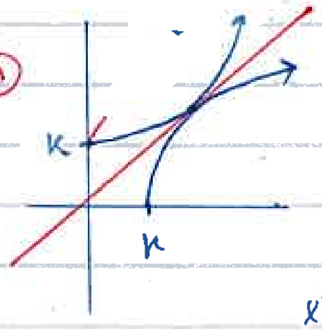
$= (-1)^3 + 6(-1)^2 + 2$

$= -1 + 6 + 2$

$= 7.$

Q40 $f: [k, \infty) \to \mathbb{R} \quad f(x) = \sqrt{x-k}$

(A)



WANT $\sqrt{x-k} = x$ TO HAVE ONE SOL^{n}

$x - k = x^2$

$x^2 - x + k = 0$

$\Delta = 0$

$(-1)^2 - 4(1)(k) = 0$

$1 - 4k = 0$

$\underline{k = \frac{1}{4}}$

(B) For f^{-1} $x \Leftrightarrow y$ $\quad x = \sqrt{y - 1/4}$

$x^2 = y - 1/4$

$y = x^2 + 1/4$

$f^{-1}: [0, \infty) \to \mathbb{R} \quad f^{-1}(x) = x^2 + 1/4$

Q41) $f(x) = 3x - 8$

$g : (-\infty, 2] \to \mathbb{R}$

$g(x) = 2x^3 + 9x^2 - 60x + 60$

$(f \circ g^{-1})(x) = -5$

$f(g^{-1}(x)) = -5$

Let $g^{-1}(x) = y$

$f(y) = -5$

$3y - 8 = -5$

$3y = 3$

$y = 1$

$g^{-1}(x) = 1$

$\Rightarrow x = g(1) = 2 \cdot 1^3 + 9 \cdot 1^2 - \cancel{60} + \cancel{60}$

$= 2 + 9$

$= 11$

Q42) $f(x) = \frac{2}{x^2}$ $\quad g(x) = x^2(x+2)$

$(f+g)(x)$

$= f(x) + g(x)$

$= \frac{2}{x^2} + x^2(x+2), \quad x \neq 0$

$(fg)(x) = f(x)\, g(x)$

$= \frac{2}{x^2}\, x^2(x+2), \quad x \neq 0$

$= 2(x+2), \quad x \neq 0$

Q43) A) $(fg)(-1)$

$= f(-1)\, g(-1)$

$= 2 \cdot (-1)$

$= -2$

$\mathrm{Dom}(fg)$
$= \mathrm{Dom}(f) \cap \mathrm{Dom}(g)$
$= \{-1, 0, 1\} \cap \{-1, 1, 3\}$
$= \{-1, 1\}$

$(fg)(1)$

$= f(1)\, g(1)$

$= 3 \cdot 0$

$= 3$

$fg = \{(-1, -2), (1, 3)\}$

B) $(g \circ f^{-1})(x) = -1$

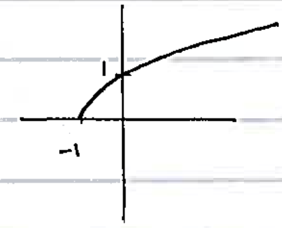
$g(f^{-1}(x)) = -1$

$f^{-1}(x) = -1$

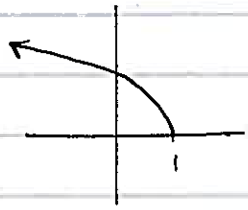
$x = f(-1) = \underline{2}$

Q44) $f(x) = (x+1)^{3/8} = \sqrt[8]{x+1}$ $\quad \frac{3}{8} < 1$

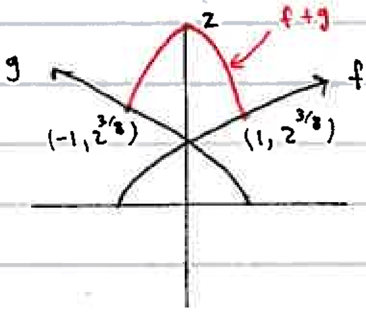
EVEN ONE BRANCH



$g(x) = (1-x)^{3/8} = \sqrt[8]{(-(x-1))^3}$ $\quad \frac{3}{8} < 1$

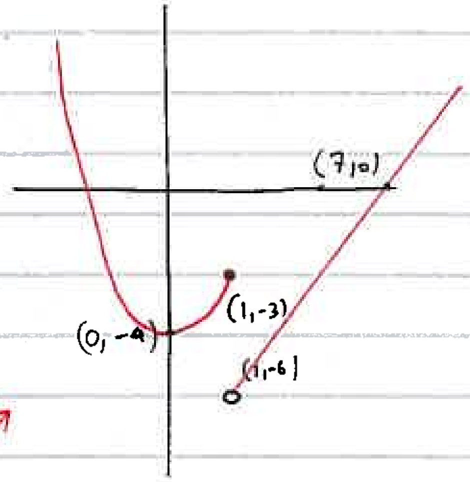


$f(x) + g(x) = \sqrt[8]{(x+1)^3} + \sqrt[8]{(1-x)^3}$ $\quad -1 \leq x \leq 1$



Q45) $f(x) = \begin{cases} x^2-4, & x \leq 1 \\ x-7, & x > 1 \end{cases}$

A) $f(1) = 1^2-4 = -3$



B)

C) $f(x) = 5$

$x^2-4=5$ OR $x-7=5$

$x^2 = 9$ $\qquad x = 12$

$x = \pm 3$

$\Rightarrow x = -3$ $\qquad \underline{x = -3, 12}$

D)

$g(x) = f(x-2) = \begin{cases} (x-2)^2-4, & x-2 \leq 1 \\ (x-2)-7, & x-2 > 1 \end{cases}$

$= \begin{cases} (x-2)^2-4, & x \leq 3 \\ x-9, & x > 3 \end{cases}$

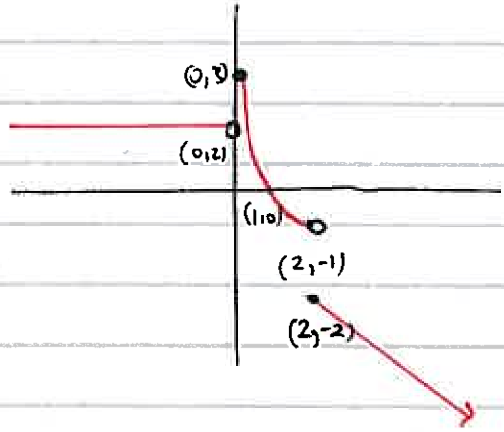
Q46)

$f(x) = \begin{cases} 2 & x \in (-\infty, 0) \\ x^2-4x+3 & x \in [0,2) \\ -x & x \in [2,\infty) \end{cases}$

NOTE : x^2-4x+3

$= x^2-4x+4-4+3$

$= (x-2)^2-1$



Q47) A) $f(-x) = -2(-x)^6 - \frac{2}{(-x)^2} = -2x^6 - \frac{2}{x^2} = f(x)$

$\Rightarrow$ EVEN

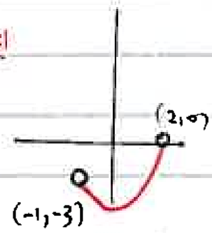
B) $f(-x) = \frac{1}{-x^3-4} \neq -f(x)$

$\Rightarrow$ NEITHER

C) $f(-x) = -x + 2(-x)^3 = -(x+2x^3) = -f(x)$

$\Rightarrow$ ODD

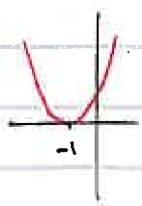
E)



NOT SYMMETRIC

$\Rightarrow$ NEITHER

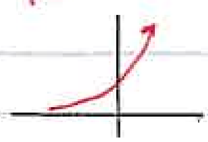
Q47 D) $f(x) = (x+1)^2$



NOT EVEN

$\Rightarrow$ NEITHER

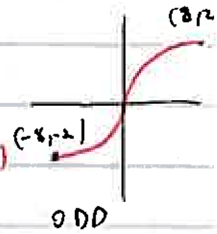
F) $f(x) = 3^x$



NOT SYMMETRIC

$\Rightarrow$ NEITHER

G)



ODD

$f(-x) = \sqrt[3]{-x} = -\sqrt[3]{x} = -f(x)$