SAMPLING + ESTIMATION SOLUTIONS

Q1) A) 3 G, 5 B

$$\#2G, 3B = {}^3C_2 \times {}^5C_3 = \underline{30}$$

B)

$$\#3G, 2B = {}^3C_3 \times {}^5C_2 = \underline{10}$$

Q2) A) $S = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$

4 EVEN 5 ODD

$$\#2E, 2O = {}^4C_2 \times {}^5C_2 = \underline{60}$$

B)

$$\#3E, 1O = {}^4C_3 \times {}^5C_1 = \underline{20}$$

Q3) 4 G, 3 B, 7 TOTAL

A) $$Pr(2G, 2B) = \frac{{}^4C_2 \times {}^3C_2}{{}^7C_4} = \underline{\frac{18}{35}}$$

B) $$Pr(3B, 1G) = \frac{{}^3C_3 \times {}^4C_1}{{}^7C_4} = \underline{\frac{4}{35}}$$

C) $Pr(3G, 1B) + Pr(4G, 0B)$

$$= \frac{{}^4C_3 \times {}^3C_1}{{}^7C_4} + \frac{{}^4C_4 \times {}^3C_0}{{}^7C_4} = \underline{\frac{13}{35}}$$

Q4) HYPERBOLA 3 V, 6 CONST.

A) $$Pr(2V, 2C) = \frac{{}^3C_2 \times {}^6C_2}{{}^9C_4} = \frac{5}{14}$$

B) $$Pr(0V, 4C) = \frac{{}^3C_0 \times {}^6C_4}{{}^9C_4} = \frac{5}{42}$$

Q5) 5 R, 5 B

A) $$Pr(0R, 5B) = \frac{{}^5C_0 \times {}^5C_5}{{}^{10}C_5} = \frac{1}{252}$$

B) $$Pr(1R, 4B) = \frac{{}^5C_1 \times {}^5C_4}{{}^{10}C_5} = \frac{25}{252}$$

C) $$Pr(2R, 3B) = \frac{{}^5C_2 \times {}^5C_3}{{}^{10}C_5} = \frac{25}{63}$$

D) $$Pr(3R, 2B) = \frac{25}{63}$$

E) $$Pr(4R, 1B) = \frac{25}{252}$$

F) $$Pr(5R, 0B) = \frac{1}{252}$$

Q6

A) (REPLACED) $X \sim Bi(4, \frac{4}{7})$

$$Pr(X=2) = {}^4C_2 \left(\frac{4}{7}\right)^2 \left(\frac{3}{7}\right)^2$$

$$= 0.36 \quad \left(= \frac{864}{2401}\right)$$

B) (NOT REPLACED)

$$Pr(2R, 2B) = \frac{{}^4C_2 \times {}^3C_2}{{}^7C_4} = \frac{18}{35}$$

$$(= 0.51)$$

Q7) (REPLACED) $X \sim Bi\left(5, \frac{5}{9}\right)$

A) $Pr(X=3) = {}^5C_3 \left(\frac{5}{9}\right)^3 \left(\frac{4}{9}\right)^2 = 0.34$

$= \frac{4000}{6561}$.

B) $Pr(3R, 2B) = \frac{{}^5C_3 \times {}^4C_2}{{}^9C_5}$

$= \frac{10}{21}$

≈ 0.48

Q8) (REPLACED) $X \sim Bi(4, 0.6)$

A) $Pr(X=2) = {}^4C_2 (0.6)^2 (0.4)^2$

$= \frac{216}{625}$

≈ 0.35.

B) (NOT REPLACED)

$Pr(\text{2 ORANGES, 2 APPLES})$

$= \frac{{}^{1200}C_2 \; {}^{800}C_2}{{}^{2000}C_4}$

≈ 0.35

Q9) (REPLACED) $X \sim Bi\left(5, \frac{9}{20}\right)$

A) $Pr(X=3) = {}^5C_3 \left(\frac{9}{20}\right)^3 \left(\frac{11}{20}\right)^2$

≈ 0.28

B) $Pr(3B, 2G) = \frac{{}^{450}C_3 \; {}^{550}C_2}{{}^{1000}C_5}$

≈ 0.28

Q10) $p = \frac{107}{107+93} = 0.535$

Q11) $\hat{p} = \frac{11}{25} = 0.44$.

Q12 A) $p = \frac{6}{16} = \frac{3}{8}$

B) (i) $\hat{p} = \frac{4}{5}$

(ii) $\hat{p} = \frac{3}{5}$

(iii) $\hat{p} = \frac{2}{5}$

(iv) $\hat{p} = \frac{3}{5}$

(v) $\hat{p} = \frac{3}{5}$

(vi) $\hat{p} = \frac{2}{5}$

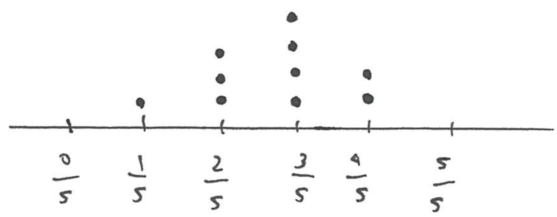
(vii) $\hat{p} = \frac{1}{5}$

(viii) $\hat{p} = \frac{3}{5}$

(ix) $\hat{p} = \frac{3}{5}$

(x) $\hat{p} = \frac{2}{5}$

c)



Q13) Let X = # of red balls taken

$\therefore \hat{p} = \frac{X}{4}$ (4R, 7B)

A) $Pr(X=0) = Pr(0R, 4B)$

$= \frac{{}^4C_0 \cdot {}^7C_4}{{}^{11}C_4}$

$= \frac{7}{66}$

$Pr(X=1) = Pr(1R, 3B)$

$= \frac{{}^4C_1 \cdot {}^7C_3}{{}^{11}C_4}$

$= \frac{14}{33}$

$Pr(X=2) = Pr(2R, 2B)$

$= \frac{{}^4C_2 \cdot {}^7C_2}{{}^{11}C_4}$

$= \frac{21}{55}$

$Pr(X=3) = Pr(3R, 1B)$

$= \frac{{}^4C_3 \cdot {}^7C_1}{{}^{11}C_4} = \frac{14}{165}$

$Pr(X=4) = Pr(4R, 0B)$

$= \frac{{}^4C_4 \cdot {}^7C_0}{{}^{11}C_4} = \frac{1}{330}$

$\hat{p}$	0	$\frac{1}{4}$	$\frac{2}{4}$	$\frac{3}{4}$	$\frac{4}{4}$
$Pr(\hat{P}=\hat{p})$	$\frac{7}{66}$	$\frac{14}{33}$	$\frac{21}{55}$	$\frac{14}{165}$	$\frac{1}{330}$

B) (i) $Pr(\hat{P} > 0.5)$

$= \frac{14}{165} + \frac{1}{330}$

$= \frac{28}{330} + \frac{1}{330}$

$= \frac{29}{330}$

(ii) $Pr(\hat{p} > 0.6 \mid \hat{p} > 0.2)$

$= \frac{Pr(\hat{p} > 0.6 \text{ AND } \hat{p} > 0.2)}{Pr(\hat{p} > 0.2)}$

$= \frac{Pr(\hat{p} > 0.6)}{Pr(\hat{p} > 0.2)}$

$= \frac{29/330}{155/330} = \frac{29}{155}$

(C) $E(\hat{p}) = 0 \cdot \frac{7}{66}$ $= \frac{4}{11}$

$+ \frac{1}{4} \cdot \frac{14}{33}$

$+ \frac{2}{4} \cdot \frac{21}{55}$

$+ \frac{3}{4} \cdot \frac{14}{165}$

$+ \frac{4}{4} \cdot \frac{1}{130}$

I.E the <u>expected</u> sample proportion is equal to the pop. proporation.

Q15) A) $\hat{p} = \frac{X}{4}$ $\quad X \sim Bi(4, 0.2)$

B)

(i) $Pr(\hat{p} \geq 0.75) = Pr(X=3) + Pr(X=4)$

$= 0.0272$

(ii) $Pr(\hat{p} \geq 0.25 \mid \hat{p} \leq 0.75)$

$= Pr(X \geq 1 \mid X \leq 3)$

$= \dfrac{Pr(X \geq 1 \text{ AND } X \leq 3)}{Pr(X \leq 3)}$

$= \dfrac{Pr(X=1) + Pr(X=2) + Pr(X=3)}{Pr(X \leq 3)}$

$= 0.5897$

C) $E(\hat{p}) = E\left(\frac{X}{4}\right) = \frac{1}{4} E(X)$

$= \frac{1}{4} \cdot 4 \cdot 0.2$

$= 0.2$

THE EXPECTED VALUE OF THE SAMPLE PROPORTION EQUALS THE POPULATION PROPORTION.

Q16) $\hat{p} \sim N(\mu, \sigma)$

$\mu = p = 0.42$

$\sigma = \sqrt{\frac{p(1-p)}{n}}$

$= \sqrt{\frac{0.42 \cdot 0.58}{1000}}$

≈ 0.01561

USING NORMCDF

A) $Pr(\hat{p} > 0.45)$

$= 0.02729.$

B) $Pr(\hat{p} < 0.41)$

$= 0.2609$

Q17) $\hat{p} \sim N(\mu, \sigma)$ (NORMAL APPROX)

$\mu = p = \frac{1}{6}$

$\sigma = \sqrt{\frac{p(1-p)}{500}}$

$= \sqrt{\frac{\frac{1}{6} \cdot \frac{5}{6}}{500}}$

$= \frac{1}{60}$

USING NORMCDF

$Pr(\hat{p} > 0.18)$

$= 0.2119$

EXACT ANSWER

18) $\hat{p} \sim N(\mu, \sigma)$

$\mu = p = 0.05$

$\sigma = \sqrt{\frac{p(1-p)}{n}} \approx 0.048734.$

A) $\Pr(\hat{p} > 0.1) \approx 0.1525$

USE NORMCDF

B) $\Pr(\hat{p} > 0.1 \mid \hat{p} > 0.05)$

$= \frac{\Pr(\hat{p} > 0.1 \text{ AND } \hat{p} > 0.05)}{\Pr(\hat{p} > 0.05)}$

$= 0.5$ SINCE $\mu = 0.05$

$= \frac{\Pr(\hat{p} > 0.1)}{0.5}$

$= 0.3049$

Q19) A) $\mu = p = \frac{1}{2}$

$\hat{p} \sim N(\mu, \sigma)$

$\sigma = \sqrt{\frac{p(1-p)}{n}}$

$= \sqrt{\frac{\frac{1}{2}\cdot\frac{1}{2}}{100}} \approx 0.05$

$\Pr(\hat{p} > 0.51) \approx 0.4207$

USE NORMCDF

B) $\sigma = \sqrt{\frac{\frac{1}{2}\cdot\frac{1}{2}}{1000}} \approx 0.01581$

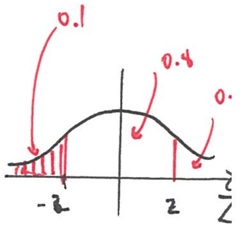
$\Pr(\hat{p} > 0.51) \approx 0.2635$

C) $\sigma = \sqrt{\frac{\frac{1}{2}\cdot\frac{1}{2}}{10000}} \approx 0.005$

$\Pr(\hat{p} > 0.51) \approx 0.0228$

Q20) $\hat{p} = \frac{210}{500} = 0.42.$

Q21) A) $\Pr(-z < Z < z) = 0.8$



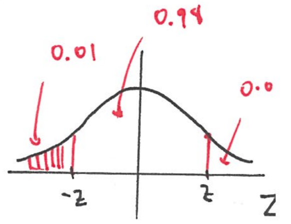
$\Rightarrow \Pr(Z < -z) = 0.1$

LEFT TAIL.

$-z = -1.28155$

$z = 1.28155 \approx 1.28$

B) $\Pr(-z < Z < z) = 0.98$

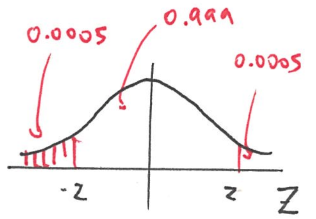


$\Pr(Z < -z) = 0.01$

$-z = -2.32635$

$z = 2.33$

C) $\Pr(-z < Z < z) = 0.999$



$\Pr(Z < -z) = 0.0005$

$-z = -3.29053$

$z = 3.29053 \approx 3.29$

Q22) $\hat{p} = \frac{12}{30} = \frac{4}{10} = 0.40$

$M = 1.96\sqrt{\frac{\hat{p}(1-\hat{p})}{30}} \approx 0.18$ (MARGIN OF ERROR)

95% CONF INTERVAL $= (0.4 - 0.18, 0.4 + 0.18)$

$= (0.22, 0.58)$

I.E. $\hat{p} = 0.40 \pm 0.18$

Q23) $\hat{p} = \frac{672}{1000} = 0.672$

$M = 1.96\sqrt{\frac{\hat{p}(1-\hat{p})}{1000}} \approx 0.0291$

95% CONF INTERVAL $= (0.672 - 0.0291,\ 0.672 + 0.029)$

$= (0.64, 0.70)$

I.E. $\hat{p} = 0.67 \pm 0.03$

Q24). $n = \left(\frac{1.96}{0.03}\right)^2 0.43\ (1-0.43)$

≈ 1046

Q25) A) $n = \left(\frac{1.96}{0.02}\right)^2 (0.2)(0.8) \approx 1537$

B) $n = \left(\frac{1.96}{0.02}\right)^2 (0.4)(0.6) \approx 2305$

C) $n = \left(\frac{1.96}{0.02}\right)^2 (0.6)(0.4) \approx 2305$

D) $n = \left(\frac{1.96}{0.02}\right)^2 (0.8)(0.2) \approx 1537$

ALL ROUNDED UP

Q26) A) $n = \left(\frac{1.96}{0.05}\right)^2 (0.18)\cdot(0.82) \approx 227$

B) $\hat{p} = \frac{50}{227} \approx 0.2202 \approx 0.22$

C) MARGIN OF ERROR

$M = 1.96\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$

$= 1.96\sqrt{\frac{0.22(1-0.22)}{227}}$

≈ 0.0539 NOTE: A LITTLE LARGER THAN THE DESIRED ERROR.

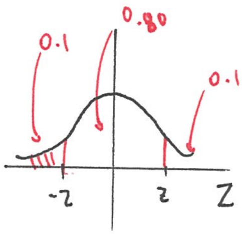
95% CONF. INTERVAL $= (0.2202 - 0.0539,\ 0.2202 + 0.0539)$

$\approx (0.17, 0.27)$

D) THE 95% CONFIDENCE INTERVAL STILL CONTAINS THE PREVIOUS POINT ESTIMATE. IT HAS PROBABLY INCREASED, THOUGH WE CANNOT SAY THIS WITH 95% CONFIDENCE.

$$\hat{p} = \frac{380}{800} \approx 0.475$$

Q27) A) FIRST STEP :



$$Pr(-z < Z < z) = 0.8$$

$$Pr(Z < -z) = 0.1$$

$$\Rightarrow z = 1.28$$

$$M = 1.28\sqrt{\frac{\hat{p}(1-\hat{p})}{n}} \approx 0.02263$$

80% CONF. INTERVAL

$$= (0.475 - 0.02263,\ 0.475 + 0.02263)$$

$$\approx (0.45,\ 0.50)$$

B) FIRST STEP :

$$Pr(-z < Z < z) = 0.98$$

$$Pr(Z < -z) = 0.01$$

$$z = 2.32635$$

$$M = 2.32635\sqrt{\frac{\hat{p}(1-\hat{p})}{n}} \approx 0.041073$$

98% CONF. INTERVAL

$$= (0.475 - 0.041073,\ 0.475 + 0.041073)$$

$$\approx (0.43,\ 0.52)$$

C) $Pr(-z < Z < z) = 0.99$

$$Pr(Z < -z) = 0.005$$

$$\Rightarrow z = 2.57583$$

$$M = 2.57583\sqrt{\frac{\hat{p}(1-\hat{p})}{n}} \approx 0.0454777$$

99% CONF. INTERVAL

$$= (0.475 - 0.454777,\ 0.475 + 0.454777)$$

$$\approx (0.43,\ 0.52)$$

To 2 D.P., THIS IS THE SAME AS IN PART (B)

D) AS THE CONFIDENCE INCREASES THE INTERVAL GETS WIDER.

Q28) A) $\hat{p} = \frac{14}{100} = 0.14.$

B) $Pr(-z < Z < z) = 0.9$

$$Pr(Z < -z) = 0.05$$

$$\Rightarrow z = 1.64485$$

$$M = 1.64485\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$

$$\approx 0.0571$$

90% CONF INT.

$$= (0.14 - 0.0571,\ 0.14 + 0.0571)$$

$$\approx (0.08,\ 0.20)$$

$$Pr(-z < Z < z) = 0.99$$

$$Pr(Z < -z) = 0.005$$

$$z = 2.57538$$

$$M = 2.57538\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$

$$= 0.08938$$

99% CONF. INT.

$$= (0.14 - 0.08938,\ 0.14 + 0.08938)$$

$$= (0.05,\ 0.23)$$

AS THE CONFIDENCE INCREASES THE INTERVAL WIDENS