Integral Calculus

(1) (a) $F(x) = 2x+3$

$\therefore F'(x) = 2$

(b) $F(x) = 3x^3 - 9x^2 + 2x + 1$

$F'(x) = 9x^2 - 18x + 2$

(c) $F(x) = x^2 - 3x + 5$

$F'(x) = 2x - 3$

(d) $F(x) = 2\sqrt{x} - x^3$

$= 2x^{1/2} - x^3$

$F'(x) = x^{-\frac{1}{2}} - 3x^2$

$= \frac{1}{\sqrt{x}} - 3x^2$

(e) $F(x) = \frac{x^2}{x-1} \begin{matrix} \leftarrow u \\ \leftarrow v \end{matrix}$

$$F'(x) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$$

$$= \frac{(x-1).2x - x^2.1}{(x-1)^2}$$

$$= \frac{2x^2 - 2x - x^2}{(x-1)^2}$$

$$= \frac{x^2 - 2x}{(x-1)^2}$$

$$= \frac{x(x-2)}{(x-1)^2}$$

(f) $F(x) = 2x\sqrt{x+1}$

$= 2x(x+1)^{1/2}$

$F'(x) = u\frac{dv}{dx} + v\frac{du}{dx}$

$= 2x . \frac{1}{2}(x+1)^{-\frac{1}{2}}.1 + (x+1)^{\frac{1}{2}}.2$

$$= \frac{x}{\sqrt{x+1}} + 2\sqrt{x+1}$$

$$= \frac{x + 2(x+1)}{\sqrt{x+1}}$$

$$= \frac{3x+2}{\sqrt{x+1}}$$

(g) $F(x) = \frac{x+1}{x-1}$

$$F'(x) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$$

$$= \frac{(x-1).1 - (x+1).1}{(x-1)^2}$$

$$= \frac{x-1-x-1}{(x-1)^2}$$

$$= \frac{-2}{(x-1)^2}$$

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(2) $F_1(x) = 2x^2 - 3x + 1$

$\therefore F_1(x) = 4x - 3$

$F_2(x) = 2x^2 - 3x + 3$

$\therefore F'_2(x) = 4x - 3$

Both $F_1(x)$ and $F_2(x)$ are antiderivatives of $f(x) = 4x - 3$

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(3) (a) $f(x) = 3x$

$\int 3x \, dx = \frac{3x^2}{2} + C$

(b) $g(x) = 5x + 2$

$\int 5x + 2 \, dx = \frac{5x^2}{2} + 2x + c$

(c) $f(x) = 4$

$\int 4 \, dx = 4x + C$

(d) $g(x) = 0$

$\int 0 \, dx = C$

(e) $f(x) = (x+2)(2x-3)$

$\int (x+2)(2x-3) \, dx$

$= \int x^2 - 3x + 4x - 6 \, dx$

$= \int x^2 + x - 6 \, dx$

$= x^3/3 + x^2/2 - 6x + C$

f) $y = 9x^2+4x-7$

$\int 9x^2+4x-7$

$= \frac{9x^3}{3} + \frac{4x^2}{2} - 7x + c$

$= 3x^3 + 2x^2 - 7x + c$

(g) $f(x) = 2\sqrt{x} - 3x^2$

$= 2x^{1/2} - 3x^2$

$\int 2x^{\frac{1}{2}} - 3x^2 \, dx$

$= \frac{2x^{\frac{3}{2}}}{\frac{3}{2}} - \frac{3x^3}{3} + c$

$= 2x^{\frac{3}{2}} \times \frac{2}{3} - x^3 + c$

$= \frac{4x^{3/2}}{3} - x^3 + c$

(h) $f(x) = 2\sqrt[3]{x} - \frac{3}{x^2}$

$= 2x^{\frac{1}{3}} - 3x^{-2}$

$\int 2x^{\frac{1}{3}} - 3x^{-2} dx$

$= \frac{2x^{4/3}}{4/3} - \frac{3x^{-1}}{-1} + c$

$= \frac{3}{\cancel{4}_2} \times \cancel{2} x^{4/3} + \frac{3}{x} + c$

$= \frac{3}{2} x^{4/3} + \frac{3}{x} + c$

(i) $\int \frac{x^3+1}{x^2} dx$

$= \int \frac{x^3}{x^2} + \frac{1}{x^2} dx$

$= \int x + x^{-2} dx$

$= \frac{x^2}{2} + \frac{x^{-1}}{-1} + c$

$= \frac{x^2}{2} - \frac{1}{x} + c$

4 (a) $f(x) = \int 4x - 3 \, dx$

$= \frac{4x^2}{2} - 3x + c$

$= 2x^2 - 3x + c$

$f(1) = 3 \quad \therefore 3 = 2 - 3 + c$

$c = 4$

$\therefore f(x) = 2x^2 - 3x + 4$

(b) $\frac{dy}{dx} = 3$

$\therefore y = \int 3 \, dx$

$= 3x + c$

$x=3, y=2 \quad \therefore 2 = 3(3) + c$

$c = -7$

$y = 3x - 7$

(c) $f(x) = \int \sqrt{x} \, dx$

$= \int x^{\frac{1}{2}} dx$

$= \frac{x^{3/2}}{3/2} + c$

$= \frac{2x^{3/2}}{3} + c$

$= \frac{2\sqrt{x^3}}{3} + c$

$f(4) = 1 \quad \therefore 1 = \frac{2\sqrt{4}^3}{3} + c$

$1 = \frac{16}{3} + c$

$c = -\frac{13}{3}$

$\therefore f(x) = \frac{2\sqrt{x^3}}{3} - \frac{13}{3}$

(d) $\frac{dy}{dx} = \sqrt[3]{x} + 2x$

$y = \int x^{\frac{1}{3}} + 2x \, dx$

$= \frac{x^{\frac{4}{3}}}{\frac{4}{3}} + \frac{2x^2}{2} + c$

$= \frac{3(\sqrt[3]{x})^4}{4} + x^2 + c$

$x=8, y=1 \quad \therefore 1 = \frac{3(\sqrt[3]{8})^4}{4} + 8^2 + c$

$1 = \frac{3 \times 16}{4} + 64 + c$

(d) $1 = 12 + 64 + c$

$c = -75$

$y = \frac{3x^{4/3}}{4} + x^2 - 75$

(e) $f'(x) = 2x(3-x)$

$= 6x - 2x^2$

$f(x) = \int 6x - 2x^2 \, dx$

$= 3x^2 - \frac{2x^3}{3} + c$

$f(3) = 2 \quad \therefore \quad 2 = 3(3)^2 - \frac{2 \times 3^3}{3} + c$

$2 = 27 - 18 + c$

$c = -7$

$f(x) = 3x^2 - \frac{2x^3}{3} - 7$

(f) $\frac{dy}{dx} = (3x+1)(x-2)$

$= 3x^2 - 6x + x - 2$

$= 3x^2 - 5x - 2$

$y = \int 3x^2 - 5x - 2 \, dx$

$= \frac{3x^3}{3} - \frac{5x^2}{2} - 2x + c$

$= x^3 - \frac{5x^2}{2} - 2x + c$

$x = 1, y = 2 \quad \therefore \quad 2 = 1 - \frac{5}{2} - 2 + c$

$c = 3 + \frac{5}{2} = \frac{11}{2}$

$y = x^3 - \frac{5x^2}{2} - 2x + \frac{11}{2}$

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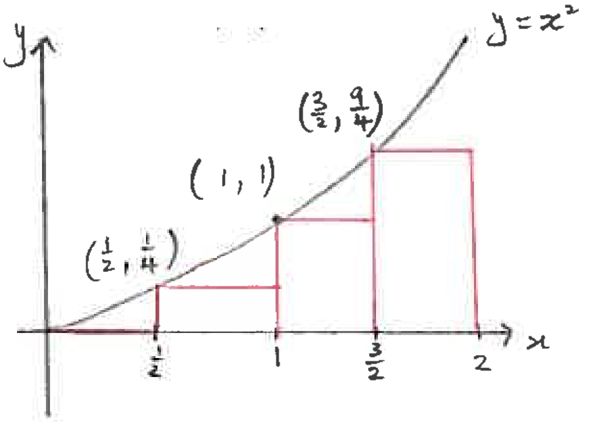
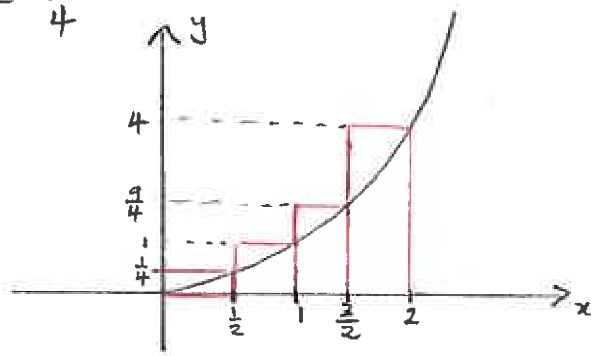
(5)



$A_L = \left(\frac{1}{2} \times 0\right) + \left(\frac{1}{2} \times \frac{1}{4}\right) + \left(\frac{1}{2} \times 1\right) + \left(\frac{1}{2} \times \frac{9}{4}\right)$

$= \frac{1}{8} + \frac{4}{8} + \frac{9}{8}$

$= \frac{7}{4}$



$A_R = \left(\frac{1}{2} \times \frac{1}{4}\right) + \left(\frac{1}{2} \times 1\right) + \left(\frac{1}{2} \times \frac{9}{4}\right) + \left(\frac{1}{2} \times 4\right)$

$= \frac{1}{8} + \frac{4}{8} + \frac{9}{8} + \frac{16}{8}$

$= \frac{30}{8}$

$= \frac{15}{4}$

$A_L < A < A_R$

$\therefore \quad \frac{7}{4} < A < \frac{15}{4}$

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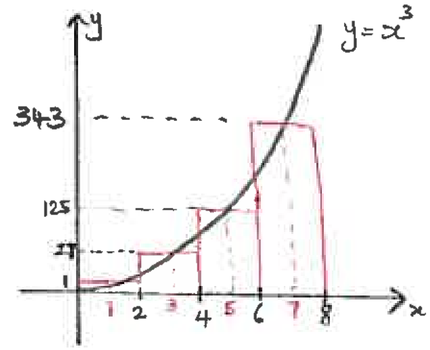
(6)



$A_m = (2 \times 1) + (2 \times 27) + (2 \times 125) + (2 \times 343)$

$= 2 + 54 + 250 + 686$

$= 992$

True area

$\approx \int_0^8 x^3 \, dx = \left[\frac{x^4}{4}\right]_0^8$

$= 1024$

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(7) $\int_1^3 (3f(x) - 2g(x)) \, dx$

$= 3 \int_1^3 f(x) \, dx - 2 \int_1^3 g(x) \, dx$

$= 3\left(\int_1^2 f(x) \, dx + \int_2^3 f(x) \, dx\right) - 2 \times 4$

$= 3(2 + 3) - 8$

$= 15 - 8$

$= 7$

(8) $\int_0^2 \left(2f(x) - 3g(x)\right) dx$

$= 2\int_0^2 f(x)\,dx - 3\int_0^2 g(x)\,dx$

$= 2\left[\int_0^1 f(x)\,dx + \int_1^2 f(x)\,dx\right] - 3\times 4$

$= 2\left(2+3\right) - 12$

$= 10 - 12$

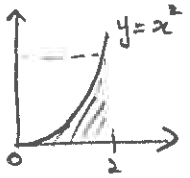
$= -2$

(9) (a)

$A = \int_0^2 x^2\,dx$

$= \left[\frac{x^3}{3}\right]_0^2$

$= \frac{8}{3}$

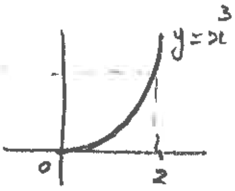


(b) $A = \int_0^2 x^3\,dx$

$= \left[\frac{x^4}{4}\right]_0^2$

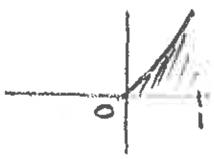
$= \frac{16}{4}$

$= 4$



(c) $y = x^2+2x$

$y = x(x+2)$



$A = \int_0^1 (x^2+2x)\,dx$

$= \left[\frac{x^3}{3} + x^2\right]_0^1$

$= \frac{1}{3} + 1$

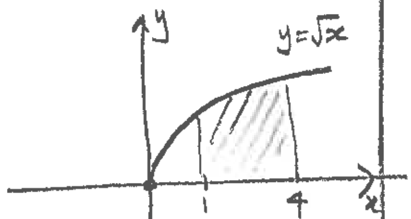
$= \frac{4}{3}$

(d) $A = \int_1^4 \sqrt{x}\,dx$

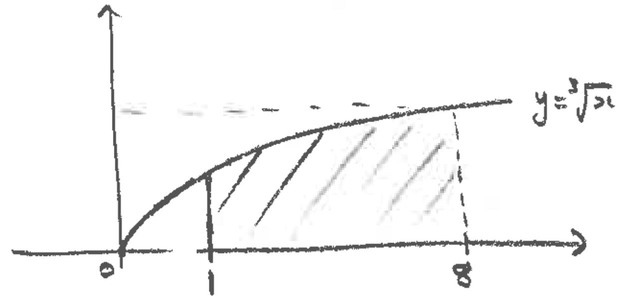
$= \int_1^4 x^{1/2}\,dx$

$= \left[\frac{x^{3/2}}{3/2}\right]_1^4$

$= \frac{2}{3}\times 4^{3/2} - \frac{2}{3} = \frac{14}{3}$



(e)



$A = \int_1^8 x^{\frac{1}{3}}\,dx$

$= \left[\frac{3}{4}x^{\frac{4}{3}}\right]_1^8$

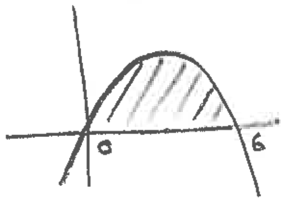
$= \frac{3}{4}\times 8^{4/3} - \frac{3}{4}\times 1$

$= 12 - \frac{3}{4}$

$= \frac{45}{4}$

(10) $y = 6x - x^2$

$= x(6-x)$



$A = \int_0^6 6x - x^2\,dx$

$= \left[3x^2 - \frac{x^3}{3}\right]_0^6$

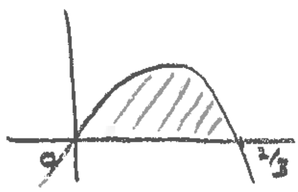
$= 108 - \frac{6^3}{3}$

$= 36$

(11) $y = 4x - 6x^2$

$= 2x(2-3x)$

Xint=0 $x = \frac{2}{3}$



$A = \int_0^{2/3} 4x - 6x^2\,dx$

$= \left[2x^2 - 2x^3\right]_0^{2/3}$

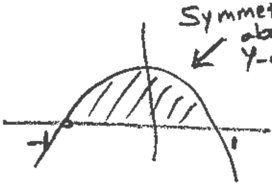
$= 2\times\frac{4}{9} - 2\times\frac{8}{27}$

$= \frac{8}{27}$

(12) $y = 1 - x^2$

$= (1-x)(1+x)$

Symmetry about Y-axis



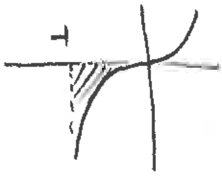
$$A = 2\int_0^1 1 - x^2\, dx$$

$$= 2\left[x - \frac{x^3}{3}\right]_0^1$$

$$= 2\left[1 - \frac{1}{3}\right]$$

$$= \frac{4}{3}$$

(13) $$\int_{-1}^{0} 4x^3\, dx$$



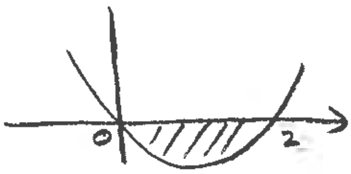
$$= \left[x^4\right]_{-1}^{0}$$

$$= 0 - (-1)^4$$

$$= -1$$

Negative because area is below the x-axis

(14) $y = 6x(x-2)$



$$A = -\int_0^2 6x(x-2)\, dx$$

$$= -\int_0^2 6x^2 - 12x\, dx$$

$$= -\left[2x^3 - 6x^2\right]_0^2$$

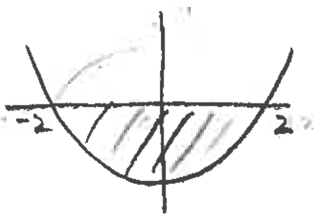
$$= -(2 \times 2^3 - 6 \times 2^2)$$

$$= -(16 - 24)$$

$$= 8$$

(15) $y = x^2 - 4$

$= (x-2)(x+2)$



$$A = -2\int_0^2 x^2 - 4\, dx$$

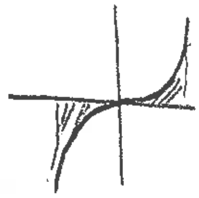
$$= -2\left[\frac{x^3}{3} - 4x\right]_0^2$$

$$= -2\left(\frac{8}{3} - 8\right)$$

$$= -2\left(\frac{-16}{3}\right)$$

$$= \frac{32}{3}$$

(16) $$\int_{-1}^{1} 4x^3\, dx$$



$$= \left[x^4\right]_{-1}^{1}$$

$$= 1 - (-1)^4$$

$$= 0$$

This is because area below x-axis is the same as area above x-axis

(19) (a) $f(x) = x(x-1)$

* For Q17, 18 go to end of solutions

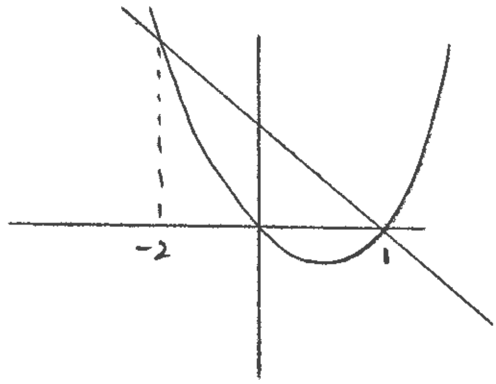
$f(x) = g(x)$

$x^2 - x = 2 - 2x$

$x^2 + x - 2 = 0$

$(x+2)(x-1) = 0$

$x = -2,\ x = 1$



$$A = \int_{-2}^{1} (2-2x) - (x^2 - x)\, dx$$

$$= \int_{-2}^{1} 2 - x - x^2\, dx$$

$$= \left[2x - \frac{x^2}{2} - \frac{x^3}{3}\right]_{-2}^{1}$$

$$= 2 - \frac{1}{2} - \frac{1}{3} - \left(-4 - \frac{4}{2} - \frac{-8}{3}\right)$$

$$= 2 - \frac{1}{2} - \frac{1}{3} + 4 + 2 - \frac{8}{3}$$

$$= 4.5$$

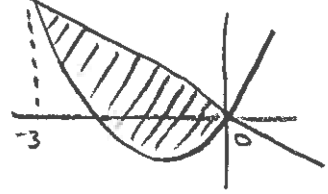
(b) Let $x^2 + 2x = -x$

$x^2 + 2x + x = 0$

$x^2 + 3x = 0$

$x(x+3) = 0$

$x = 0, -3$



$$A = \int_{-3}^{0} -x - (x^2 + 2x)\, dx$$

$$A = \int_{-3}^{0} (-x - x^2 - 2x)\, dx$$

$$= \int_{-3}^{0} -3x - x^2 \; dx$$

$$= \left[-\frac{3x^2}{2} - \frac{x^3}{3} \right]_{-3}^{0}$$

$$= -\left(-\frac{3(-3)^2}{2} - \frac{(-3)^3}{3} \right)$$

$$= -\left(\frac{-27}{2} + \frac{27}{3} \right)$$

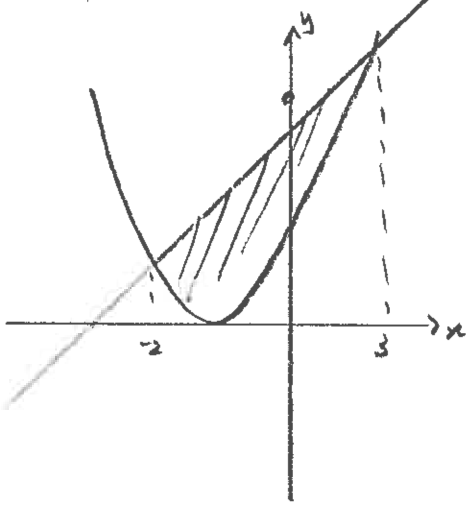
$$= 4.5$$

(c) Let $x^2 + 2x + 1 = 3x + 7$

$x^2 - x - 6 = 0$

$(x-3)(x+2) = 0$

$x = 3, -2$



$$A = \int_{-2}^{3} 3x + 7 - (x^2 + 2x + 1)\, dx$$

$$= \int_{-2}^{3} 3x + 7 - x^2 - 2x - 1 \; dx$$

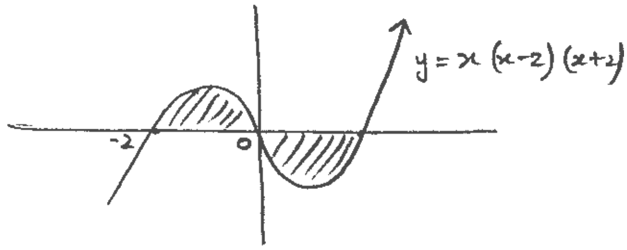
$$= \int_{-2}^{3} x + 6 - x^2 \; dx$$

$$= \left[\frac{x^2}{2} + 6x - \frac{x^3}{3} \right]_{-2}^{3}$$

$$= \left[\frac{9}{2} + 18 - \frac{27}{3} - \left(\frac{4}{2} - 12 - \frac{-8}{3} \right) \right]$$

$$= \frac{9}{2} + 18 - 9 - 2 + 14 - \frac{8}{3} = \frac{137}{6}$$

(20)



$x(x-2)(x+2)$

$= x(x^2 - 4)$

$= x^3 - 4x$

$$A = 2\int_{-2}^{0} x^3 - 4x \; dx$$

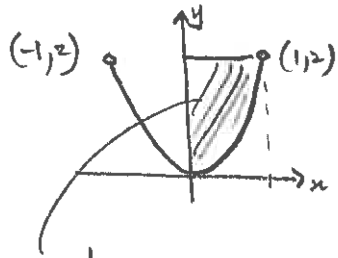
$$= 2\left[\frac{x^4}{4} - 2x^2 \right]_{-2}^{0}$$

$$= -2\left[\frac{(-2)^4}{4} - 2(-2)^2 \right]$$

$$= -2\,(4 - 8)$$

$$= 8$$

(21) (a)



$$A = \int_{0}^{1} 2 - 2x^2 \; dx$$

$$= \left[2x - \frac{2x^3}{3} \right]_{0}^{1}$$

$$= 2 - \frac{2}{3}$$

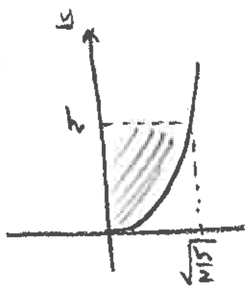
$$= \frac{4}{3}$$

$h = 2x^2$

$x = \sqrt{\frac{h}{2}}$

$$V = \frac{8}{3} \times 6 = 16$$

(b)



$$\int_{0}^{\sqrt{\frac{h}{2}}} h - 2x^2 \; dx = \frac{2}{3}$$

$$\left[hx - \frac{2x^3}{3} \right]_{0}^{\sqrt{\frac{h}{2}}} = \frac{2}{3}$$

$$h\sqrt{\frac{h}{2}} - \frac{2\left(\sqrt{\frac{h}{2}}\right)^3}{3} = \frac{2}{3}$$

$$3h\sqrt{\frac{h}{2}} - 2\,\frac{h}{2}\sqrt{\frac{h}{2}} = 2$$

$$\frac{3h^{\frac{3}{2}}}{\sqrt{2}} - \frac{h^{3/2}}{\sqrt{2}} = 2$$

$$3h^{\frac{3}{2}} - h^{\frac{3}{2}} = 2\sqrt{2}$$

$$2h^{\frac{3}{2}} = 2\sqrt{2}$$

$$h^{3/2} = \sqrt{2}$$

$$h^{\frac{3}{2}} = 2^{\frac{1}{2}}$$

$$h = \left(2^{\frac{1}{2}}\right)^{\frac{2}{3}}$$

$$h = 2^{1/3}$$

$$h = \sqrt[3]{2}$$

(22) (a)

$$\frac{d}{dx}(x^2+3)^5 = 5(x^2+3)^4 \times 2x$$

$$= 10x(x^2+3)^4$$

(b) $$\int_1^2 10x(x^2+3)^4\,dx$$

$$= \left[(x^2+3)^5\right]_1^2$$

$$= (2^2+3)^5 - (1^2+3)^5$$

$$= 7^5 - 4^5$$

$$= 15783$$

(23) (a) $$\frac{d}{dx}\,x\sqrt{x+1}$$

$$= \frac{d}{dx}\,x(x+1)^{\frac{1}{2}}$$

$$= u\frac{dv}{dx} + v\frac{du}{dx}$$

$$= x \cdot \tfrac{1}{2}(x+1)^{-\frac{1}{2}} + (x+1)^{\frac{1}{2}} \cdot 1$$

$$= \frac{x}{2\sqrt{x+1}} + \sqrt{x+1}$$

$$= \frac{x + 2(x+1)}{2\sqrt{x+1}}$$

$$= \frac{3x+2}{2\sqrt{x+2}}$$

(b) $$\int_1^3 \frac{3x+2}{2\sqrt{x+2}}\,dx$$

$$= \left[x\sqrt{x+1}\right]_1^3$$

$$= 3\sqrt{4} - \sqrt{2}$$

$$= 6 - \sqrt{2}$$

(24) (a) $$\frac{d}{dx}\,\frac{x}{x^2+1}$$

$$= \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$$

$$= \frac{(x^2+1)\cdot 1 - x \cdot 2x}{(x^2+1)^2}$$

$$= \frac{x^2+1-2x^2}{(x^2+1)^2}$$

$$= \frac{1-x^2}{(x^2+1)^2}$$

(b) $$\int_1^3 \frac{1-x^2}{(x^2+1)^2}\,dx$$

$$= \left[\frac{x}{x^2+1}\right]_1^3$$

$$= \frac{3}{10} - \frac{1}{2}$$

$$= -\frac{1}{5}$$

(25) $$\int_0^1 4x^n\,dx = \frac{2}{3}$$

$$\left[\frac{4x^{n+1}}{n+1}\right]_0^1 = \frac{2}{3}$$

$$\frac{4}{n+1} = \frac{2}{3}$$

$$12 = 2n+2$$

$$n = 5$$

(26) $\int_1^2 x^n \, dx = \frac{31}{5}$

$\left[\frac{x^{n+1}}{n+1}\right]_1^2 = \frac{31}{5}$

$\frac{2^{n+1}}{n+1} - \frac{1}{n+1} = \frac{31}{5}$

$\frac{2^{n+1}-1}{n+1} = \frac{31}{5}$

Try $n=4$ since $n+1=5$

$\frac{2^5-1}{4+1} = \frac{32-1}{5} = \frac{31}{5}$

So $n=4$

---

(27) $\int_0^2 ax(2-x)\,dx = 1$

$\int_0^2 2ax - ax^2 \, dx = 1$

$\left[ax^2 - \frac{ax^3}{3}\right]_0^2 = 1$

$4a - \frac{8a}{3} = 1$

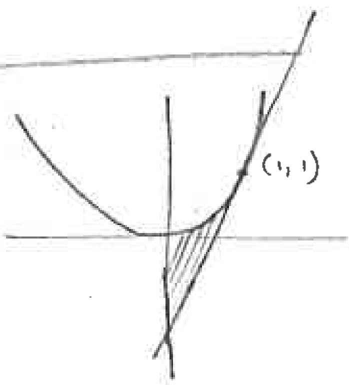
$\frac{12a-8a}{3} = 1$

$4a = 3$

$a = \frac{3}{4}$

---

(28)



$y = x^2$

$\frac{dy}{dx} = 2x$

$x=1, \ \frac{dy}{dx} = 2$

Eq of tangent

$y - y_1 = m(x - x_1)$

$y - 1 = 2(x-1)$

$y = 2x - 1$

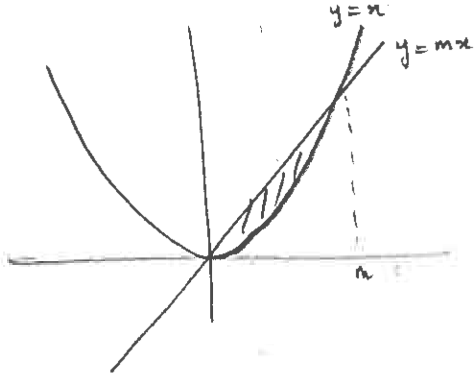
$A = \int_0^1 x^2 - (2x-1)\,dx$

$= \int_0^1 x^2 - 2x + 1 \, dx$

$= \left[\frac{x^3}{3} - x^2 + x\right]_0^1$

$= \frac{1}{3} - 1 + 1 = \frac{1}{3}$

(29)



Let $x^2 = mx$

$x^2 - mx = 0$

$x(x-m) = 0$

$x = 0, m$

$\int_0^m mx - x^2 \, dx = 1$

$\left[\frac{mx^2}{2} - \frac{x^3}{3}\right]_0^m = 1$

$\frac{m^3}{2} - \frac{m^3}{3} = 1$

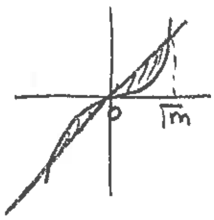
$\frac{m^3}{6} = 1$

$m^3 = 6$

$m = \sqrt[3]{6}$

---

(30) Let $x^3 = mx$



$x^3 - mx = 0$

$x(x^2 - m) = 0$

$x = 0, \ x = \pm\sqrt{m}$

$2\int_0^{\sqrt{m}} mx - x^3 \, dx = 1$

$\left[\frac{mx^2}{2} - \frac{x^4}{4}\right]_0^{\sqrt{m}} = \frac{1}{2}$

$\frac{m^2}{2} - \frac{m^2}{4} = \frac{1}{2}$

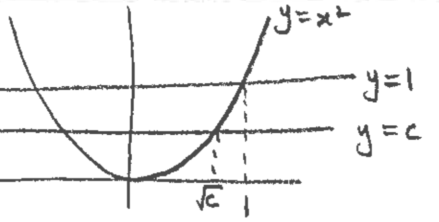
$\frac{m^2}{4} = \frac{1}{2}$

$m^2 = 2$

$m = \sqrt{2}$ since $m > 0$

---

(31)



$x^2 = c$

$\therefore x = \pm\sqrt{c}$

$$\int_0^1 1-x^2\,dx = 2\int_0^{\sqrt{c}} c-x^2\,dx$$

$$\left[x-\frac{x^3}{3}\right]_0^1 = 2\left[cx-\frac{x^3}{3}\right]_0^{\sqrt{c}}$$

$$\frac{2}{3} = 2\left(c\sqrt{c}-\frac{c\sqrt{c}}{3}\right)$$

$$\frac{2}{3} = 2\left(\frac{2c\sqrt{c}}{3}\right)$$

$$c\sqrt{c} = \frac{1}{2}$$

$$c^{\frac{3}{2}} = \frac{1}{2}$$

$$c = \left(\frac{1}{2}\right)^{2/3}$$

---

(32) $\int_0^a x(a-x)\,dx = 30$

$\therefore \int_0^a xa-x^2\,dx = 30$

$\therefore \left[\frac{ax^2}{2}-\frac{x^3}{3}\right]_0^a = 30$

$\frac{a^3}{2}-\frac{a^3}{3} = 30$

$\frac{a^3}{6} = 30$

$a^3 = 180$

$a = \sqrt[3]{180}$

---

(33) $\int_{-1}^1 (x^2+ax+b)(cx+d)\,dx = 0$

$\therefore \int_{-1}^1 cx^3+(d+ac)x^2+(ad+bc)x+bd\,dx = 0$

$\therefore \left[\frac{cx^4}{4}+\frac{(d+ac)x^3}{3}+\frac{(ad+bc)x^2}{2}+bdx\right]_{-1}^1 = 0$

$\frac{c}{4}+\frac{(d+ac)}{3}+\frac{(ad+bc)}{2}+bd$

$-\frac{c}{4}+\frac{(d+ac)}{3}-\frac{(ad+bc)}{2}+bd = 0$

$\frac{2(d+ac)}{3}+2bd = 0$

$\frac{2d}{3}+\frac{2ac}{3}+2bd = 0$

Let $a=0$

$\frac{2d}{3}+2bd = 0$

$d\left(\frac{2}{3}+2b\right) = 0$

Let $2b+\frac{2}{3} = 0$

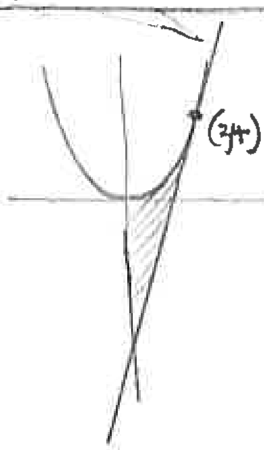
$b = -\frac{1}{3}$

$a=0,\ b=-\frac{1}{3}$

A surprising result.

---

(34) $y = x^2$



$\frac{dy}{dx} = 2x$

$x=2,\ \frac{dy}{dx} = 4$

$y-y_1 = m(x-x_1)$

$y-4 = 4(x-2)$

$y = 4x-4$

$A = \int_0^2 x^2-(4x-4)\,dx$

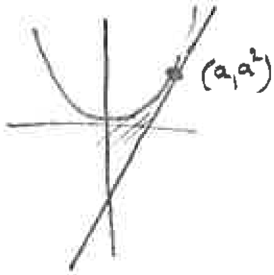
$= \left[\frac{x^3}{3}-\frac{4x^2}{2}+4x\right]_0^2$

$= \frac{8}{3}-\frac{16}{2}+8$

$= \frac{8}{3}$

---

(35) $y = x^2$



$\frac{dy}{dx} = 2x$

$x=a,\ \frac{dy}{dx} = 2a$

$y-y_1 = m(x-x_1)$

$y-a^2 = 2a(x-a)$

$y = 2ax-a^2$

$\int_0^a x^2-(2ax-a^2)\,dx = 4$

$\left[\frac{x^3}{3}-ax^2+a^2x\right]_0^a = 4$

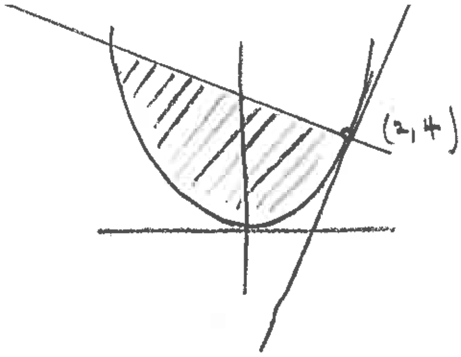
$\frac{a^3}{3}-a^3+a^3 = 4$

$a^3 = 12$

$a = \sqrt[3]{12}$

Or by symmetry $a = -\sqrt[3]{12}$

(36)



$y = x^2$

$\frac{dy}{dx} = 2x$

$x = 2,\ \frac{dy}{dx} = 4$

$m_\perp = -\frac{1}{4}$

$y - 4 = -\frac{1}{4}(x-2)$

$y = -\frac{1}{4}x + \frac{9}{2}$

Let $x^2 = -\frac{1}{4}x + \frac{9}{2}$

$4x^2 = -x + 18$

$4x^2 + x - 18 = 0$

$4x \quad 9$

$x \quad -2$

$(4x+9)(x-2) = 0$

$x = -\frac{9}{4}$

$$A = \int_{-\frac{9}{4}}^{2} -\frac{1}{4}x + \frac{9}{2} - x^2 \, dx$$

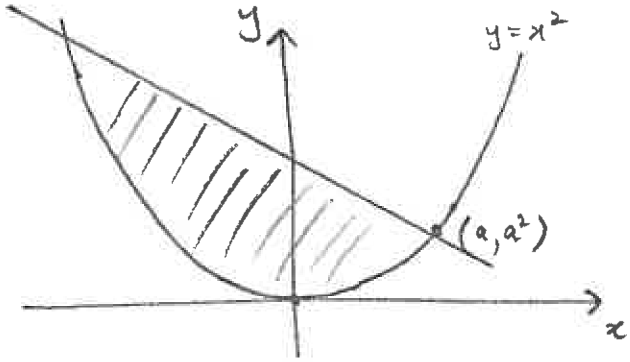
$$= \left[-\frac{x^2}{8} + \frac{9x}{2} - \frac{x^3}{3} \right]_{-\frac{9}{4}}^{2}$$

$$= -\frac{1}{2} + 9 - \frac{8}{3} - \left(-\frac{81}{128} - \frac{81}{8} + \frac{243}{64} \right)$$

$$= -\frac{1}{2} + 9 - \frac{8}{3} + \frac{81}{128} + \frac{81}{8} - \frac{243}{64}$$

$$= \frac{4913}{384}$$

(37)



$y = x^2$

$\frac{dy}{dx} = 2x$

$x = a,\ \frac{dy}{dx} = 2a$

$m_\perp = -\frac{1}{2a}$ (normal)

$y - a^2 = -\frac{1}{2a}(x - a)$

$y = -\frac{1}{2a}x + \frac{1}{2} + a^2$

Point of intersection

$x^2 = -\frac{1}{2a}x + \frac{1}{2} + a^2$

$x^2 + \frac{1}{2a}x - \frac{1}{2} - a^2 = 0$

$\left(x + \frac{1}{4a}\right)^2 - \frac{1}{16a^2} - \frac{1}{2} - a^2 = 0$

$\left(x + \frac{1}{4a}\right)^2 = \frac{1}{16a^2} + \frac{1}{2} + a^2$

$\left(x + \frac{1}{4a}\right)^2 = \frac{1 + 8a^2 + 16a^4}{16a^2}$

$\left(x + \frac{1}{4a}\right)^2 = \frac{(1+4a^2)^2}{16a^2}$

$x + \frac{1}{4a} = \pm \frac{1+4a^2}{4a}$

In this case we will take − refer to diagram

$x = -\frac{1}{4a} - \frac{1+4a^2}{4a}$

$x = -\frac{2+4a^2}{4a} = -\frac{1+2a^2}{2a}$

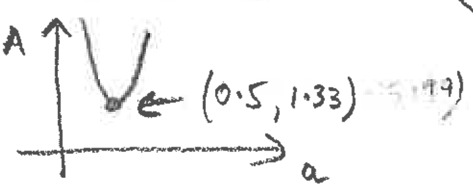
$$A = \int_{-\frac{1+2a^2}{2a}}^{a} -\frac{1}{2a}x + \frac{1}{2} + a^2 - x^2 \, dx$$

$$A = \left[-\frac{x^2}{4a} + \frac{1}{2}x + a^2 x - \frac{x^3}{3} \right]_{-\frac{1+2a^2}{2a}}^{a}$$

$$= -\frac{a^2}{4a} + \frac{a}{2} + a^3 - \frac{a^3}{3} + \frac{(1+2a^2)^2}{16a^3} + \frac{1+2a^2}{4a} + \frac{a+2a^3}{2} - \frac{(1+2a)^3}{192a^3}$$

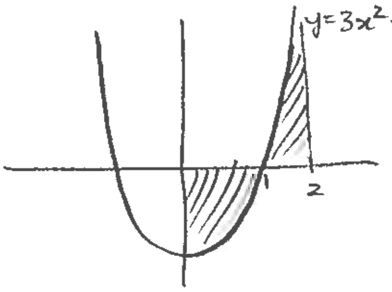
$$A = \frac{3a}{4} + \frac{5a^3}{3} + \frac{(1+2a^2)^2}{16a^3} + \frac{1+2a^2}{4a} - \frac{(1+2a^2)^3}{24a^3}$$

Graph on calculator



$a \simeq 0{\cdot}5$

(Q17)



$x=2,\ y=3\times4-3=9$

X int: $y=0$

$3x^2-3=0$

$x^2=1$

$x=1$

$$A=-\int_0^1 3x^2-3\,dx+\int_1^2 3x^2-3\,dx$$

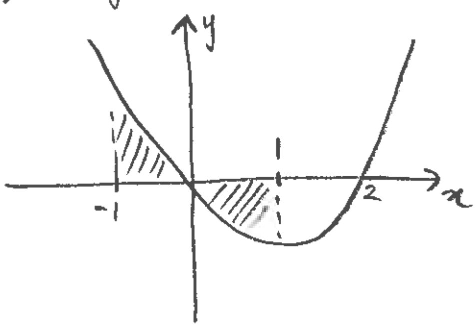
$$=-\left[x^3-3x\right]_0^1+\left[x^3-3x\right]_1^2$$

$$=-\left[1-3\right]+\left[8-6-(1-3)\right]$$

$$=2+\left[2+2\right]$$

$$=6$$

(18) $y=x(x-2)$



$$A=\int_{-1}^{0} x^2-2x\,dx-\int_0^1 x^2-2x\,dx$$

$$=\left[\frac{x^3}{3}-x^2\right]_{-1}^{0}-\left[\frac{x^3}{3}-x^2\right]_0^1$$

$$=-\left(\frac{-1}{3}-1\right)-\left[\frac{1}{3}-1\right]$$

$$=\frac{1}{3}+1-\frac{1}{3}+1$$

$$=2$$