

CIRCULAR FUNCTIONS

$$\begin{aligned} \text{Q1) a) } 240^\circ &= \frac{240}{360} \times 2\pi^\circ \\ &= \frac{2}{3} \times 2\pi^\circ \\ &= \frac{4\pi}{3} \end{aligned}$$

$$\begin{aligned} \text{b) } 330^\circ &= \frac{330}{360} \times 2\pi^\circ \\ &= \frac{11}{12} \times 2\pi^\circ \\ &= \frac{11\pi}{6} \end{aligned}$$

$$\begin{aligned} \text{c) } 12^\circ &= \frac{12}{360} \times 2\pi^\circ \\ &= \frac{1}{30} \times 2\pi^\circ \\ &= \frac{\pi}{15} \end{aligned}$$

$$\begin{aligned} \text{d) } 15^\circ &= \frac{15}{360} \times 2\pi^\circ \\ &= \frac{1}{24} \times 2\pi^\circ \\ &= \frac{\pi}{12} \end{aligned}$$

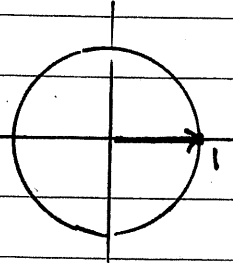
$$\begin{aligned} \text{Q2) a) } \frac{B\pi}{6}^\circ &= \frac{B\pi/6}{2\pi} \times 360^\circ \\ &= \frac{B}{12} \times 360^\circ \\ &= 30B^\circ \end{aligned}$$

$$\begin{aligned} \text{b) } \frac{\pi}{25}^\circ &= \frac{\pi/25}{2\pi} \times 360^\circ \\ &= \frac{1}{50} \times 360^\circ \\ &= \frac{36^\circ}{5} \\ &= 7\frac{1}{5}^\circ \end{aligned}$$

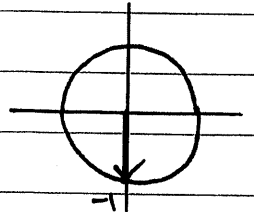
$$\begin{aligned} \text{c) } \frac{\pi}{24}^\circ &= \frac{\pi/24}{2\pi} \times 360^\circ \\ &= \frac{360^\circ}{48} \\ &= \frac{60^\circ}{8} \\ &= \frac{15^\circ}{2} \\ &= 7\frac{1}{2}^\circ \end{aligned}$$

$$\begin{aligned} \text{d) } 2^\circ &= \frac{2}{2\pi} \times 360^\circ \\ &= \frac{360^\circ}{\pi} \\ &\approx 114.59^\circ \end{aligned}$$

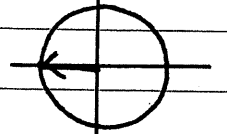
$$\text{Q3) a) } \cos 0 = 1$$



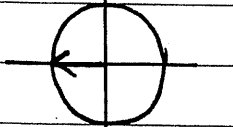
$$\text{b) } \sin \frac{3\pi}{2} = -1$$



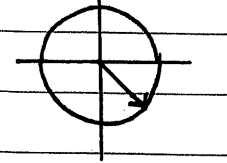
$$\text{c) } \tan 180^\circ = 0$$



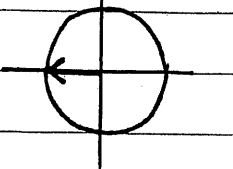
$$\text{d) } \sin \pi = 0$$



$$\text{e) } \tan 315^\circ = -1$$



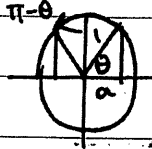
$$\text{f) } \cos 90^\circ = 0$$



Q4) $\cos \theta = x = -3/5$
 $\sin \theta = y = 4/5$
 $\tan \theta = y/x = \frac{4/5}{-3/5} = -4/3$
 $\cos \phi = -5/13$
 $\sin \phi = -12/13$
 $\tan \phi = y/x = \frac{-12/13}{-5/13} = +12/5$

Q7) $\cos \theta = a \quad \theta \in [0, \pi/2]$

$\cos^2 \theta + \sin^2 \theta = 1$
 $a^2 + \sin^2 \theta = 1$
 $\sin^2 \theta = 1 - a^2$
 $\sin \theta = \pm \sqrt{1 - a^2}$
 $= +\sqrt{1 - a^2}, \quad \theta \in [0, \pi/2]$
 $\therefore \sin(\pi - \theta) = +\sqrt{1 - a^2}, \quad \pi - \theta \in [\pi/2, \pi]$

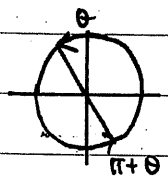


Q5) QUADRANT

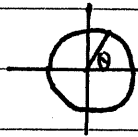
QUADRANT	cos θ	sin θ	tan θ
1	+	+	+
2	-	+	-
3	-	-	+
4	+	-	-

Q8) $\sin \theta = a, \quad \theta \in [\pi/2, \pi]$

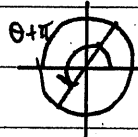
$\cos^2 \theta + \sin^2 \theta = 1$
 $\cos^2 \theta + a^2 = 1$
 $\cos^2 \theta = 1 - a^2$
 $\cos \theta = \pm \sqrt{1 - a^2}$
 $= -\sqrt{1 - a^2}, \quad \theta \in [0, \pi/2]$
 $\therefore \cos(\theta + \pi) = +\sqrt{1 - a^2}, \quad \theta + \pi \in [3\pi/2, 2\pi]$



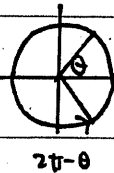
Q6) $\sin \theta = 4/5$



a) $\sin(\theta + \pi) = -4/5$



b) $\sin(2\pi - \theta) = -4/5$



c) $\cos^2 \theta + \sin^2 \theta = 1$

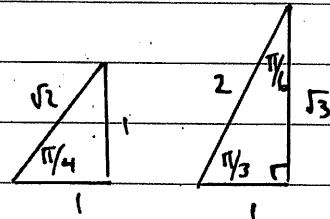
$\cos^2 \theta + 16/25 = 1$

$\cos^2 \theta = 1 - \frac{16}{25} = \frac{9}{25}$

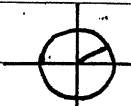
$\cos \theta = \pm \frac{3}{5}$

d) $\tan \theta = \frac{\sin \theta}{\cos \theta} = \frac{-4/5}{\pm 3/5} = \pm \frac{4}{3}$

Q9)



A) $\sin 30^\circ = \frac{1}{2}$



B) $\cos \frac{\pi}{3} = \frac{1}{2}$



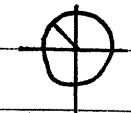
C) $\tan(-\frac{\pi}{4}) = -1$



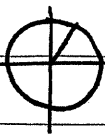
D) $\sin(\frac{3\pi}{4}) = \frac{1}{\sqrt{2}}$



E) $\cos(-225^\circ) = -\frac{1}{\sqrt{2}}$



f) $\sin \frac{7\pi}{3} = \sin \frac{\pi}{3} = \frac{\sqrt{3}}{2}$

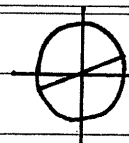


g) $\sin \frac{13\pi}{6} = -\sin \frac{\pi}{6} = -\frac{1}{2}$



h) $\tan(-\frac{13\pi}{3}) = \tan(-\frac{\pi}{3}) = -\sqrt{3}$

d) $\tan(2\theta + \pi/4) = \frac{1}{\sqrt{3}}$



NOTE: $\theta \in [0, 2\pi)$

PA = $\pi/6$

$\therefore 2\theta \in [0, 4\pi)$

$2\theta + \pi/4 \in [\pi/4, 17\pi/4)$

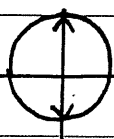
$\therefore 2\theta + \frac{\pi}{4} = \frac{7\pi}{6}, \frac{13\pi}{6}, \frac{19\pi}{6}, \frac{25\pi}{6}$

$\therefore 2\theta = \frac{11\pi}{12}, \frac{23\pi}{12}, \frac{35\pi}{12}, \frac{47\pi}{12}$

$\therefore \theta = \frac{11\pi}{24}, \frac{23\pi}{24}, \frac{35\pi}{24}, \frac{47\pi}{24}$

Q10) $\cos \theta = 0$

a) $\theta = \pi/2, 3\pi/2$

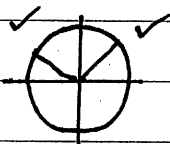


b) $\sin \theta = \frac{1}{2}$

$\theta = 30^\circ, 180 - 30$

$= 30^\circ, 150^\circ$

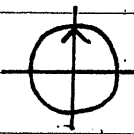
PA = 30°



Q11) a) $\sin \theta = 1$

$\theta = -\frac{3\pi}{2}, \frac{\pi}{2}$

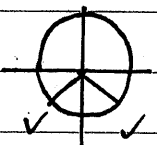
PA = $\frac{\pi}{2}$



b) $\sin(\theta - \pi/2) = -\frac{\sqrt{2}}{2} = -\frac{1}{\sqrt{2}}$

NOTE: $\theta \in [0, 2\pi)$

$\therefore \theta - \frac{\pi}{2} \in [-\pi/2, 3\pi/2)$



PA = $\pi/4$

$\therefore \theta - \pi/2 = -\frac{\pi}{4}, \frac{5\pi}{4}$

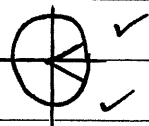
$\theta = \frac{\pi}{4}, \frac{7\pi}{4}$

c) $\cos 2\theta = \frac{\sqrt{3}}{2}$ $\theta \in [0, 2\pi)$

$2\theta \in [0, 4\pi)$

$\therefore 2\theta = \frac{\pi}{6}, \frac{11\pi}{6}, \frac{13\pi}{6}, \frac{23\pi}{6}$

$\therefore \theta = \frac{\pi}{12}, \frac{11\pi}{12}, \frac{13\pi}{12}, \frac{23\pi}{12}$

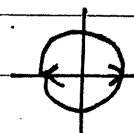


PA = $\pi/6$

Q12) a) $\cos^2 \theta = 1$

$\cos \theta = \pm 1$

$\therefore \theta = 0, \pi, 2\pi$



b) $2\cos^2 \theta + \cos \theta = 1$

$2\cos^2 \theta + \cos \theta - 1 = 0$

$(2\cos \theta - 1)(\cos \theta + 1) = 0$

$\therefore \cos \theta = \frac{1}{2}$ or $\cos \theta = -1$

$\theta = \pi/3, 5\pi/3$ or $\theta = \pi$

$\therefore \theta = \pi/3, \pi, 5\pi/3$

c) $\cos^2 \theta = \cos \theta \sin \theta$

$\cos^2 \theta - \cos \theta \sin \theta = 0$

$\cos \theta (\cos \theta - \sin \theta) = 0$

$\cos \theta = 0$ or $\cos \theta = \sin \theta$

$\theta = \pi/2, 3\pi/2$ or $\tan \theta = 1$

$\theta = \pi/2, 3\pi/2$ or $\theta = \pi/4, 5\pi/4$

$\theta = \pi/4, \pi/2, 5\pi/4, 3\pi/2$

(12) $\sin \theta + \tan \theta = \cos \theta$

$\sin \theta \frac{\sin \theta}{\cos \theta} = \cos \theta$
 $\frac{\sin^2 \theta}{\cos \theta} = \cos \theta$

$\sin^2 \theta - \cos^2 \theta = 0$

$(\sin \theta - \cos \theta)(\sin \theta + \cos \theta) = 0$

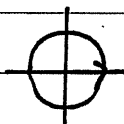
$\sin \theta = \cos \theta$ or $\sin \theta = -\cos \theta$

$\tan \theta = 1$ or $\tan \theta = -1$

$\theta = \pi/4, 5\pi/4$ or $\theta = 3\pi/4, 7\pi/4$

$\theta = \pi/4, 3\pi/4, 5\pi/4, 7\pi/4$

Q13) (A) $\cos x = 1$



$x = 0 + 2n\pi$

$= 2n\pi, n \in \mathbb{Z}$

(B) $\sin x = -\frac{1}{2}$



PA = $\pi/6$

$\therefore x = -\frac{\pi}{6} + 2n\pi,$

or $x = \frac{7\pi}{6} + 2n\pi, n \in \mathbb{Z}$

CAN COMBINE SOLUTIONS

$x = n\pi + (-1)^{n+1} \frac{\pi}{6}, n \in \mathbb{Z}$

(c) $\cos 2x = \frac{1}{\sqrt{2}}$



$\therefore 2x = \frac{\pi}{4} + 2n\pi, n \in \mathbb{Z}$

or $2x = -\frac{\pi}{4} + 2n\pi, n \in \mathbb{Z}$

$\therefore x = +\frac{\pi}{8} + n\pi$ or $x = -\frac{\pi}{8} + n\pi, n \in \mathbb{Z}$

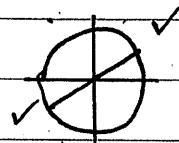
CAN COMBINE SOLUTIONS:

$2x = (2n+1) \frac{\pi}{2} + (-1)^{n+1} \frac{\pi}{4}, n \in \mathbb{Z}$

$\therefore x = (2n+1) \frac{\pi}{4} + (-1)^{n+1} \frac{\pi}{8}, n \in \mathbb{Z}$

(D) $\tan(2x + \pi/3) = \frac{1}{\sqrt{3}}$

$\therefore 2x + \frac{\pi}{3} = \frac{\pi}{6} + n\pi$

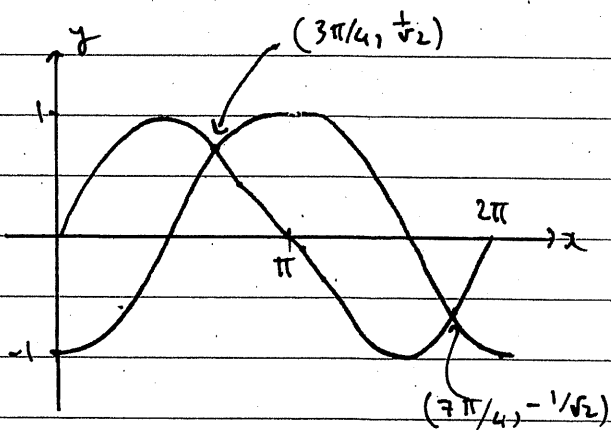


$2x = -\frac{\pi}{6} + n\pi$

PA = $\pi/6$

$x = -\frac{\pi}{12} + \frac{n\pi}{2}, n \in \mathbb{Z}$

Q14)



$y = \sin x$

$y = -\cos x \therefore \sin x = -\cos x$

$\tan x = -1$

$x = \frac{3\pi}{4}, \frac{7\pi}{4}$

WHEN $x = \frac{3\pi}{4}$

$x = \frac{7\pi}{4}$

$y = \sin \frac{3\pi}{4}$

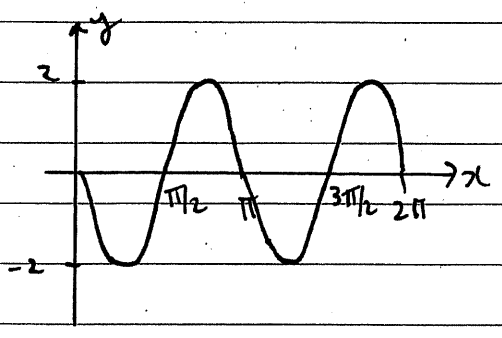
$y = \sin \frac{7\pi}{4}$

$= \frac{1}{\sqrt{2}}$

$= -\frac{1}{\sqrt{2}}$

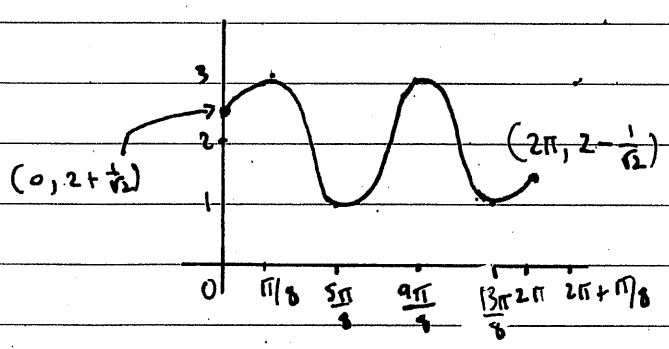
Q15) (A) $y = -2 \sin 2x \quad x \in [0, 2\pi)$

PERIOD = $\frac{2\pi}{2} = \pi$



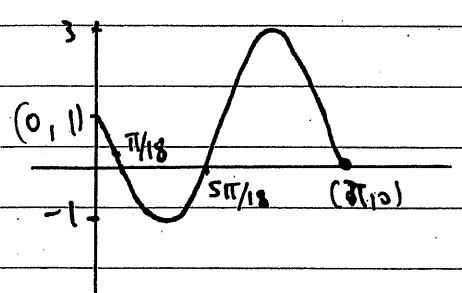
(B) $y = 2 + \cos(2x - \pi/4)$
 $= 2 + \cos(2(x - \pi/8))$

PERIOD = $\frac{2\pi}{2} = \pi$



(C) $f(x) = -2 \sin 3x \quad x \in [0, \pi)$

PERIOD = $\frac{2\pi}{3}$

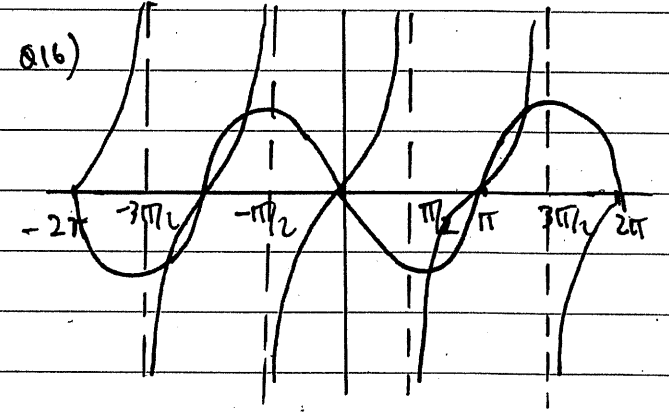
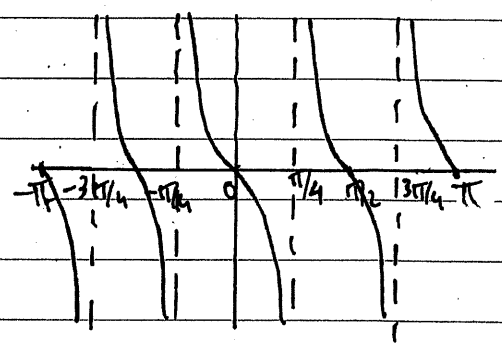


when $f(x) = 0 \quad 0 = 1 - 2 \sin 3x$
 $\sin 3x = 1/2$

$3x = \pi/6, 5\pi/6$
 $x = \pi/18, 5\pi/18$

(D) $y = -\tan(2x - \pi) \quad x \in [-\pi, \pi)$
 $= -\tan(2(x - \pi/2))$

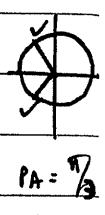
PERIOD = $\frac{\pi}{2}$



$y = \tan x \quad \tan x = -2 \sin x$
 $y = -2 \sin x \quad \frac{\sin x}{\cos x} = -2 \sin x$
 $\sin x = -2 \sin x \cos x$

$\sin x + 2 \sin x \cos x = 0$
 $\sin x (1 + 2 \cos x) = 0$

$\sin x = 0 \quad \text{or} \quad 1 + 2 \cos x = 0$
 $x = n\pi, n \in \mathbb{Z} \quad \cos x = -1/2$
 $x = \frac{2\pi}{3} + 2n\pi$
 $\text{or } x = \frac{4\pi}{3} + 2n\pi$



$x = n\pi, \frac{2\pi}{3} + 2n\pi, \frac{4\pi}{3} + 2n\pi, n \in \mathbb{Z}$

(F) $y = a + b \sin(n\pi)$

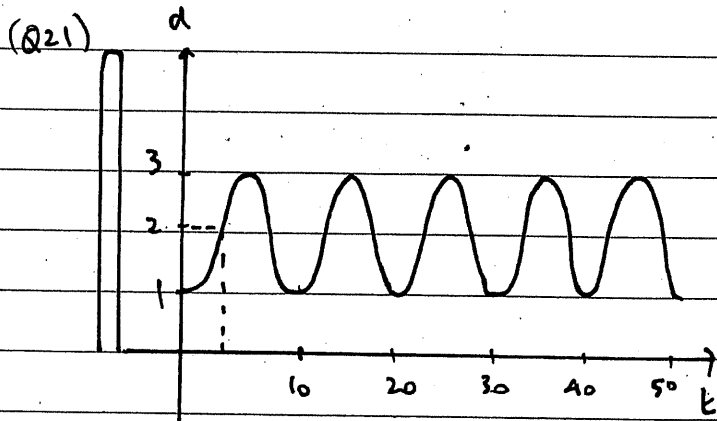
$$\begin{cases} a = -1 & T = \frac{2\pi}{n} \\ b = 3 & \frac{2\pi}{3} = \frac{2\pi}{n} \\ n = 3 & n = 3 \end{cases}$$

(G) $y = a + b \cos nx$

$$\begin{cases} a = -2 & T = \frac{2\pi}{n} \\ b = -3 & 4 = \frac{2\pi}{n} \\ n = \frac{\pi}{2} & n = \frac{\pi}{2} \end{cases}$$

(H) $y = a + b \sin(n\pi)$

$$\begin{cases} a = -1 & 8 = \frac{2\pi}{n} \\ b = 2 & n = \frac{\pi}{4} \\ n = \frac{\pi}{4} & \end{cases}$$



(A) $\begin{cases} a = 2 & \text{PERIOD} = \frac{2\pi}{n} \\ b = -1 & 10 = \frac{2\pi}{n} \\ n = \frac{\pi}{5} & n = \frac{\pi}{5} \end{cases}$

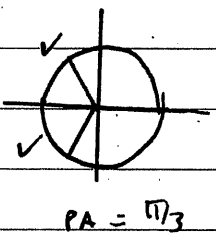
$d(t) = 2 - \cos\left(\frac{\pi t}{5}\right)$

(B) SEE EARLIER

(C) WHEN $t=5, d=2$

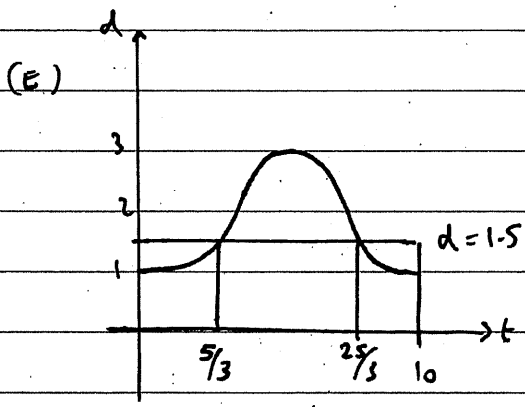
(D) $d = 2 - \cos\left(\frac{\pi t}{5}\right)$
 $2.5 = 2 - \cos\left(\frac{\pi t}{5}\right)$
 $-\frac{1}{2} = -\cos\left(\frac{\pi t}{5}\right)$

$\frac{\pi t}{5} = \frac{2\pi}{3} + 2\pi n$ OR $\frac{\pi t}{5} = \frac{4\pi}{3} + 2\pi n$
 $t = \frac{10}{3} + 10\pi n$ OR $t = \frac{20}{3} + 20\pi n$
 $n \in \mathbb{Z}$



OR COMBINED ANSWER

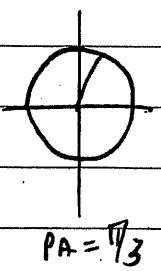
$\frac{\pi t}{5} = \frac{(2n+1)\pi}{2} + (-1)^n \frac{\pi}{6}$
 $\therefore t = \frac{5(2n+1)}{2} + (-1)^n \frac{5}{6}, n \in \mathbb{Z}$



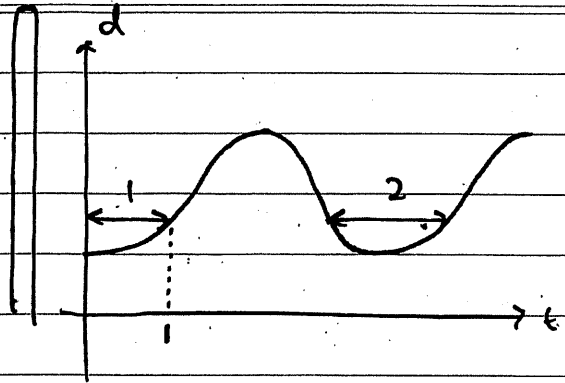
FIND FIRST SOLUTION TO

$1.5 = 2 - \cos\left(\frac{\pi t}{5}\right)$
 $\frac{1}{2} = \cos\left(\frac{\pi t}{5}\right)$

$\frac{\pi t}{5} = \frac{\pi}{3}$
 $t = \frac{5}{3}$



% TIME = $\frac{2 \times \frac{5}{3}}{10} \times 100\%$
 $= 33 \frac{1}{3} \%$

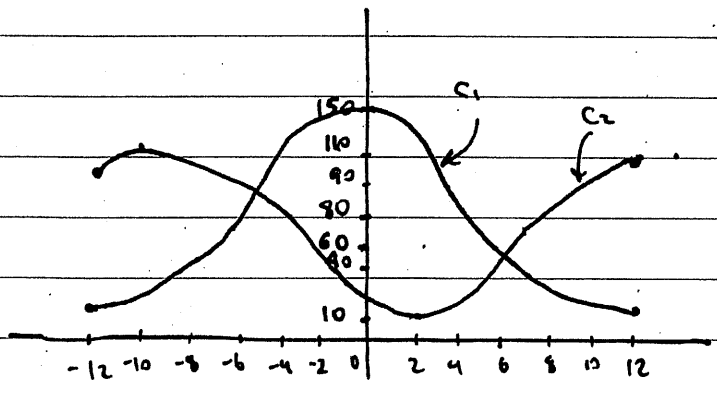


(F) WHEN $t=1$,

$$d(1) = 2 - \cos\left(\frac{\pi \times 1}{5}\right) \approx 1.19$$

\therefore MUSSELS SURVIVE TO A HEIGHT OF APPROX 1.19 m.

(Q22) $C_1 = 80 + 70 \cos(\pi t / 12)$
 $C_2 = 60 + 50 \cos(\pi(t-2) / 12)$



$$P_{100} = \frac{2\pi}{\pi} = \frac{2\pi}{\pi/12} = 24$$

(B) MELATONIN MAX @ $t=0$
 (I.E. MIDNIGHT)

(C) CORTISOL MIN @ $t=2$
 (I.E. 2 AM)

(D) $C_1 \geq 8 \Leftrightarrow t \in [-6, 6]$
 $C_2 \leq 60 \Leftrightarrow t \in [-4, 8]$

INTERSECT THIS DOMAINS TO GIVE

$$t \in [-4, 6]$$

I.E. BETWEEN 8 PM AND 6 AM.

Q23

(A) WHEN $h=10$

$$10 = 10 - 3e^{-t/10} \cos \frac{\pi t}{2}$$

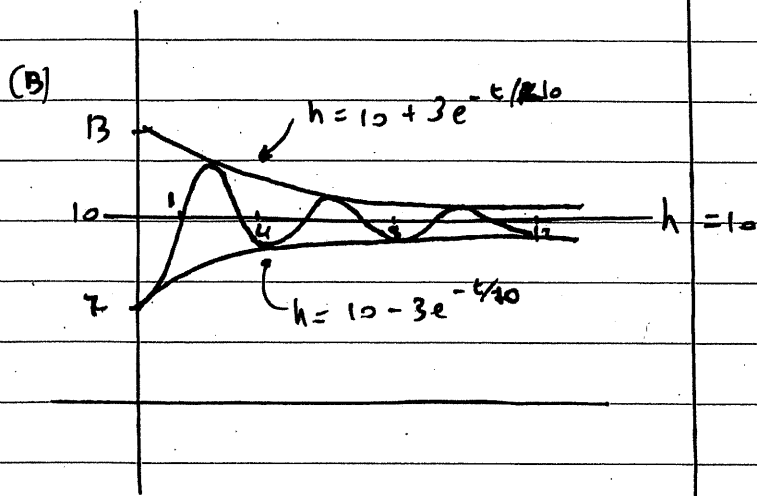
$$0 = 3e^{-t/10} \cos \frac{\pi t}{2}$$

$$0 = \cos \frac{\pi t}{2}$$

$$\Rightarrow \frac{\pi t}{2} = \frac{\pi}{2} \quad (\text{1ST SOLN})$$

$$\Rightarrow t = 1 \text{ SECOND}$$

TURN OVER



(C) $h(t) = 10 - 3e^{-t/10} \cos\left(\frac{\pi t}{2}\right)$

PERIOD = $\frac{2\pi}{\pi/2} = 4$

SEE ABOVE DIAGRAM