Topic 2 : Quadratic Equations

(1) (a) $(x+2)(x-3)=0$

$x+2=0$ or $x-3=0$

$x=-2$ or $x=3$

(b) $6x^2-7x=3$

$6x^2-7x-3=0$

$\begin{array}{cc} 2x & -3 \\ 3x & 1 \end{array}$

$(2x-3)(3x+1)=0$

$x=\frac{3}{2}$ or $x=-\frac{1}{3}$

(c) $(x-3)^2-3=0$

$(x-3-\sqrt{3})(x-3+\sqrt{3})=0$

$x-3-\sqrt{3}=0$, $x-3+\sqrt{3}=0$

$x=3+\sqrt{3}$, $x=3-\sqrt{3}$

(d) $\frac{5}{x+2} - \frac{x}{2}$

$10=x(x+2)$

$10=x^2+2x$

$x^2+2x-10=0$

$(x+1)^2-1-10=0$

$(x+1)^2-11=0$

$(x+1-\sqrt{11})(x+1+\sqrt{11})=0$

$x=-1+\sqrt{11}$, $x=-1-\sqrt{11}$

(e) $x^4-13x^2+36=0$

Let $m=x^2$

$\therefore m^2-13m+36=0$

$\begin{array}{cc} m & -9 \\ m & -4 \end{array}$

$(m-9)(m-4)=0$

$m=9$, $m=4$

$x^2=9$, $x^2=4$

$x=\pm 3$, $x=\pm 2$

(f) $\sqrt{x+3}=x-3$

$x+3=(x-3)^2$

$x+3=x^2-6x+9$

$x^2-7x+6=0$

$\begin{array}{cc} x & -6 \\ x & -1 \end{array}$

$(x-6)(x-1)=0$

$x=6$, $x=1$

---

(2) $x+y=3 \Rightarrow y=3-x$

$\frac{1}{x}+\frac{1}{y}=\frac{27}{14}$

$\frac{1}{x}+\frac{1}{3-x}=\frac{27}{14}$

$\frac{3-x+x}{x(3-x)}=\frac{27}{14}$

$\frac{3}{x(3-x)}=\frac{27}{14}$

$42=27x(3-x)$

$42=81x-27x^2$

$27x^2-81x+42=0$

$9x^2-27x+14=0$

$\begin{array}{cc} 3x & -7 \\ 3x & -2 \end{array}$

$(3x-7)(3x-2)=0$

$x=\frac{7}{3}$, $x=\frac{2}{3}$

$x=\frac{7}{3}$, $y=3-\frac{7}{3}=\frac{2}{3}$

$x=\frac{2}{3}$, $y=\frac{7}{3}$

The two numbers are $\frac{2}{3}$, $\frac{7}{3}$

---

(3) (a) $\sqrt{2}x^2+\sqrt{3}x-\sqrt{5}=0$

$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$

$=\frac{-\sqrt{3}\pm\sqrt{3+4(\sqrt{2}\sqrt{5})}}{2\sqrt{2}}$

$=\frac{-\sqrt{3}\pm\sqrt{3+4\sqrt{10}}}{2\sqrt{2}}$

(b) $\pi x^2 + \sqrt{2}x - 2 = 0$

$a = \pi, \quad b = \sqrt{2}, \quad c = -2$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-\sqrt{2} \pm \sqrt{2 - 4\pi(-2)}}{2\pi}$$

$$= \frac{-\sqrt{2} \pm \sqrt{2 + 8\pi}}{2\pi}$$

---

(4) (a) $x^2 - 2x - 4 = 0$

$\Delta = b^2 - 4ac$

$= (-2)^2 - 4(1)(-4)$

$= 4 + 16$

$= 20 > 0$

$\therefore$ 2 solutions

(b) $2x^2 - 2x + 3 = 0$

$\Delta = b^2 - 4ac$

$= (-2)^2 - 4(2)(3)$

$= 4 - 24$

$= -20 < 0$

$\therefore$ No solution

(c) $25x^2 - 20x + 4 = 0$

$\Delta = b^2 - 4ac$

$= (-20)^2 - 4(25)(4)$

$= 400 - 400$

$= 0$

$\therefore$ 1 solution

---

(5) $kx^2 - x - k = 0$

$\Delta = b^2 - 4ac$

$= (-1)^2 - 4(k)(-k)$

$= 1 + 4k^2 > 0 \quad \therefore$ 2 solutions

---

(6) (a) $y = x^2 + 1$

(b) $y = (x+2)^2$

(c) $y = (x-3)^2$

(d) $y = x^2 - 4$

(e) $y = -x^2$

(f) $y = x^2$

(g) $y = 2x^2$

(h) $y = \frac{1}{3}x^2$

(i) $y = \left(\frac{1}{4}x\right)^2 = \frac{1}{16}x^2$

(j) $y = (5x)^2 = 25x^2$

---

7 (a) $y = 2(x+1)^2$

- dilation factor 2 from the x-axis
- translate by 1 unit left

(b) $y = -x^2 + 2$

- reflect in the x-axis
- translate 2 units up

(c) $y = 3(x-2)^2 - 3$

- dilate by factor 3 from the x-axis
- translate 3 units right
- translate 3 units down

(d) $y = -(x-3)^2 - 2$

- reflect in the x-axis
- translate 3 units right
- translate 2 units down

---

(8) (a) $y = x^2 + 2x - 7$

$y = (x+1)^2 - 1 - 7$

$y = (x+1)^2 - 8$

$TP = (-1, -8)$

(b) $y = 2x^2 + 4x - 9$

$y = 2[x^2 + 2x] - 9$

$y = 2[(x+1)^2 - 1] - 9$

$y = 2(x+1)^2 - 2 - 9$

8(b) continued

$y = 2(x+1)^2 - 11$

$TP = (-1, -11)$

(c) $y = -x^2 + 6x - 9$

$y = -[x^2 - 6x + 9]$

$= -[(x-3)^2 - 9 + 9]$

$= -(x-3)^2$

$TP = (3, 0)$

(d) $y = -2x^2 + x - 9$

$= -2\left[x^2 - \frac{1}{2}x\right] - 9$

$= -2\left[\left(x - \frac{1}{4}\right)^2 - \frac{1}{16}\right] - 9$

$= -2\left(x - \frac{1}{4}\right)^2 + \frac{1}{8} - \frac{72}{8}$

$= -2\left(x - \frac{1}{4}\right)^2 - \frac{71}{8}$

$TP = \left(\frac{1}{4}, \frac{-71}{8}\right)$

9 (a) $y = \frac{1}{3}x^2 + \frac{2}{4}x - 7$

$x_{TP} = -\frac{b}{2a} = \frac{-\frac{2}{4}}{2 \times \frac{1}{3}}$

$= \frac{-\frac{1}{2}}{\frac{2}{3}}$

$= -\frac{1}{2} \times \frac{3}{2}$

$= -\frac{3}{4}$

(b) $y = \sqrt{2}x^2 + \sqrt{10}x + 1$

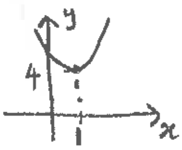
$x_{TP} = -\frac{b}{2a} = -\frac{\sqrt{10}}{2\sqrt{2}} = \frac{-\sqrt{5}}{2}$

(10) Let $y = x^2 - 2x + 5$

$= (x-1)^2 - 1 + 5$

$= (x-1)^2 + 4$

Min value $= 4$



11 (a) $y = x^2 - 6x + 8$

$y = (x-4)(x-2)$

Y int: $x = 0$, $y = 8$

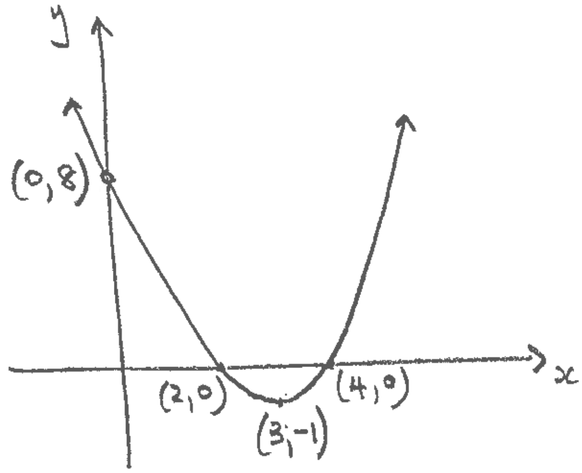
X int: $y = 0$, $(x-4)(x-2) = 0$

$x = 4, 2$

$x_{TP} = \frac{2+4}{2} = 3$

$y_{TP} = (3-4)(3-2) = -1 \times 1 = -1$

$TP = (3, -1)$



(b) $y = x^2 + 3x + 2$

$y = (x+2)(x+1)$

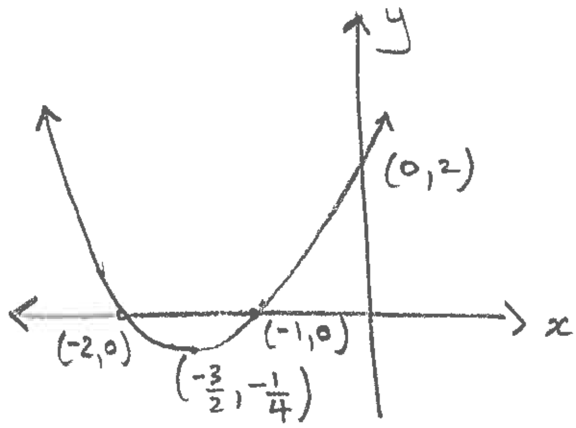
Y int: $x = 0$, $y = 2$

X int: $y = 0$, $(x+2)(x+1) = 0$

$x = -2, -1$

$x_{TP} = \frac{-2-1}{2} = \frac{-3}{2}$

$y_{TP} = \left(\frac{-3}{2} + 2\right)\left(\frac{-3}{2} + 1\right) = \frac{1}{2} \times -\frac{1}{2} = -\frac{1}{4}$



11(c) $y = 4x^2 - 4x - 3$

$2x \quad -3$

$2x \quad 1$

$y = (2x-3)(2x+1)$

Yint: $x = 0,\ y = -3$

Xint: $y = 0,\ (2x-3)(2x+1) = 0$

$x = \frac{3}{2},\ x = -\frac{1}{2}$

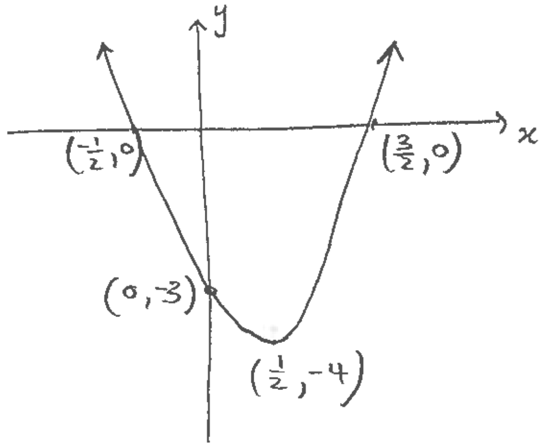
$x_{TP} = \dfrac{\frac{3}{2} + -\frac{1}{2}}{2} = \frac{1}{2}$

$y_{TP} = \left(2\left(\tfrac{1}{2}\right) - 3\right)\left(2\left(\tfrac{1}{2}\right) + 1\right)$

$= -2 \times 2$

$= -4$

$TP = \left(\frac{1}{2}, -4\right)$



(d) $y = -4x^2 + 8x - 2$

Yint: $x = 0,\ y = 2$

Xint: $y = 0,\ -4x^2 + 8x - 2 = 0$

$2x^2 - 4x + 1 = 0$

$2[x^2 - 2x] + 1 = 0$

$2[(x-1)^2 - 1] + 1 = 0$

$2(x-1)^2 - 2 + 1 = 0$

$2(x-1)^2 - 1 = 0$

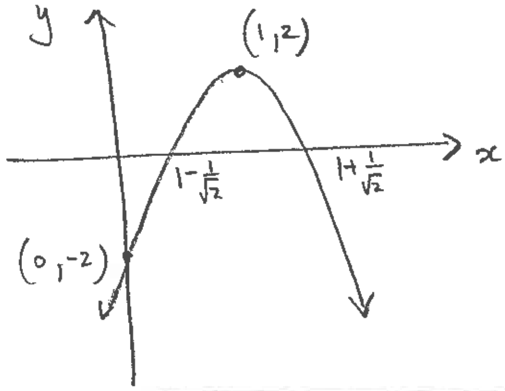
$(x-1)^2 = \frac{1}{2}$

$x - 1 = \pm\frac{1}{\sqrt{2}}$

$x = 1 \pm \frac{1}{\sqrt{2}}$

$\therefore x_{TP} = 1$

$y_{TP} = -4 + 8 - 2 = 2 \qquad TP = (1, 2)$



(12) $x_{TP} = \dfrac{a+b}{2} = 2$

$\therefore a + b = 4 \quad ① \Rightarrow b = 4 - a$

when $x = 2,\ y = -8$

$(2-a)(2-b) = -8 \quad — ②$

sub $b = 4 - a$ into ②

$(2-a)(2-(4-a)) = -8$

$(2-a)(2-4+a) = -8$

$(2-a)(-2+a) = -8$

$-4 + 2a + 2a - a^2 = -8$

$a^2 - 4a - 4 = 0$

$(a-2)^2 - 4 - 4 = 0$

$(a-2)^2 = 8$

$a - 2 = \pm\sqrt{8}$

$a = 2 \pm 2\sqrt{2}$

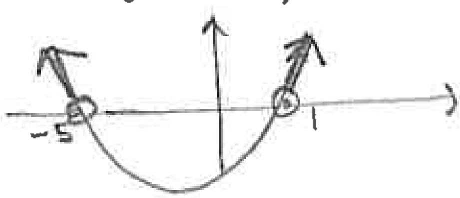
$a = 2 + 2\sqrt{2},\ b = 4 - (2 + 2\sqrt{2})$

$= 2 - 2\sqrt{2}$

$a = 2 + 2\sqrt{2},\ b = 2 - 2\sqrt{2}$

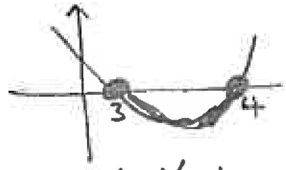
$a = 2 - 2\sqrt{2},\ b = 2 + 2\sqrt{2}$

13(a) $(x-1)(x+5) > 0$



$x < -5$ or $x > 1$

(b) $(x-3)(x-4) \leq 0$

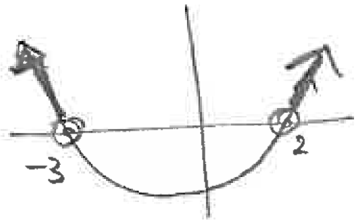


$3 \leq x \leq 4$

13(c) $x^2+x>6$

$x^2+x-6>0$

$(x+3)(x-2)>0$



$x<-3$ or $x>2$

(d) $-2x^2+9x-9\geqslant 0$

$2x^2-9x+9\leqslant 0$

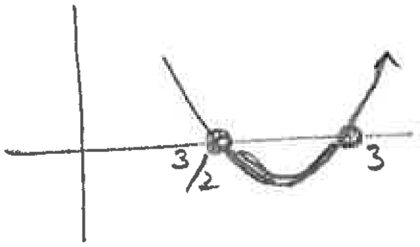
Consider $y=2x^2-9x+9$

Xint: $y=0$, $2x^2-9x+9=0$

$$\begin{array}{ccc} 2x & \times & -3 \\ x & & -3 \end{array}$$

$(2x-3)(x-3)=0$

$x=\frac{3}{2}$, $x=3$



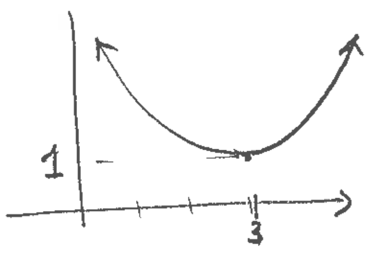
$\frac{3}{2}\leqslant x\leqslant 3$

(e) $x^2-6x+10>0$

Consider $y=x^2-6x+10$

$=(x-3)^2-9+10$

$y=(x-3)^2+1$



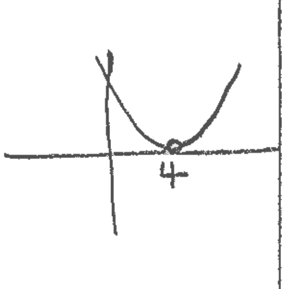
$x\in R$

(f) $x^2\leq 8x-16$

$x^2-8x+16\leq 0$

$(x-4)(x-4)\leq 0$

$x=4$



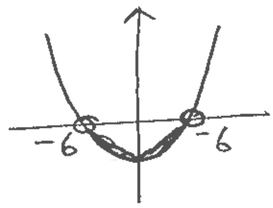
14 (a) $x^2+kx+9=0$

$\Delta=b^2-4ac$

$=k^2-4(9)$

$=k^2-36$

Let $k^2-36<0$



$-6<k<6$

(b) $kx^2+kx+1=0$

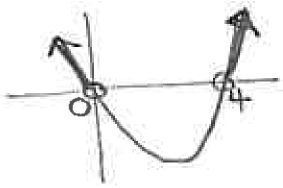
$\Delta=b^2-4ac$

$=k^2-4(k)$

$=k^2-4k$

Let $k^2-4k>0$

$k(k-4)>0$



$k<0$ or $k>4$

---

(15) $y=x^2+2$ ①

$y=kx-2$ ②

Let $x^2+2=kx-2$

$x^2-kx+4=0$

$\Delta=0$

$\Delta=b^2-4ac$

$=(-k)^2-4(1)(4)$

$=k^2-16$

Let $k^2-16=0$

$k^2=16$

$k=\pm 4$

Two possible lines

$y=4x-2$ or $y=-4x-2$

Also the vertical line $x=0$

(16) $y = mx$ ①

$y = (x-1)^2 + 1$ ②

Point(s) of intersection

$(x-1)^2 + 1 = mx$

$x^2 - 2x + 1 + 1 = mx$

$x^2 - 2x - mx + 2 = 0$

$x^2 - (2+m)x + 2 = 0$

$\Delta = b^2 - 4ac$

$= (-(2+m))^2 - 4(2)$

$= (2+m)^2 - 8$

$= 4 + 4m + m^2 - 8$

$= m^2 + 4m - 4$

Let $m^2 + 4m - 4 = 0$

$(m+2)^2 - 4 - 4 = 0$

$(m+2)^2 - 8 = 0$

$(m + 2 - \sqrt{8})(m + 2 + \sqrt{8}) = 0$

$m = -2 + \sqrt{8}, \; m = -2 - \sqrt{8}$

$m = -2 + 2\sqrt{2}, \; m = -2 - 2\sqrt{2}$

---

(17) $y = x^2 + 2x + 1$ ①

$y = mx + c$ ②

$x = 1, y = 4 \;\; \therefore \; 4 = m + c$

$\therefore \; c = 4 - m$

② $y = mx + 4 - m$

Point of intersection

Let $x^2 + 2x + 1 = mx + 4 - m$

$x^2 + (2-m)x - 3 + m = 0$

$\Delta = 0$

$b^2 - 4ac = 0$

$(2-m)^2 - 4(m-3) = 0$

$4 - 4m + m^2 - 4m + 12 = 0$

$m^2 - 8m + 16 = 0$

$(m-4)^2 = 0$

$m = 4$

$m = 4, \; c = 4 - m = 0$

$\therefore$ Tangent is $y = 4x$

---

(18) $h = -10t^2 + 15t + 25, \; t \geq 0$

(a) Yint: $t = 0, h = 25$

Xint: $h = 0$

$-10t^2 + 15t + 25 = 0$

$(\div -5)$ $2t^2 - 3t - 5 = 0$

$2t \quad -5$

$t \quad\;\; 1$

$(2t-5)(t+1) = 0$

$t = \frac{5}{2}, \; t = -1$

$\therefore t = \frac{5}{2}$ since $t \geq 0$

$x_{TP} = \dfrac{\frac{5}{2} + -1}{2} = \dfrac{\frac{3}{2}}{2} = \frac{3}{4}$

$y_{TP} = -10\left(\frac{3}{4}\right)^2 + 15\left(\frac{3}{4}\right) + 25$

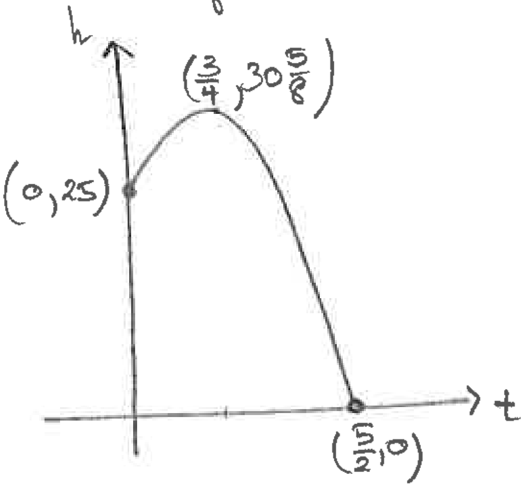
$= -\cancel{10}^{5} \times \frac{9}{\cancel{16}_{8}} + \frac{45}{4} + 25$

$= -\frac{45}{8} + \frac{90}{8} + \frac{200}{8}$

$= \frac{245}{8}$

$= 30\frac{5}{8}$

$8 \overline{)245} = 30\frac{5}{8}$



(b) 25 m

(c) $2\frac{1}{2}$ seconds

(d) Let $-10t^2 + 15t + 25 = 10$

$2t^2 - 3t - 5 = -2$

$2t^2 - 3t - 3 = 0$

$t = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$= \dfrac{3 \pm \sqrt{9 - 4(2)(-3)}}{4}$

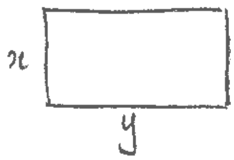
$= \dfrac{3 \pm \sqrt{33}}{4}$

18 (d) $\frac{3+\sqrt{33}}{4}$ seconds

(e) $30\frac{5}{8}$ metres

---

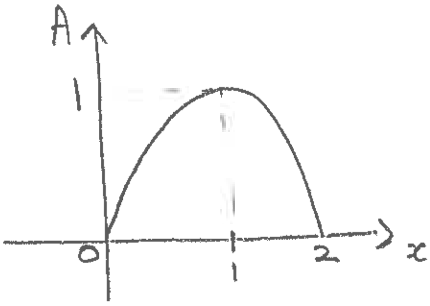
(19)



$$2x + 2y = 4$$
$$\therefore x + y = 2$$
$$\therefore y = 2 - x \quad ①$$
$$A = xy$$
$$= x(2-x)$$
$$= 2x - x^2$$

x int: $x = 0, 2$

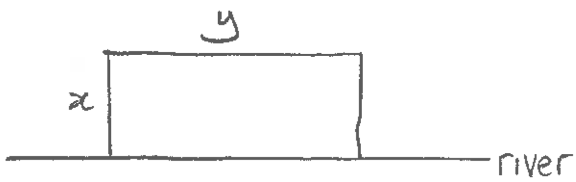
$$x_{TP} = \frac{0+2}{2} = 1$$
$$A_{TP} = 1(2-1) = 1$$



Max Area = 1 m^2

---

(20)



$$2x + y = 360$$
$$y = 360 - 2x$$
$$A = xy$$
$$= x(360 - 2x)$$
$$= 2x(180 - x)$$

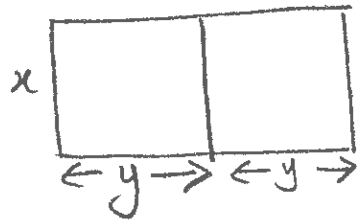
x-int; $x = 0, 180$

$$x_{TP} = \frac{180}{2} = 90$$
$$A_{TP} = 2(90)(180-90) = 2 \times 8100$$
$$= 16200 \text{ m}^2$$
$$x = 90, y = 360 - 2 \times 90 = 180$$

$\therefore$ Max area is 16200 m^2 obtained when $x = 90$, $y = 180$

---

(21)



$$3x + 4y = 90 \Rightarrow y = \frac{90-3x}{4}$$

Let A = area of one paddock

$$\therefore A = xy$$
$$= x\left(\frac{90-3x}{4}\right)$$
$$= \frac{1}{4}x(90-3x)$$

x-int: $x = 0$, $x = 30$

$$x_{TP} = \frac{0+30}{2} = 15$$
$$A_{TP} = \frac{1}{4} \times 15 \times (90 - 45)$$
$$= \frac{15 \times 45}{4}$$
$$= 168.75$$

Max Area = 168.75 m^2

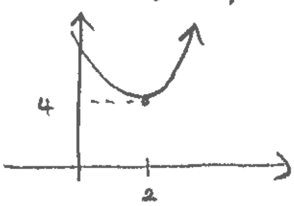
---

(22) $x + y = 2$

$$\Rightarrow y = 2 - x$$

Let $z = x^2 + 4y$

$$= x^2 + 4(2-x)$$
$$= x^2 - 4x + 8$$
$$= (x-2)^2 - 4 + 8$$
$$= (x-2)^2 + 4$$

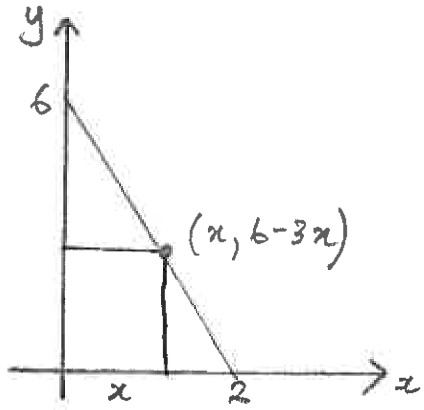


Minimum value = 4

(23) $y = 6 - 3x$

Y int : $y = 6$

X int : $6 - 3x = 0 \Rightarrow x = 2$

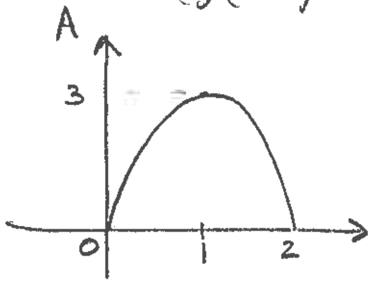


$$A = xy$$
$$= x(6-3x)$$
$$= 3x(2-x)$$

X int : $x = 0$, $x = 2$

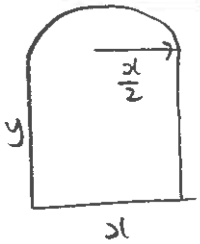
$$x_{TP} = \frac{0+2}{2} = 1$$

$$A_{TP} = 3(1)(2-1) = 3$$



Max Area = 3 units^2

(24)



$$2y + x + \frac{\pi x}{2} = 1$$
$$2y = 1 - x - \frac{\pi x}{2}$$
$$2y = \frac{2 - 2x - \pi x}{2}$$
$$y = \frac{2 - 2x - \pi x}{4}$$

$$A = xy + \frac{1}{2}\pi\left(\frac{x}{2}\right)^2$$
$$= x\left(\frac{2-2x-\pi x}{4}\right) + \frac{\pi x^2}{8}$$
$$= \frac{2x(2-2x-\pi x) + \pi x^2}{8}$$
$$= \frac{4x - 4x^2 - 2\pi x^2 + \pi x^2}{8}$$
$$= \frac{4x - 4x^2 - \pi x^2}{8}$$

X int : $\frac{4x - 4x^2 - \pi x^2}{8} = 0$

$$4x - 4x^2 - \pi x^2 = 0$$
$$x(4 - 4x - \pi x) = 0$$
$$x = 0, \quad 4 - 4x - \pi x = 0$$
$$\therefore x = 0, \quad 4 = x(4+\pi)$$
$$x = 0, \quad \frac{4}{4+\pi}$$

$$x_{TP} = \frac{0 + \frac{4}{4+\pi}}{2} = \frac{2}{4+\pi}$$

$$y = \frac{2 - 2 \times \frac{2}{4+\pi} - \pi \times \frac{2}{4+\pi}}{4}$$

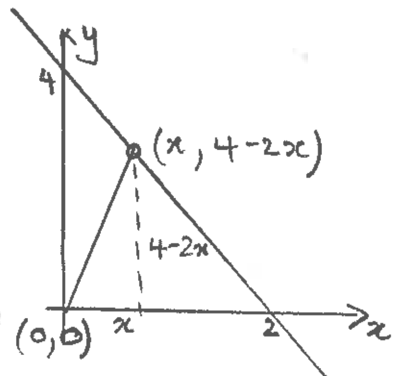
$$= \frac{2 - \frac{4}{4+\pi} - \frac{2\pi}{4+\pi}}{4}$$

$$= \frac{\frac{2(4+\pi) - 4 - 2\pi}{4+\pi}}{4}$$

$$= \frac{4}{4(4+\pi)}$$

$$= \frac{1}{4+\pi}$$

(25) $y = 4 - 2x$



$$D^2 = x^2 + (4-2x)^2$$
$$= x^2 + 16 - 16x + 4x^2$$
$$= 5x^2 - 16x + 16$$
$$= 5\left[x^2 - \frac{16}{5}x\right] + 16$$
$$= 5\left[\left(x - \frac{8}{5}\right)^2 - \frac{64}{25}\right] + 16$$
$$= 5\left(x - \frac{8}{5}\right)^2 - \frac{64}{5} + 16$$
$$= 5\left(x - \frac{8}{5}\right)^2 - \frac{64}{5} + \frac{80}{5}$$
$$= 5\left(x - \frac{8}{5}\right)^2 + \frac{16}{5}$$

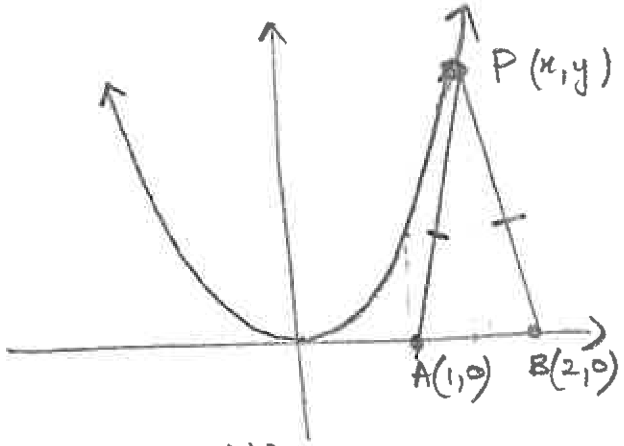
Min TP occurs when $x = \frac{8}{5}$

$x = \frac{8}{5}$, $y = 4 - 2 \times \frac{8}{5} = 4 - \frac{16}{5} = \frac{4}{5}$

Point closest to origin is

$\left(\frac{8}{5}, \frac{4}{5}\right)$

(26)



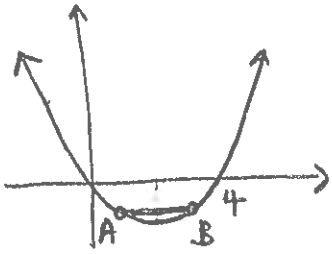
At P, $x = \frac{1+2}{2} = \frac{3}{2}$

$\therefore y = x^2 = \left(\frac{3}{2}\right)^2 = \frac{9}{4}$

$P = \left(\frac{3}{2}, \frac{9}{4}\right)$

---

(27) $y = x(x-4)$



At A, let $x = a$

$\therefore y = a(a-4)$

$A(a, a(a-4))$

$\therefore B(4-a, a(a-4))$

$\therefore 4-a-a = 2$

$4-2a = 2$

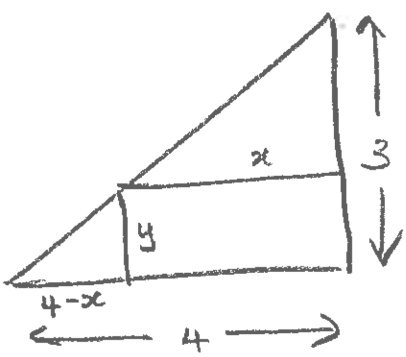
$a = 1$

$x = 1, y = 1(1-4) = -3$

$A = (1, -3)$

---

(28)



Using similar triangles

$\frac{y}{4-x} = \frac{3}{4}$

$4y = 3(4-x)$

$4y = 12-3x$

$y = \frac{12-3x}{4}$

$A = xy$

$= x\left(\frac{12-3x}{4}\right)$

$= \frac{1}{4}x(12-3x)$

$= \frac{3}{4}x(4-x)$

X int: $x = 0, 4$

$x_{TP} = \frac{0+4}{2} = 2$

$A_{TP} = \frac{3}{4}(2)(2) = 3$

$x = 2, y = \frac{12-3(2)}{4} = \frac{3}{2}$

Max Area $= 3$

---

(29) (a) $\sqrt{x+a} = x-a$

$x+a = (x-a)^2$

$x+a = x^2 - 2xa + a^2$

$x^2 - 2xa - x + a^2 - a = 0$

$x^2 - (2a+1)x + a^2 - a = 0$

$\Delta = b^2 - 4ac$

$= (2a+1)^2 - 4(a^2-a)$

$= 4a^2 + 4a + 1 - 4a^2 + 4a$

$= 8a+1$

Let $8a+1 > 0$

$a > -\frac{1}{8}$

29 (b) For a quadratic equation with 2 solutions the solutions are

$$\frac{-b+\sqrt{b^2-4ac}}{2a} \text{ and } \frac{-b-\sqrt{b^2-4ac}}{2a}$$

So the sum of the solutions is

$$\frac{-b+\sqrt{b^2-4ac}}{2a} + \frac{-b-\sqrt{b^2-4ac}}{2a}$$

$$= \frac{-b+\sqrt{b^2-4ac} -b-\sqrt{b^2-4ac}}{2a}$$

$$= \frac{-2b}{2a}$$

$$= \frac{-b}{a}$$

So for the equation

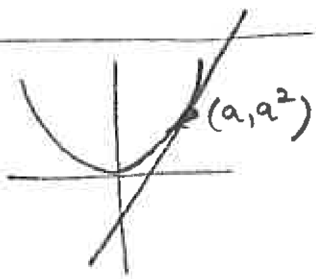
$$x^2-(2a+1)x+a^2-a=0$$

the sum of the solutions is

$$-\frac{-(2a+1)}{1} = 2a+1$$

---

(30) Let equation of tangent be



$y = mx+c$

$x=a,\ y=a^2$

$a^2 = ma+c$

$c = a^2-ma$

$\therefore y = mx+a^2-ma$ ①

$y = x^2$ ②

At point of intersection

$x^2 = mx+a^2-ma$

$x^2-mx-a^2+ma=0$

$\Delta = b^2-4ac$

$= (-m)^2-4(ma-a^2)$

$= m^2-4ma+4a^2$

tangent $\Rightarrow$ 1 point of intersection $\Rightarrow \Delta=0$

$\therefore m^2-4ma+4a^2=0$

$(m-2a)^2=0$

$m=2a$

$\therefore$ Now from ① the equation of the tangent line is

$y = mx+a^2-ma$

Yint: $x=0,\ y=a^2-ma$

$\therefore y = a^2-2a\times a$

$= a^2-2a^2$

$= -a^2$

$\therefore$ Y int $= (0,-a^2)$

(b) $A(a,a^2)$ $B(0,-a^2)$

$$m_{AB} = \frac{a^2--a^2}{a-0} = \frac{2a^2}{a} = 2a$$

$\therefore$ Eq of line through AB is

$y = 2ax-a^2$ ①

$y = x^2$ ②

At point of intersection

$x^2 = 2ax-a^2$

$x^2-2ax+a^2=0$

$\Delta = b^2-4ac$

$= (-2a)^2-4(a^2)$

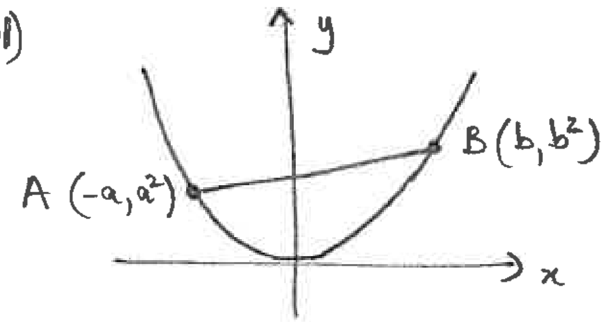
$= 4a^2-4a^2$

$= 0$

$\therefore y=2ax-a^2$ only touches the parabola $y=x^2$

$\therefore$ It must be tangent.

(31)



$$m_{AB} = \frac{b^2-a^2}{b+a} = \frac{(b-a)(b+a)}{b+a} = b-a$$

$$y-y_1 = m(x-x_1)$$

$$y-a^2 = (b-a)(x - {}^-a)$$

$$y-a^2 = (b-a)(x+a)$$

Y int: Let $x=0$

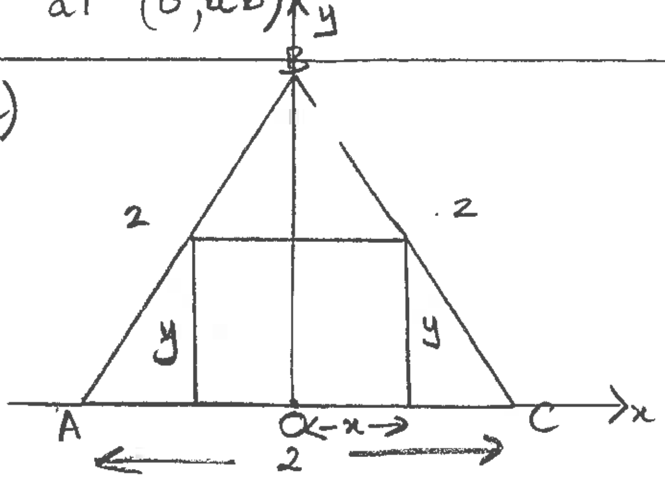
$$y-a^2 = (b-a)(a)$$

$$\therefore y = ba - a^2 + a^2$$

$$y = ba$$

$\therefore$ AB intersects the y-axis at $(0,ab)$

(32)



$$OB = \sqrt{4-1} = \sqrt{3}$$

$$\frac{OB}{y} = \frac{1}{1-x} \quad \text{(Similar } \Delta\text{s)}$$

$$\frac{\sqrt{3}}{y} = \frac{1}{1-x}$$

$$y = \sqrt{3}(1-x)$$

$$A = 2xy$$

$$= 2x\sqrt{3}(1-x)$$

X int: $x=0$, $x=1$

$$x_{TP} = \frac{0+1}{2} = \frac{1}{2}$$

$$\therefore A_{TP} = 2\left(\tfrac{1}{2}\right)\sqrt{3}\left(1-\tfrac{1}{2}\right)$$

$$= \frac{\sqrt{3}}{2}$$

Maximum Area $= \frac{\sqrt{3}}{2}$

(33) (a) $(a-b)^2 \geq 0$

$$\therefore a^2 - 2ab + b^2 \geq 0$$

$$\therefore a^2 + 2ab + b^2 \geq 4ab \quad \text{(+4ab both sides)}$$

$$\therefore (a+b)^2 \geq 4ab$$

$$\therefore a+b \geq 2\sqrt{ab} \quad \text{since } a,b \geq 0$$

$$\therefore \frac{a+b}{2} \geq \sqrt{ab}$$

(b) $x, \frac{1}{x} \geq 0$

So from (a)

$$\frac{x+\frac{1}{x}}{2} \geq \sqrt{x \times \frac{1}{x}}$$

$$\frac{x+\frac{1}{x}}{2} \geq \sqrt{1}$$

$$\frac{x+\frac{1}{x}}{2} \geq 1$$

$$x + \frac{1}{x} \geq 2$$