Probability Exercise Solutions

1 (a) $\xi = \{R, B, G\}$

(b) $\xi = \{2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12\}$

(c) $\xi = \{1, 2, 3, 4, 5, 6, 8, 9, 10, 12, 15, 16, 18, 20, 24, 25, 30, 36\}$

(d) $\xi = \{HHH, HHT, HTH, THH, HTT, THT, TTH, TTT\}$

---

(2) $Pr(\text{Vowel}) = \frac{2}{7}$

---

(3)

	1	2	3	4	5	6
1	(1,1)	(1,2)	(1,3)	(1,4)	(1,5)	(1,6)
2	(2,1)	(2,2)	(2,3)	(2,4)	(2,5)	(2,6)
3	(3,1)	(3,2)	(3,3)	(3,4)	(3,5)	(3,6)
4	(4,1)	(4,2)	(4,3)	(4,4)	(4,5)	(4,6)
5	(5,1)	(5,2)	(5,3)	(5,4)	(5,5)	(5,6)
6	(6,1)	(6,2)	(6,3)	(6,4)	(6,5)	(6,6)

(a) $Pr(6) = \frac{4}{36} = \frac{1}{9}$

(b) $Pr(\text{Sum} = 5) = \frac{4}{36} = \frac{1}{9}$

(c) $Pr(\text{Diff} = 1) = \frac{10}{36} = \frac{5}{18}$

(d) $Pr(\text{Diff} \leq 2) = \frac{24}{36} = \frac{2}{3}$

---

(4)

Product	Other 2	Arrange
1	1,1	1
2	1,2	3
3	1,3	3
4	1,4	3
4	2,2	3
5	1,5	3
6	1,6	3
6	2,3	6
		25

Total # Arrangements $= 6 \times 6 \times 6 = 216$

Prob $= \frac{25}{216}$

---

(5)

Largest	Other	Arrangements
3	1,1,1	4
4	2,1,1	12
5	1,2,2	12
5	3,1,1	12
6	1,1,4	12
6	1,2,3	24
6	2,2,2	4
		80

Total # Outcomes $= 6^4 = 1296$

$$\text{Prob} = \frac{80}{1296} = \frac{5}{81}$$

---

(6) (a) $Pr(A) = \frac{4}{10} = \frac{2}{5}$

(b) $Pr(B) = \frac{5}{10} = \frac{1}{2}$

(c) $A \cap B = \{5, 7\}$

$Pr(A \cap B) = \frac{2}{10} = \frac{1}{5}$

(d) $A \cup B = \{1, 2, 5, 6, 7, 8, 9\}$

$Pr(A \cup B) = \frac{7}{10}$

Also $Pr(A \cup B) = Pr(A) + Pr(B) - Pr(A \cap B)$

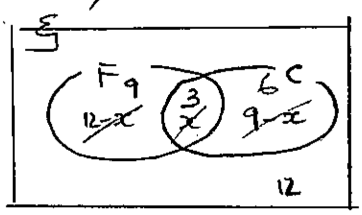
$$= \frac{2}{5} + \frac{1}{2} - \frac{1}{5}$$

$$= \frac{4 + 5 - 2}{10}$$

$$= \frac{7}{10} \checkmark$$

---

(7) 12 F, 9 C, 12 neither



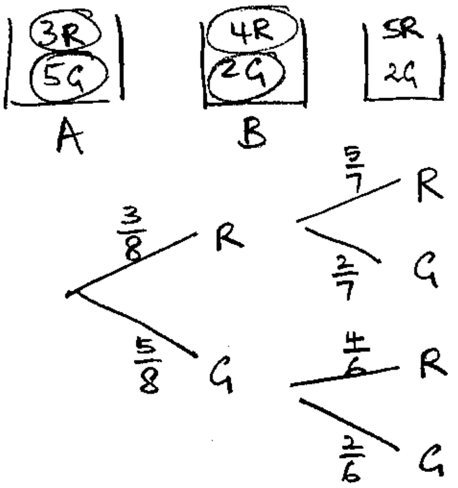
$12 - x + x + 9 - x + 12 = 30$

$33 - x = 30$

$x = 3$

$$Pr(F \cap C') = \frac{9}{30} = \frac{3}{10}$$

(8)



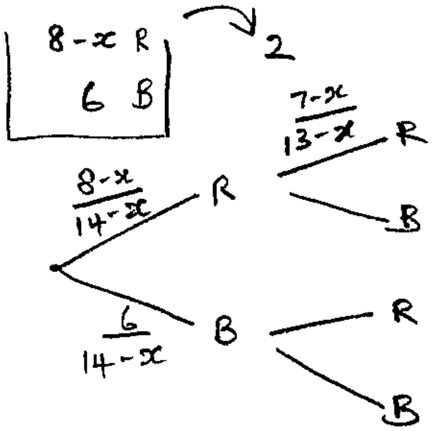
$Pr(RG) + Pr(GR)$

$= \frac{3}{\cancel{8}_4} \times \frac{\cancel{2}^1}{7} + \frac{5}{\cancel{8}_4} \times \frac{\cancel{2}^1}{3}$

$= \frac{3}{28} + \frac{5}{12}$

$= \frac{11}{21}$

(9) Let x = # red removed



$Pr(RR) = \frac{2}{15}$

$\frac{8-x}{14-x} \times \frac{7-x}{13-x} = \frac{2}{15}$

$15(8-x)(7-x) = 2(14-x)(13-x)$

$15(56 - 15x + x^2) = 2(182 - 27x + x^2)$

$840 - 225x + \cancel{15}x^2 = 364 - 54x + \cancel{2}x^2$

$13x^2 - 171x + 476 = 0$

$13x \quad -119$

$x \quad -4$

$(13x - 119)(x - 4) = 0$

$x = \frac{119}{13}, 4$

$\therefore x = 4$

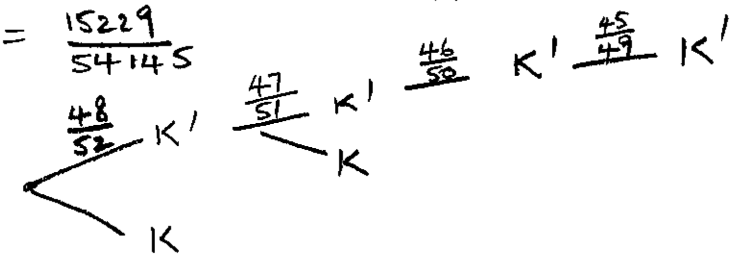
Four red removed

(10) Pr(At least 1 King)

$= 1 - Pr$(No Kings)

$= 1 - \frac{48}{52} \times \frac{47}{51} \times \frac{46}{50} \times \frac{45}{49}$

$= \frac{15229}{54145}$



(11) Pr(Even) $= 1 - Pr$(odd)

$= 1 - Pr$(All odd #s)

$= 1 - \left(\frac{1}{2}\right)^5$

$= 1 - \frac{1}{32}$

$= \frac{31}{32}$

(12) Pr(Mult 3) $= Pr$(Atleast one 3 or 6)

$= 1 - Pr$(No 3s or 6s)

$= 1 - \left(\frac{4}{6}\right)^5$

$= 1 - \left(\frac{2}{3}\right)^5$

$= 1 - \frac{32}{243}$

$= \frac{211}{243}$

(13) Let n = # tosses req

Pr(At least one 6) > 0.95

$1 - Pr$(No 6s) > 0.95

$1 - \left(\frac{5}{6}\right)^n > 0.95$

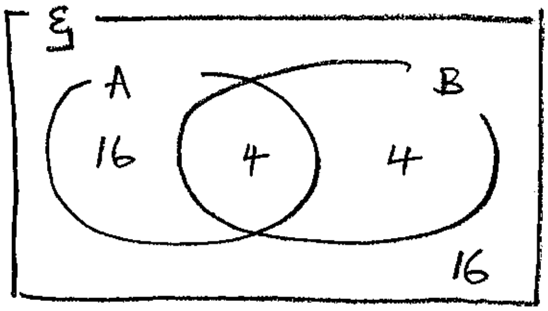
$\left(\frac{5}{6}\right)^n < 0.05$

$\left(\frac{5}{6}\right)^{16} = 0.054$

$\left(\frac{5}{6}\right)^{17} = 0.045$

$\therefore$ 17 rolls of the die

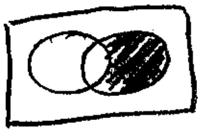
(14)

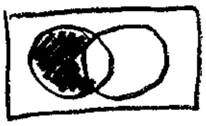


$n(A) = 20 \qquad n(B) = 8$

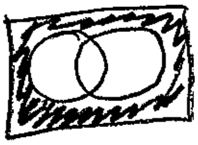
(a) 

$Pr(A \cap B) = \frac{4}{40} = \frac{1}{10}$

(b)  $Pr(A' \cap B) = \frac{4}{40} = \frac{1}{10}$

(c)  $Pr(A \cap B') = \frac{16}{40} = \frac{2}{5}$

(d)  $Pr(A' \cap B') = \frac{16}{40} = \frac{2}{5}$

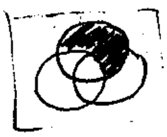
(e)  $Pr((A \cup B)') = \frac{2}{5}$

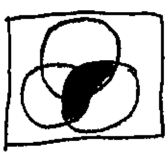
(f)  $Pr(A' \cup B') = \frac{36}{40} = \frac{9}{10}$

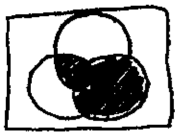
(15)

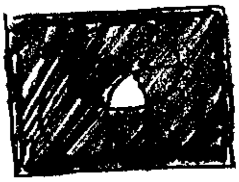
(a)  $Pr(A) = \frac{15}{40} = \frac{3}{8}$

(b)  $Pr(A \cup B) = \frac{24}{40} = \frac{3}{5}$

(c)  $Pr(A \cap B') = \frac{9}{40}$

(d)  $Pr((A \cup B) \cap C) = \frac{12}{40} = \frac{3}{10}$

(e)  $Pr((A \cap B) \cup C) = \frac{21}{40}$

(f)  $Pr((A \cap B \cap C)') = \frac{37}{40}$

(16)

	E	E′	
M	0·5	0·3	0·8
M′	0·1	0·1	0·2
	0·6	0·4	1

(a) $Pr(M \cap E') = 0.3$

(b) $Pr(M \cup E')$

$= Pr(M) + Pr(E') - Pr(M \cap E')$

$= 0.8 + 0.4 - 0.3$

$= 0.9$

(17)

	C	C′	
T	x	3	$x+3$
T′	3	6	9
	$x+3$	9	$x+12$

$Pr(C \cap T) = \frac{2}{5}$

$\frac{x}{x+12} = \frac{2}{5}$

$5x = 2x + 24$

$3x = 24$

$x = 8$

(18) $Pr(A|B)$

$= \frac{Pr(A \cap B)}{Pr(B)}$

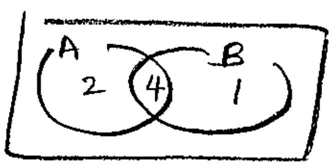
$= \frac{4}{5}$

$Pr(B|A)$

$= \frac{Pr(B \cap A)}{Pr(A)}$

$= \frac{4}{6}$

$= \frac{2}{3}$



(19)

	A	A'	
B	0·1	0·3	0·4
B'	0·4	0·2	0·6
	0·5	0·5	1

(a) $P_r(A|B)$

$= \dfrac{P_r(A\cap B)}{P_r(B)}$

$= \dfrac{0\cdot1}{0\cdot4}$

$= \dfrac{1}{4}$

(b) $P_r(B|A')$

$= \dfrac{P_r(B\cap A')}{P_r(A')}$

$= \dfrac{0\cdot3}{0\cdot5}$

$= \dfrac{3}{5}$

(20) $P_r(A\cap B) = P_r(A|B)P_r(B)$

$= 0\cdot3 \times 0\cdot2$

$= 0\cdot06$

$\therefore P_r(B|A) = \dfrac{P_r(B\cap A)}{P_r(A)}$

$= \dfrac{0\cdot06}{0\cdot4}$

$= \dfrac{6}{40}$

$= \dfrac{3}{20}$

(21) $P_r(B|A) = \dfrac{P_r(A\cap B)}{P_r(A)}$

$0\cdot6 = \dfrac{0\cdot3}{P_r(A)}$

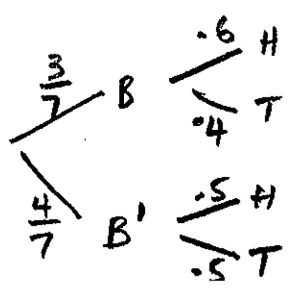
$P_r(A) = \dfrac{0\cdot3}{0\cdot6} = 0\cdot5$

$P_r(A\cup B) = P_r(A) + P_r(B) - P_r(A\cap B)$

$= 0\cdot5 + 0\cdot7 - 0\cdot3$

$= 0\cdot9$

(22) 3B 4B'



$\frac{3}{7}\times\frac{3}{5} = \frac{9}{35}$

$\frac{3}{7}\times\frac{2}{5} = \frac{6}{35}$

$\frac{4}{7}\times\frac{1}{2} = \frac{2}{7}$

$\frac{4}{7}\times\frac{1}{2} = \frac{2}{7}$

$P_r(B|H) = \dfrac{P_r(B\cap H)}{P_r(H)}$

$= \dfrac{9/35}{\frac{9}{35} + \frac{2}{7}}$

$= \dfrac{\frac{9}{35}}{\frac{19}{35}}$

$= \dfrac{9}{19}$

(23) $P_r(A\cup B) = P_r(A) + P_r(B) - P_r(A\cap B)$

$0\cdot65 = 0\cdot3 + 0\cdot5 - P_r(A\cap B)$

$\Rightarrow P_r(A\cap B) = 0\cdot15$

And $P_r(A)P_r(B)$

$= 0\cdot3 \times 0\cdot5$

$= 0\cdot15$

$\therefore P_r(A\cap B) = P_r(A)P_r(B)$

$\therefore$ A and B are independent events

(24) $P_r(A\cup B) = P_r(A) + P_r(B) - P_r(A\cap B)$

$\frac{1}{3} = \frac{1}{4} + P_r(B) - P_r(A)P_r(B)$ since indep.

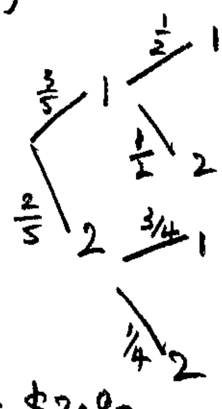
$\therefore \frac{1}{3} = \frac{1}{4} + P_r(B) - \frac{1}{4}P_r(B)$

$\frac{1}{3} - \frac{1}{4} = \frac{3}{4}P_r(B)$

$\frac{1}{12} = \frac{3}{4}P_r(B)$

$\therefore P_r(B) = \frac{1}{{}_3\cancel{12}} \times \frac{\cancel{4}^1}{3} = \frac{1}{9}$

(25)



$\frac{3}{10}$

$\frac{3}{10}$

$\frac{3}{10}$

$\frac{1}{10}$

$2 \leftarrow \boxed{\begin{matrix}3 & \$1 \\ 2 & \$2\end{matrix}}$

x	2	3	4
$P_r(X=x)$	$\frac{3}{10}$	$\frac{6}{10}$	$\frac{1}{10}$

Expected return

$= 2\times\frac{3}{10} + 3\times\frac{6}{10} + 4\times\frac{1}{10}$

$= \dfrac{6+18+4}{10} = \dfrac{28}{10}$

$= \$2\cdot80$

Less than $\$2\cdot80$

(27)

x	1	2	10
$P(x)$	$\frac{1}{4}$	k	k^2

$\frac{1}{4}+k+k^2=1$

$1+4k+4k^2=4$

$4k^2+4k-3=0$

$2k \quad 3$

$2k \quad -1$

$(2k+3)(2k-1)=0$

$k=-\frac{3}{2}, \frac{1}{2}$

But $k>0 \therefore k=\frac{1}{2}$

$E(X)=1\times\frac{1}{4}+2k+10k^2$

$=\frac{1}{4}+2\times\frac{1}{2}+10\times\left(\frac{1}{2}\right)^2$

$=\frac{1}{4}+1+10\times\frac{1}{4}$

$=\frac{15}{4}$

(28) (a) ${}^7C_3=\frac{7!}{3!4!}=\frac{7\times\cancel{6}\times5\times\cancel{4!}}{\cancel{3\times2}\times\cancel{4!}}=35$

(b) ${}^8C_2=\frac{8!}{2!6!}=\frac{\cancel{8}^4\times7\times\cancel{6!}}{\cancel{2}\times\cancel{6!}}=28$

(c) ${}^{100}C_1=100$

(d) ${}^{100}C_{99}=100$

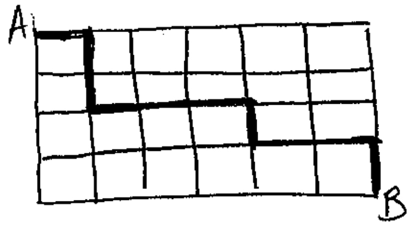
(e) ${}^{10}C_0=1$

(f) ${}^{10}C_{10}=1$

(29) (a) ${}^6C_2=\frac{6!}{2!4!}=\frac{6\times5}{2}=15$

(b) ${}^6C_3=\frac{6!}{3!3!}=\frac{6\times5\times4}{6}=20$

(c) ${}^6C_4=15$

(30) 

RDDRRRDRRD

Every path is a rearrangement of the above letters. 10 positions from which 4 must be allocated as down

${}^{10}C_4=\frac{10!}{6!\,4!}=\frac{10\times\cancel{9}^3\times\cancel{8}\times7}{\cancel{4}\times\cancel{3}\times\cancel{2}}=210$

(31) Let X = the number of goals out of 6

$Pr(GGG\,G'G'G')=\left(\frac{3}{4}\right)^3\left(\frac{1}{4}\right)^3$

But there are ${}^6C_3$ different ways she could have scored the goals.

$\therefore Pr(X=3)={}^6C_3\times\left(\frac{3}{4}\right)^3\left(\frac{1}{4}\right)^3$

$=0.131836$

(32) X = the number of 6s out of 3

$X\sim Bi\left(n=3, p=\frac{1}{6}\right)$

$E(X)=np$

$=3\times\frac{1}{6}$

$=\frac{1}{2}$

The expected value does not need to be a possible value.

(33) X = the # of red out of 5

4R 3B

$X\sim Bi\left(n=5, p=\frac{4}{7}\right)$

$Pr(X=3)=\binom{n}{x}p^x(1-p)^{n-x}$

$=\binom{5}{3}\left(\frac{4}{7}\right)^3\left(\frac{3}{7}\right)^2$

≈ 0.3427

34) X = the # bulleyes out of 5

$X \sim Bi(n=5, p=\frac{2}{5})$

$Pr(X=2) = \binom{n}{x} p^x (1-p)^{n-x}$

$= \binom{5}{2}\left(\frac{2}{5}\right)^2\left(\frac{3}{5}\right)^3$

$= 0.3456$

---

35) X = the # undersized eggs

$X \sim Bi(n=6, p=0.35)$

$Pr(X \geq 4)$

$= Pr(X=4) + Pr(X=5) + Pr(X=6)$

$= \binom{6}{4} 0.35^4\, 0.65^2 + \binom{6}{5} 0.35^5\, 0.65 + \binom{6}{6} 0.35^6$

$= 15 \times 0.35^4 \times 0.65^2 + 6 \times 0.35^5 \times 0.65 + 0.35^6$

$= 0.1174$

---

36) (a) $Pr = \left(\frac{1}{6}\right)^5 \simeq 0.000129$

(b) X = # even

$X \sim Bi(n=5, p=\frac{1}{2})$

$Pr(X=3) = \binom{5}{3}\left(\frac{1}{2}\right)^3\left(\frac{1}{2}\right)^2$

$\simeq 0.3125$

(c) X = # greater than 4

$X \sim Bi(n=5, p=\frac{1}{3})$

$Pr(X=4) = \binom{5}{4}\left(\frac{1}{3}\right)^4\left(\frac{2}{3}\right)$

$= \frac{10}{243}$

---

37) (a) X = # right moves

$X \sim Bi(n=6, p=\frac{1}{6})$

$Pr(X=3) = \binom{6}{3}\left(\frac{1}{6}\right)^3\left(\frac{5}{6}\right)^3 \simeq 0.0536$

(b) $Pr(X>3)$

$= Pr(X=4) + Pr(X=5) + Pr(X=6)$

$= \binom{6}{4}\left(\frac{1}{6}\right)^4\left(\frac{5}{6}\right)^2 + \binom{6}{5}\left(\frac{1}{6}\right)^5\left(\frac{5}{6}\right) + \left(\frac{1}{6}\right)^6$

$= 0.008702$

---

(38) X = # 6s out of n

$X \sim Bi(n, p=\frac{1}{6})$

$Pr(X \geq 1) \geq 0.95$

$1 - Pr(X=0) \geq 0.95$

$1 - \left(\frac{5}{6}\right)^n \geq 0.95$

Use solve on calculator

$n \geq 17$

At least 17 Rolls

---

(39) $Pr(X \geq 2) \geq 0.95$

$1 - Pr(X=0) - Pr(X=1) \geq 0.95$

$1 - \left(\frac{5}{6}\right)^n - \binom{n}{1}\left(\frac{1}{6}\right)\left(\frac{5}{6}\right)^{n-1} \geq 0.95$

$1 - \left(\frac{5}{6}\right)^n - n\left(\frac{1}{6}\right)\left(\frac{5}{6}\right)^{n-1} \geq 0.95$

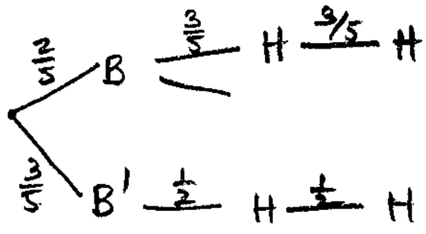
Solve $n \geq 26.5$ (Use calculator)

$\therefore n \geq 27$

At least 27 trials

---

(40) $Pr(B \mid HH) = \dfrac{Pr(B \cap HH)}{Pr(HH)}$



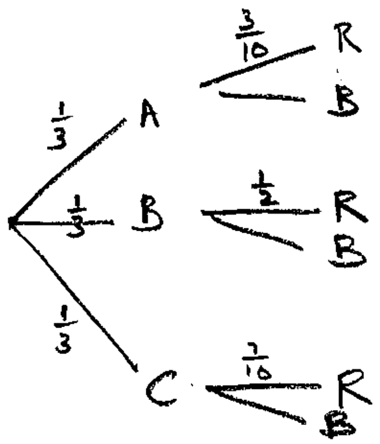
$= \dfrac{\frac{2}{5} \times \left(\frac{3}{5}\right)^2}{\frac{2}{5} \times \left(\frac{3}{5}\right)^2 + \frac{3}{5} \times \left(\frac{1}{2}\right)^2}$

$= 0.489796$

(41)

$\dfrac{3R\ 7B}{A}$ $\quad \dfrac{5R\ 5B}{B}$ $\quad \dfrac{7R\ 3B}{C}$

$$Pr(A|R) = \frac{Pr(A \cap R)}{Pr(R)}$$



$$Pr(A|R) = \frac{\frac{1}{3} \times \frac{3}{10}}{\frac{1}{3}\times\frac{3}{10} + \frac{1}{3}\times\frac{1}{2} + \frac{1}{3}\times\frac{7}{10}}$$

$$= \frac{\frac{3}{10}}{\frac{3}{10} + \frac{1}{2} + \frac{7}{10}}$$

$$= \frac{\frac{3}{10}}{\frac{15}{10}}$$

$$= \frac{1}{5}$$

(42) X = # games Jenny wins out of n

$X \sim Bi(n, p=0.3)$

(a) $Pr(X=2) = \binom{n}{2} p^2 (1-p)^{n-2}$

$$= \binom{n}{2} \times 0.3^2 \times 0.7^{n-2}$$

$$= \frac{n!}{(n-2)!\,2!} \times \left(\frac{3}{10}\right)^2 \times 0.7^{n-2}$$

$$= \frac{n(n-1)\cancel{(n-2)!}}{\cancel{(n-2)!}\,2} \times \frac{9}{100} \times 0.7^{n-2}$$

$$= n(n-1) \times \frac{9}{200} \times 0.7^{n-2}$$

$$= \frac{9 \times 0.7^{n-2}}{200} n(n-1)$$

(b) $Pr(X=2) = 0.2668$

$$\frac{9 \times 0.7^{n-2}}{200} n(n-1) = 0.2668$$

$n = 9$ (Use calculator)

(43) $\boxed{\begin{smallmatrix}3D\\2M\end{smallmatrix}}$ 1st M

a) $\xi_x = \{1, 2, 3, 4\}$

$Pr(X=1) = \frac{2}{5}$

$Pr(X=2) = \frac{3}{5} \times \frac{2}{4} = \frac{3}{10}$

$Pr(X=3) = \frac{3}{5} \times \frac{2}{4} \times \frac{2}{3} = \frac{1}{5}$

$Pr(X=4) = \frac{3}{5} \times \frac{2}{4} \times \frac{1}{3} \times 1 = \frac{1}{10}$

x	1	2	3	4
$p(x)$	$\frac{2}{5}$	$\frac{3}{10}$	$\frac{1}{5}$	$\frac{1}{10}$

(b) $E(X) = \frac{2}{5} + 2\times\frac{3}{10} + 3\times\frac{1}{5} + 4\times\frac{1}{10}$

$$= \frac{4}{10} + \frac{6}{10} + \frac{6}{10} + \frac{4}{10}$$

$$= 2$$

(44) $Pr(A|B) = \dfrac{Pr(A \cap B)}{Pr(B)}$

$$= \frac{Pr(B|A)\,Pr(A)}{Pr(B)}$$

$$= \frac{Pr(A)\,Pr(B|A)}{Pr(B)}$$

As required

PTO.

(45) Let x = # cards in pack

$Pr(4 \text{ Aces}) = \frac{1}{1001}$

$$\frac{4}{x} \times \frac{3}{x-1} \times \frac{2}{x-2} \times \frac{1}{x-3} = \frac{1}{1001}$$

$$\frac{24}{x(x-1)(x-2)(x-3)} = \frac{1}{1001}$$

$$x(x-1)(x-2)(x-3) = 24024$$

$x = 14$ (Solve on calc)

$\therefore$ $52 - 14 = 38$

38 cards removed.

---

(46) Must have at least 'one' and one even.

$FEE' + FEE + FE'E$

$$\frac{1}{6}\frac{1}{2}\frac{1}{2} + \frac{1}{6}\frac{1}{2}\frac{1}{2} + \frac{1}{6}\frac{1}{2}\frac{1}{2}$$

$= \frac{1}{8}$

This repeats for F in 2nd + 3rd position.

$FFE + FEF + EFF$

$$= 3 \times \left(\frac{1}{6}\right)^2 \times \frac{1}{2}$$

$$= \frac{1}{2} \times \frac{1}{2} \times \frac{1}{6} = \frac{1}{24}$$

$\therefore$ Prob $= 3 \times \frac{1}{8} + \frac{1}{24}$

$$= \frac{9}{24} + \frac{1}{24}$$

$$= \frac{10}{24}$$

$$= \frac{5}{12}$$