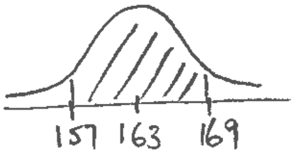
Normal Distribution

(1) X = the height of adult female

$X \sim N(\mu = 163, \sigma = 6)$

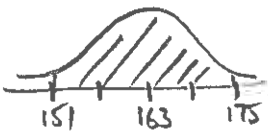
(a) (i)



$Pr(157 < X < 169)$

$\simeq 0.683$

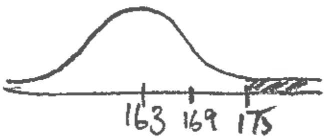
(ii)



$Pr(151 < X < 175)$

$\simeq 0.954$

(iii)

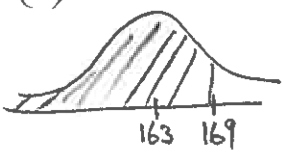


$Pr(X > 175)$

$\simeq \dfrac{1 - 0.954}{2}$

$= 0.023$

(iv)

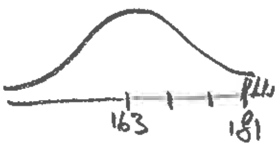


$Pr(X < 169)$

$\simeq 0.5 + \dfrac{0.683}{2}$

$= 0.8415$

(b)



$Pr(X > 181)$

$\simeq \dfrac{1 - 0.997}{2}$

$= 0.0015$

Expected number $= 4000 \times 0.0015$
$= 6$

2 (a) $z = \dfrac{x - \mu}{\sigma} = \dfrac{42 - 40}{0.4} = 5$

(b) $z = \dfrac{x - \mu}{\sigma} = \dfrac{94 - 100}{10} = -0.6$

(c) $z = \dfrac{x - \mu}{\sigma}$

$\Rightarrow 0.6 = \dfrac{x - 12}{2.5}$

$\Rightarrow x = 2.5 \times 0.6 + 12 \simeq 13.5$

(d) $z = \dfrac{x - \mu}{\sigma}$

$\Rightarrow x = z\sigma + \mu$

$= -1.5 \times 1.4 + 60$

$= 57.9$

(3) A: $N(\mu = 70, \sigma = 4.5)$

$z = \dfrac{x - \mu}{\sigma} = \dfrac{75 - 70}{4.5} = 1.11$

B: $N(\mu = 44, \sigma = 5.5)$

$z = \dfrac{x - \mu}{\sigma} = \dfrac{50 - 44}{5.5} = 1.09$

In test A her score is 1.11 standard deviations above the mean whereas in test B it is 1.09 standard deviations above the mean. So relative to the population she has done better in test A.

(4) $X \sim N(\mu = 5, \sigma = 0.7)$

$Pr(X < 6) = Pr\left(Z < \dfrac{6 - 5}{0.7}\right)$

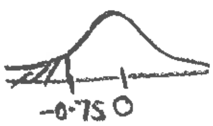
$= Pr(Z < 1.4286)$

$\simeq 0.9236$

(b) $X \sim N(\mu = 13, \sigma = 4)$

$Pr(X < 10) = Pr\left(Z < \dfrac{10 - 13}{4}\right)$

$= Pr(Z < -0.75)$



$= Pr(Z > 0.75)$

$= 1 - Pr(Z < 0.75)$

$= 1 - 0.7734$

$= 0.2266$

(c) $X \sim N(18, 1.4)$

$Pr(X > 20) = Pr\left(Z > \dfrac{20 - 18}{1.4}\right)$

$= Pr(Z > 1.42857)$

$= 1 - Pr(Z < 1.42857)$

$= 1 - 0.9236 = 0.0764$

(d) $X \sim N(95, 4)$

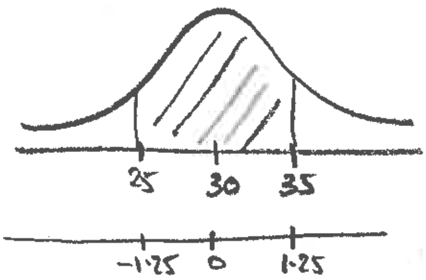
$\Pr(X > 90) = \Pr\left(Z > \frac{90-95}{4}\right)$

$= \Pr(Z > -1.25)$

$= \Pr(Z < 1.25)$

≈ 0.8944

(e) $\Pr(25 < X < 35)$ $\quad X \sim N(30, 4)$



$z_1 = \frac{35-30}{4}$
$= 1.25$

$z_2 = \frac{25-30}{4}$
$= -1.25$

$\Pr(Z < 1.25) = 0.8944$

$\Pr(Z < -1.25) = \Pr(Z > 1.25)$

$= 1 - \Pr(Z < 1.25)$

$= 1 - 0.8944$

$= 0.1056$

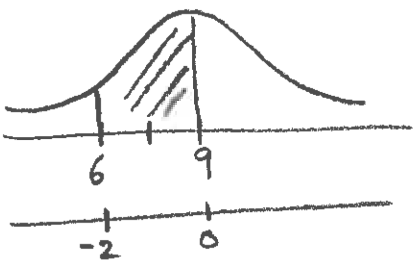
$\Pr(25 < X < 35)$

$= 0.8944 - 0.1056$

$= 0.7888$

(f) $X \sim N(9, 1.5)$

$\Pr(6 < X < 9)$



$z_1 = \frac{x-\mu}{6}$

$= \frac{6-9}{1.5}$

$= -2$

$z_2 = 0$

$\Pr(Z < -2) = \Pr(X > 2)$

$= 1 - \Pr(X < 2)$

$= 1 - 0.9772$

$= 0.0228$

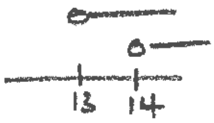
$\Pr(Z < 0) = 0.5$

$\therefore \Pr(6 < X < 9)$

$= 0.5 - 0.0228$

$= 0.4772$

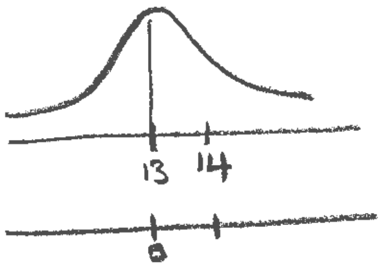
(g) $X \sim N(13, 4)$



$\Pr(X > 14 \mid X > 13)$

$= \frac{\Pr(X > 14 \cap X > 13)}{\Pr(X > 13)}$

$= \frac{\Pr(X > 14)}{\Pr(X > 13)}$



$z_1 = \frac{x-\mu}{\sigma}$

$= \frac{14-13}{4}$

$= 0.25$

$\Pr(X > 14)$

$= \Pr(Z > 0.25)$

$= 1 - \Pr(Z < 0.25)$

$= 1 - 0.5987$

$= 0.4013$

$\Pr(X > 13)$

$= \Pr(Z > 0)$

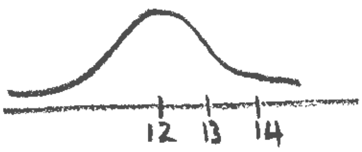
$= 0.5$

$\therefore \Pr(X > 14 \mid X > 13) = \frac{0.4013}{0.5}$

$= 0.8036$

(h) $\Pr(X > 14 \mid X > 13)$ $\quad X \sim N(12, 3)$

$= \frac{\Pr(X > 14)}{\Pr(X > 13)}$



$z_1 = \frac{x-\mu}{\sigma}$

$= \frac{13-12}{3}$

$= \frac{1}{3}$

$= 0.333$

$\Pr(X > 13) = \Pr(Z > 0.333)$

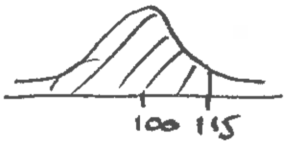
$= 1 - \Pr(Z < 0.333)$

$= 1 - 0.6293$

$= 0.3707$

(7) $X \sim N(\mu = 100, \sigma = 10)$

(a)



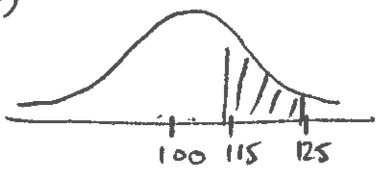
$z = \frac{115-100}{10}$

$= 1.5$

$P(Z < 1.5)$

$= 0.9332$

(b)



$z_1 = \frac{125-100}{10}$

$= 2.5$

$z_2 = 1.5$

$P(X < 115) = Pr(Z < 1.5) = 0.9332$

$Pr(X > 125) = Pr(Z > 2.5)$

$= 1 - Pr(Z < 2.5)$

$= 1 - 0.9938$

$= 0.0062$

$Pr(115 < X < 125)$

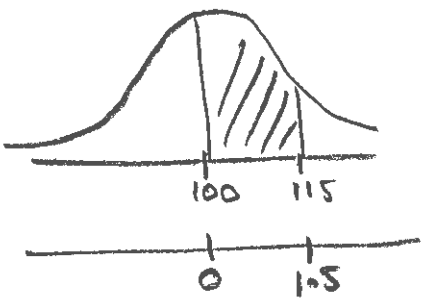
$= 1 - 0.9332 - 0.0062$

$= 0.0606$

(c) $Pr(X < 115 \mid X > 100)$

$= \frac{Pr(X < 115 \cap X > 100)}{Pr(X > 100)}$

$= \frac{Pr(100 < X < 115)}{Pr(X > 100)}$



$Pr(X < 100) = Pr(Z < 0) = 0.5$

$Pr(X > 115) = P(Z > 1.5) = 1 - 0.9332$

$= 0.0668$

$Pr(100 < X < 115) = 1 - 0.5 - 0.0668$

$= 0.4332$

$Pr(X > 100) = 0.5$

$\therefore Pr(X < 115 \mid X > 100)$

$= \frac{0.4332}{0.5}$

$= 0.8664$

(d) $Pr(115 < X < 125 \mid X > 115)$

$= \frac{Pr(115 < X < 125 \cap X > 115)}{Pr(X > 115)}$

$= \frac{Pr(115 < X < 125)}{Pr(X > 115)}$

$= \frac{0.0606}{1 - 0.9332}$ (see part (b))

$= 0.9072$

(8)(a) $X \sim N(\mu = 7, \sigma = 2)$

$Pr(X < x) = 0.70$

$Pr(Z < z_1) = 0.7$

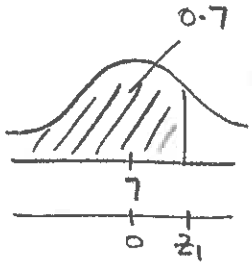
$z_1 = 0.5244$

$z = \frac{x - \mu}{\sigma}$

$\therefore x = z\sigma + \mu$

$x = 0.5244 \times 2 + 7$

$= 8.0488$



(b) $Pr(X < x) = 0.45$

$Pr(Z < z) = 0.45$

$Pr(Z > -z) = 0.45$

$Pr(Z < -z) = 0.55$

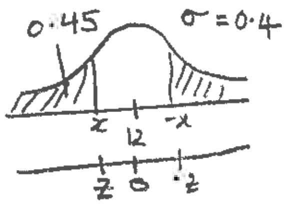
$-z = 0.1257$

$z = -0.1257$

$x = z\sigma + \mu$

$= -0.1257 \times 0.4 + 12$

$= 11.94972$



(c) $Pr(X \geq x) = 0.15$

$P(X \leq x) = 0.85$

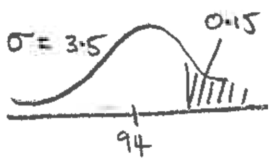
$P(Z < z) = 0.85$

$z = 1.0364$

$x = z\sigma + \mu$

$= 1.0364 \times 3.5 + 94$

$= 97.6274$



$z_2 = \frac{x-\mu}{\sigma} = \frac{14-12}{3} = \frac{2}{3} \approx 0.67$

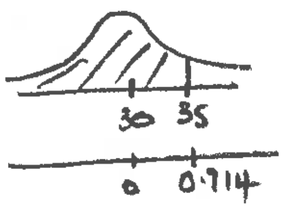
$\Pr(X>14) = \Pr(Z>0.67)$
$= 1-\Pr(Z<0.67)$
$= 1-0.7486$
$= 0.2514$

$\Pr(X>14 \mid X>13) = \frac{0.2514}{0.3707}$
$= 0.6782$

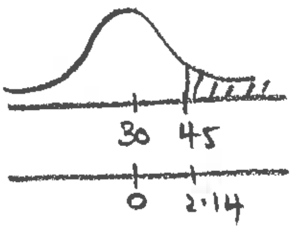
---

5) $X \sim N(\mu=30, \sigma=7)$

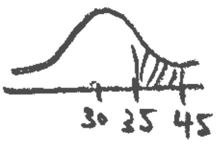
$\Pr(X<35)$
$= \Pr\left(Z < \frac{35-30}{7}\right)$
$\approx \Pr(Z<0.714)$
$= 0.7611$



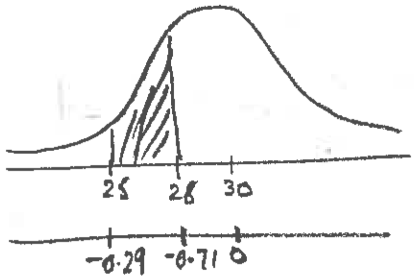
(b) $\Pr(X>45)$
$= \Pr\left(Z > \frac{45-30}{7}\right)$
$= \Pr(Z>2.14)$
$= 1-\Pr(Z<2.14)$
$= 1-0.9838$
$= 0.0162$



(c) $\Pr(35<X<45)$
$= \Pr(X<45) - \Pr(X<35)$
$= \Pr(Z<2.14) - \Pr(Z<.714)$
$= 0.9838 - 0.7611$
$= 0.2227$



(d)



$z_1 = \frac{25-30}{7}$
$= -0.714$

$z_2 = \frac{28-30}{7}$
$= -0.286$

$\Pr(X<25)$
$= \Pr(Z<-0.714)$
$= \Pr(Z>0.714)$
$= 1-\Pr(Z<.714)$
$= 1-0.7611$
$= 0.2389$

$\Pr(X>28)$
$= \Pr(Z>-0.286)$
$= P(Z<.286)$
$= 0.6141$

$\Pr(25<X<28)$
$= 1-0.2389-0.6141$
$= 0.147$

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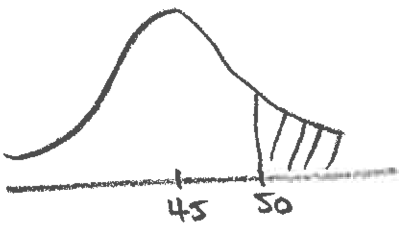
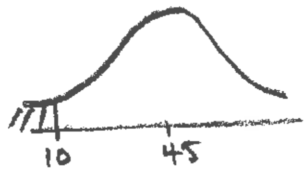
(6) (a) $X \sim N(\mu=45, \sigma=11)$



$z_1 = \frac{50-45}{11}$
$= 0.45$

$\Pr(X>50) = \Pr(Z>0.45)$
$= 1-\Pr(Z<0.45)$
$= 1-0.6736$
$= 0.3264$

(b)



$z = \frac{10-45}{11}$
$= -3.18$

$\Pr(X<10) = \Pr(Z<-3.18)$
$= \Pr(Z>3.18)$
$= 1-\Pr(Z<3.18)$
$= 1-0.9993$
$= 0.0007$

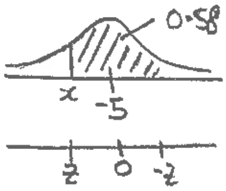
3000×0.0007
$= 2.1$
Expect this to happen on 2 occasions

(d) $X \sim N(-5, 0.1)$

$Pr(X \geq x) = 0.58$

$Pr(Z \geq z) = 0.58$

$P(Z \leq -z) = 0.58$



$-z = 0.2019$

$z = -0.2019$

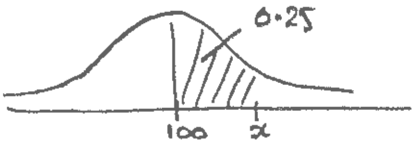
$x = z\sigma + \mu$

$= -0.2019 \times 0.1 + {}^{-}5$

$= -5.02019$

8(e) $X \sim N(100, 10)$

$Pr(100 < x < x) = 0.25$



$Pr(X < x) = 0.75$

$Pr(Z < z) = 0.75$

$z = 0.6745$

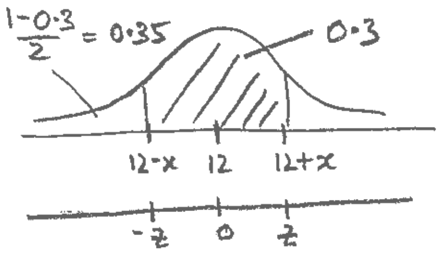
$x = z\sigma + \mu$

$= 0.6745 \times 10 + 100$

$= 106.745$

(f) $X \sim N(12, 2)$

$Pr(12 - x < X < 12 + x) = 0.3$



$Pr(X < x) = 0.65$

$Pr(Z < z) = 0.65$

$z = 0.3853$

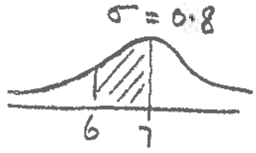
$x = z\sigma + \mu$

$= 0.3853 \times 2 + 12$

$= 12.7706$

9 (a) $Pr(6 < X < 7)$

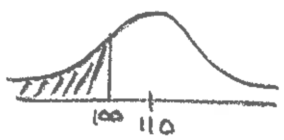
$= 0.3944$



(b) $X \sim N(110, 5)$

$Pr(X < 100)$

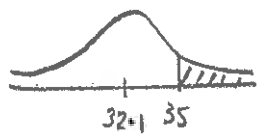
$= 0.0228$



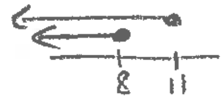
(c) $X \sim N(32.1, 3.2)$

$Pr(X \geq 35)$

$= 0.1824$



(d) $Pr(X \leq 8 \mid X \leq 11)$



$X \sim N(11, 3)$

$= \dfrac{Pr(X \leq 8 \cap X \leq 11)}{Pr(X \leq 11)}$

$= \dfrac{Pr(X \leq 8)}{Pr(X \leq 11)}$

$= \dfrac{0.158655}{0.5}$

$= 0.317$

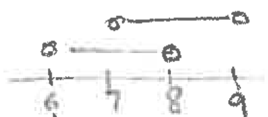
(e) $Pr(X \geq 10 \mid X \leq 11)$ $\quad X \sim N(10, 2)$

$= \dfrac{Pr(X \geq 10 \cap X \leq 11)}{Pr(X \leq 11)}$

$= \dfrac{Pr(10 \leq X \leq 11)}{Pr(X \leq 11)}$

$= \dfrac{0.191462}{0.691462}$

$= 0.2769$



(f) $P(6 < X < 8 \mid 7 < X < 9)$ $\quad X \sim N(8.5, 2.5)$

$= \dfrac{Pr(7 < X < 8)}{Pr(7 < X < 9)}$

$= \dfrac{0.1464872}{0.3050066}$

$= 0.4803$

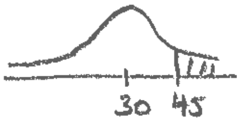
(10) $X \sim N(\mu=30, \sigma=7)$

(a) $Pr(X < 35)$

$\simeq 0.7625$

(b) $Pr(X > 45)$

$= 0.0161$



(c) $Pr(25 < X\ 35)$

$\simeq 0.5249$

(d) $Pr(X > 45 \mid X > 25)$

$$= \frac{Pr(X > 45 \cap X > 25)}{Pr(X > 25)}$$

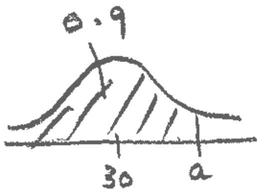
$$= \frac{Pr(X > 45)}{Pr(X > 25)}$$

$$= \frac{0.016062}{0.76247}$$

$= 0.0211$

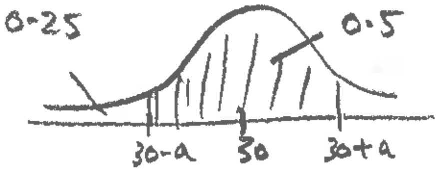
(11) $X \sim N(\mu=30, \sigma=7)$

(a) $Pr(X < a) = 0.9$



$a = 38.97$

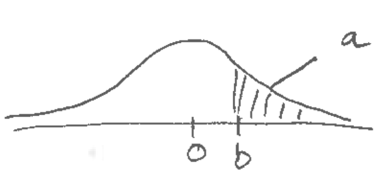
(b) $Pr(30-a < X < 30+a)$



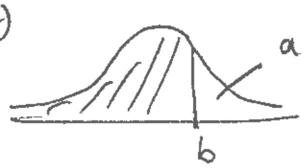
$Pr(X < a) = 0.75$

$a = 34.72$

(12) $X \sim N(0,1)$. $P(X \geqslant b) = a$



(a)

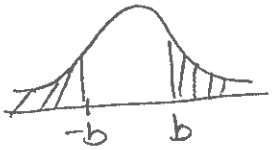


$Pr(X \le b)$

$= 1 - Pr(X > b)$

$= 1 - a$

(b)

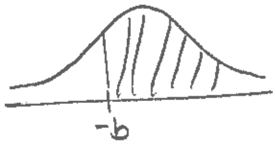


$Pr(X \le -b)$

$= P(X \geqslant b)$

$= a$

(c)

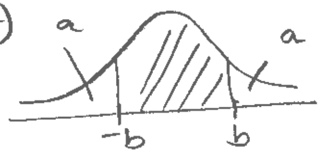


$Pr(X \geqslant -b)$

$= Pr(X \le b)$

$= 1 - a$

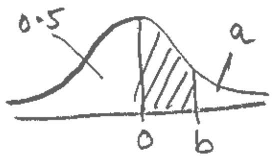
(d)



$Pr(-b \le X \le b)$

$= 1 - 2a$

(e)

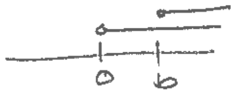


$Pr(0 \le X \le b)$

$= 1 - 0.5 - a$

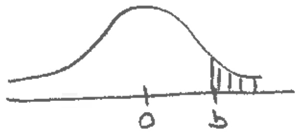
$= 0.5 - a$

(f)



$Pr(X \geqslant b \mid X \geqslant 0)$

$$= \frac{Pr(X \geqslant b \cap X \geqslant 0)}{Pr(X \geqslant 0)}$$



$$= \frac{Pr(X \geqslant b)}{Pr(X \geqslant 0)}$$

$$= \frac{a}{0.5}$$

$= 2a$

(g) $Pr(X \geqslant 0 \mid X \geqslant -b) = \dfrac{Pr(X \geqslant 0 \cap X \geqslant -b)}{Pr(X \geqslant -b)}$

$$= \frac{Pr(X \geqslant 0)}{Pr(X \geqslant -b)}$$

$$= \frac{\frac{1}{2}}{1-a}$$

$$= \frac{1}{2(1-a)}$$

(h) $Pr(X \le b \mid X \geqslant -b) = \dfrac{Pr(X \le b \cap X \geqslant -b)}{Pr(X \geqslant -b)}$

$$= \frac{Pr(-b \le X \le b)}{Pr(X \geqslant -b)}$$

$$= \frac{1-2a}{1-a}$$

(i) $\Pr(X \geq b \mid X \geq -b)$

$= \dfrac{\Pr(X \geq b \cap X \geq -b)}{\Pr(X \geq -b)}$

$= \dfrac{\Pr(X \geq b)}{\Pr(X \geq -b)}$

$= \dfrac{a}{1-a}$

(j) $\Pr(X \geq 0 \mid -b \leq X \leq b)$

$= \dfrac{\Pr(X \geq 0 \cap -b \leq X \leq b)}{\Pr(-b \leq X \leq b)}$

$= \dfrac{\Pr(0 \leq X \leq b)}{1-2a}$

$= \dfrac{\frac{1}{2} - a}{1-2a}$

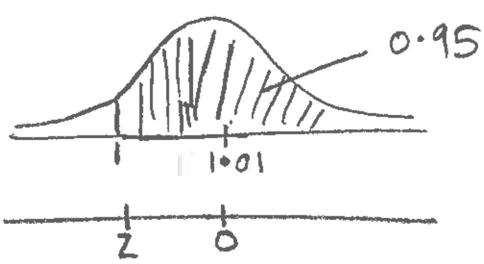
$= \dfrac{\frac{1-2a}{2}}{1-2a}$

$= \dfrac{1}{2}$

(13) X = volume in a bottle

$X \sim N(\mu = 1.01, \sigma)$

$\Pr(X > 1) = 0.95$



$\Pr(Z > z) = 0.95$

$\Pr(Z < z) = 0.05$

$z = -1.64485$

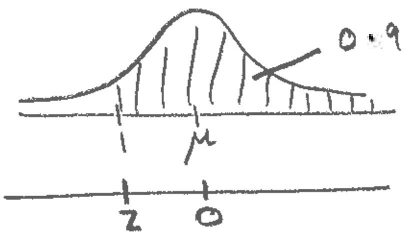
$z = \dfrac{x-\mu}{\sigma}$

$\sigma = \dfrac{x-\mu}{z} = \dfrac{1-1.01}{-1.64485} \simeq 0.006$

(14) X = volume in bottle

$X \sim N(\mu, \sigma = 0.006)$

$\Pr(X > 1) = 0.95$



$\Pr(Z < z) = 0.05$

$z = -1.64485$

$z = \dfrac{x-\mu}{\sigma}$

$z\sigma = x - \mu$

$\mu = x - z\sigma$

$= 1 - {}^{-}1.64485 \times 0.006$

$= 1 + 0.0098691$

$= 1.0099$

b) $X \sim N(\mu = 1.00987, \sigma = 0.006)$

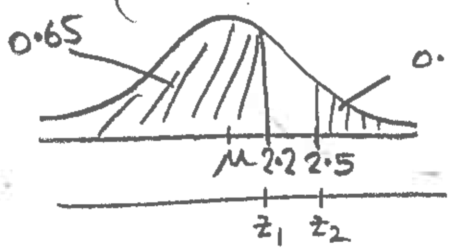
$\Pr(X < 0.99) \simeq 0.000464$

(15) W = Weight of a chicken

$W \sim N(\mu, \sigma)$

$\Pr(W > 2.5) = 0.1$

$\Pr(W < 2.2) = 0.65$



$P(Z < z_1) = 0.65 \Rightarrow z_1 = 0.38532$

$\Pr(Z > z_2) = 0.1 \Rightarrow \Pr(Z < z_2) = 0.9 \Rightarrow z_2 = 1.28155$

$z_1 = \dfrac{x_1 - \mu}{\sigma} \Rightarrow z_1\sigma = x_1 - \mu$ ①

$z_2 = \dfrac{x_2 - \mu}{\sigma} \Rightarrow z_2\sigma = x_2 - \mu$ ②

② − ① : $(z_2 - z_1)\sigma = x_2 - x_1$

$\Rightarrow \sigma = \dfrac{x_2 - x_1}{z_2 - z_1} = \dfrac{2.5 - 2.2}{1.28155 - 0.38532}$

$= 0.33474$

(1) $z_1\sigma = x_1 - \mu$

$\therefore \mu = x_1 - z_1\sigma$

$= 2.2 - 0.38532 \times 0.33474$

$= 2.071$

$\mu \simeq 2.07$, $\sigma \simeq 0.33$

---

(16) X = lifespan of battery

$X \sim N(\mu = 5.5, \sigma = 2.2)$

(a) (i) $\Pr(X \geq 6) = 0.4101$

(ii) $\Pr(7 \leq X \leq 8) \simeq 0.1198$

(iii) $\Pr(X \geq 7 \mid X \leq 8)$

$$= \frac{\Pr(7 \leq X \leq 8)}{\Pr(X \leq 8)}$$

$$= \frac{0.1198}{0.872}$$

$\simeq 0.1373$

(b) $\Pr(\text{At least one lasts} \geq 6)$

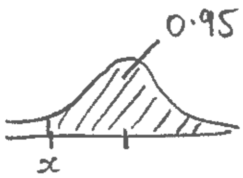
$= 1 - \Pr(\text{none})$

$= 1 - (1 - 0.4101)^3$

$\simeq 0.79$

(c) $\Pr(X > x) = 0.95$

$\Pr(X < x) = 0.05$



$x = 1.88$

$x \simeq 1.9$

---

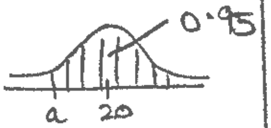
(17) X = lifespan of solar panel

$X \sim N(\mu = 20, \sigma = 5.5)$

(a) $\Pr(X \geq 10) \simeq 0.9655$

(b) $\Pr(X \geq a) = 0.95$

$\Pr(X < a) = 0.05$



$a = 10.95$

$\simeq 11.0$

(c) $\Pr(\text{at least 3 working})$

$= \Pr(\text{3 working}) + \Pr(\text{4 working})$

$= \binom{4}{3} \times 0.5^3 \times 0.5 + 0.5^4$

$= 4 \times 0.5^4 + 0.5^4$

$= 5 \times 0.5^4$

$= 0.3125$

---

(18) X = speed of cars

$X \sim N(\mu = 42.3, \sigma = 4.4)$

(a) (i) $\Pr(X < 52) \simeq 0.9863$

(ii) $\Pr(52 < X < 60) \simeq 0.0137$

(iii) $\Pr(X > 60) \simeq 0.0000289$

(b) $480000 \times 0.0137 \times 200$

$+ 480000 \times 0.0000289 \times 400$

$\simeq \$1{,}322{,}086$

(c) $\Pr(X > 50) \simeq 0.04$

(d) $\Pr(X > 52 \mid X > 50)$

$$= \frac{\Pr(X > 52)}{\Pr(X > 50)}$$

$$= \frac{0.0137}{0.04}$$

$\simeq 0.3431$

---

(19) X = weight of banana

$X \sim W(\mu = 180, \sigma = 45)$

>210	140 – 210	<140
\$2	\$1.50	0

(a) (i) $\Pr(X > 210) \simeq 0.2525$

(ii) $\Pr(140 < X < 210) \simeq 0.5605$

(b) $80 \times 0.2525 \times 2 + 80 \times 0.5605 \times 1.5$

$= 107.66$

(19) (c) $\Pr(\text{all worthless})$

$= (0.18703)^5$

$\simeq 0.00023$

(ii) $\Pr(\text{at least one worthless})$

$= 1 - \Pr(\text{none})$

$= 1 - (1-0.18703)^5$

$\simeq 0.6449$

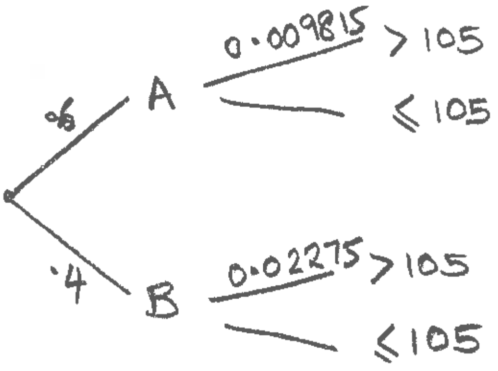
(20) $A \sim N(\mu = 98, \sigma = 3)$

$B \sim N(\mu = 95, \sigma = 5)$

$\Pr(A \mid IQ > 105)$

$= \dfrac{\Pr(A \cap IQ > 105)}{\Pr(IQ > 105)}$

$= \dfrac{\Pr(A \cap IQ > 105)}{\Pr(IQ > 105)}$



$= \dfrac{.6 \times 0.009815}{.6 \times 0.009815 + .4 \times .02275}$

$= 0.39289$

$\simeq 0.3929$

(21) $X \sim N(\mu = 2.2, \sigma = 0.3)$

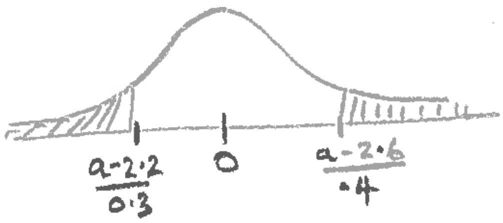
$Y \sim N(\mu = 2.6, \sigma = 0.4)$

(a) (i) $\Pr(X < 2.4) \simeq 0.7475$

(ii) $\Pr(Y > 2.4) \simeq 0.6915$

$\Pr(X < a) = \Pr\left(Z < \dfrac{a-2.2}{0.3}\right)$

$\Pr(Y > a) = \Pr\left(Z > \dfrac{a-2.6}{0.4}\right)$



$\therefore \dfrac{a-2.2}{0.3} = -\dfrac{a-2.6}{0.4}$

$0.4a - 0.88 = -0.3a + 0.78$

$0.7a = 1.66$

$a = 2.37$

(22) $X \sim N(\mu_1, \sigma_1)$ $\quad Y \sim N(\mu_2, \sigma_2)$

$\Pr(X < a) = \Pr\left(Z < \dfrac{a-\mu_1}{\sigma_1}\right)$

$\Pr(Y > a) = \Pr\left(Z > \dfrac{a-\mu_2}{\sigma_2}\right)$

$\dfrac{a-\mu_1}{\sigma_1} = -\dfrac{a-\mu_2}{\sigma_2}$

$a\sigma_2 - \mu_1\sigma_2 = -a\sigma_1 + \sigma_1\mu_2$

$a(\sigma_1 + \sigma_2) = \mu_1\sigma_2 + \sigma_1\mu_2$

$a = \dfrac{\mu_1\sigma_2 + \mu_2\sigma_1}{\sigma_1 + \sigma_2}$