#1

INTEGRAL CALCULUS

(Q1)

(A) $F'(x) = 4(3x+4)^3 \times 3$

$= 12(3x+4)^3$

$= f(x)$

(B) $F(x) = (3x-1)^{1/2}$

$\therefore F'(x) = \frac{1}{2}(3x-1)^{-1/2} \cdot 3$

$= \frac{3}{2\sqrt{3x-1}}$

$= f(x)$

(C) $F(x) = 2(3x-2)^{-3}$

$\therefore F'(x) = -3 \times 2(3x-2)^{-4} \cdot 3$

$= \frac{-18}{(3x-2)^4}$

$= f(x)$

(D) $F(x) = 2e^{3x^2-5}$

$F'(x) = 2 \times 6x \cdot e^{3x^2-5}$

$= 12x\, e^{3x^2-5}$

$= f(x)$

(E) $F(x) = x(\cos x + \sin x)$

$F'(x) = x(-\sin x + \cos x) + (\cos x + \sin x)$

$= (x+1)\cos x + (1-x)\sin x$

(F) $F(x) = \frac{\sin x}{x+1}$

$F'(x) = \frac{(x+1)\cos x - \sin x}{(x+1)^2}$

$= f(x)$

(G) $F(x) = 2\log_e(2x-3)$

$F'(x) = 2 \cdot \frac{2}{2x-3}$

$= \frac{4}{2x-3}$

$= f(x)$

(Q2) $F(x) = \log_e 5x$ $\quad G(x) = \log_e(4x)$

$F'(x) = \frac{5}{5x}$ $\quad G'(x) = \frac{4}{4x}$

$= \frac{1}{x}$ $\quad = \frac{1}{x}$

$= f(x)$ $\quad = f(x)$

(Q3) (A) $\int \frac{2x+3}{x^3}\, dx$

$= \int 2x^{-2} + 3x^{-3}\, dx$

$= \frac{2x^{-1}}{-1} + \frac{3x^{-2}}{-2} + c$

$= -\frac{2}{x} - \frac{3}{2x^2} + c$

(B) $\int 4x^{1/2} - 2x^{1/3}\, dx$

$= \frac{4x^{3/2}}{3/2} - \frac{2x^{4/3}}{4/3} + c$

$= \frac{8}{3}x^{3/2} - \frac{3}{2}x^{4/3} + c$

(C) $\int \frac{1}{x+2} - \frac{1}{x-2}\, dx$

$= \log_e(x+2) - \log_e(x-2) + c$

$= \log_e\left(\frac{x+2}{x-2}\right) + c$

(D) $\int \frac{1}{3x+2}\, dx$

$= \frac{1}{3}\log_e(3x+2) + c$

(E) $\int \frac{1}{2x-1} + (2x-1)^{-2}\, dx$

$= \frac{1}{2}\log_e(2x-1) + \frac{(2x-1)^{-1}}{-2} + c$

$= \frac{1}{2}\log_e(2x-1) - \frac{1}{2x-1} + c$

(F) $\int e^{2x} - e^{-2x}\, dx$

$= \frac{1}{2}e^{2x} + \frac{1}{2}e^{-2x} + c$

(G) $\int (3x-2)^{-1/2} + e^{3x}\, dx$

$= \frac{(3x-2)^{1/2}}{1/2} + \frac{1}{3}e^{3x} + c$

$= 2\sqrt{3x-2} + \frac{1}{3}e^{3x} + c$

(H) $\int \cos x + \sin 3x\, dx$

$= +\sin x - \frac{1}{3}\cos 3x + c$

(I) $\int 4\cos 4x - 9\sin 3x\, dx$

$= +\sin 4x + 3\cos 3x + c$

(Q4) (A) $\frac{dy}{dx} = 3x^2 - 5x - 2$

$y = \int 3x^2 - 5x - 2 \, dx$

$= x^3 - \frac{5x^2}{2} - 2x + c$

$x=1, y=2 \quad \therefore \quad 2 = 1 - \frac{5}{2} - 2 + c$

$2 = -\frac{7}{2} + c$

$c = 2 + \frac{7}{2}$

$= \frac{11}{2}$

$\therefore \quad y = x^3 - \frac{5}{2}x^2 - 2x + \frac{11}{2}$

(B) $f'(x) = \cos x$

$f(x) = \int \cos x \, dx$

$= \sin x + c$

$f(0)=1 \quad \therefore \quad 1 = \sin 0 + c$

$c = 1$

$\therefore \quad f(x) = \sin x + 1$

(C) $\frac{dy}{dx} = e^x - x$

$y = \int e^x - x \, dx$

$= e^x - \frac{x^2}{2} + c$

$x=0, y=2 \quad \therefore \quad 2 = e^0 - 0^2/2 + c$

$2 = 1 + c$

$c = 1$

$\therefore \quad y = e^x - \frac{x^2}{2} + 1$

(D) $f'(x) = \frac{1}{2x-5}$

$f(x) = \int \frac{1}{2x-5} \, dx$

$= \frac{1}{2}\log_e(2x-5) + c$

$f(3)=2 \quad \therefore \quad 2 = \frac{1}{2}\log_e(1) + c$

$\therefore \quad c = 2$

$\therefore \quad f(x) = \frac{1}{2}\log_e(2x-5) + 2$

(E) $\frac{dy}{dx} = 2\cos 2x - 3\sin 3x$

$y = \int 2\cos 2x - 3\sin 3x \, dx$

$= +\sin 2x + \cos 3x + c$

$x = \pi/6, y = 1 \qquad 1 = \sin\frac{\pi}{3} + \cos\frac{\pi}{2} + c$

$1 = \frac{\sqrt{3}}{2} + 0 + c$

$c = 1 - \sqrt{3}/2$

$\therefore \quad y = \sin 2x + \cos 3x + 1 - \frac{\sqrt{3}}{2}$

(F) $\frac{dy}{dx} = (2x+3)^{1/2}$

$y = \int (2x+3)^{1/2} \, dx$

$= \frac{(2x+3)^{3/2}}{3/2} + c$

$= \frac{2}{3}(2x+3)^{3/2} + c$

$x=3, y=-2 \qquad -2 = \frac{2}{3}(2\cdot 3+3)^{3/2} + c$

$-2 = \frac{2}{3}\cdot 9^{3/2} + c$

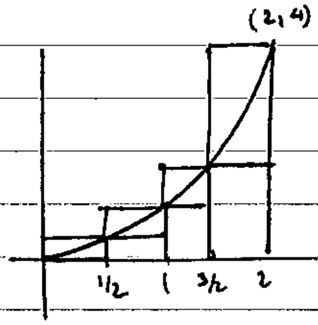
$-2 = \frac{2}{3}\cdot 3^3 + c$

$-2 = 2\cdot 9 + c$

$c = -20$

$\therefore \quad y = \frac{2}{3}(2x+3)^{3/2} - 20.$

(Q5)



LEFT SUM $= \frac{1}{2}\cdot 0^2 + \frac{1}{2}\cdot\left(\frac{1}{2}\right)^2 + \frac{1}{2}\cdot 1^2 + \frac{1}{2}\cdot\left(\frac{3}{2}\right)^2$

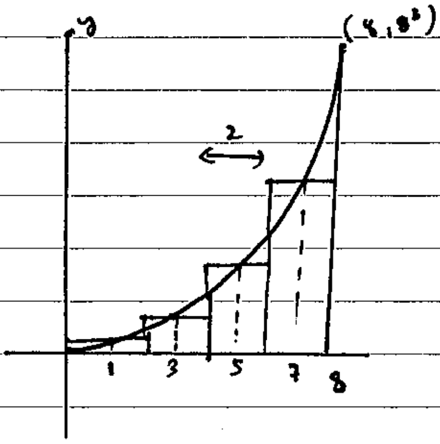
$= \frac{7}{4} \longleftarrow$ UNDER ESTIMATE

RIGHT SUM $= \frac{1}{2}\cdot\left(\frac{1}{2}\right)^2 + \frac{1}{2}\cdot 1^2 + \frac{1}{2}\cdot\left(\frac{3}{2}\right)^2 + \frac{1}{2}\cdot 2^2$

$= \frac{15}{4} \longleftarrow$ OVER ESTIMATE

$\therefore \quad \frac{7}{4} < A < \frac{15}{4}.$

(Q6)



MID POINT SUM $= 2\cdot 1^3 + 2\cdot 3^3 + 2\cdot 5^3 + 2\cdot 7^3$

$= 992$

(Q7) $\int_0^3 3f(x) + 2g(x)\ dx$

$= 3\int_0^3 f(x) + 2\int_0^3 g(x)\ dx$

$= 3\left(\int_0^1 f(x)\ dx + \int_1^2 f(x)\ dx\right)$

$+ 2\int_0^3 g(x)\ dx$

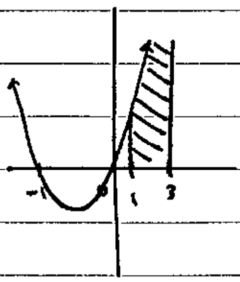
$= 3\cdot(3+4) + 2\cdot 3$

$= 3\cdot 7 + 6$

$= 27.$

(Q8) (a) $y = x^2 + x$

$= x(x+1)$



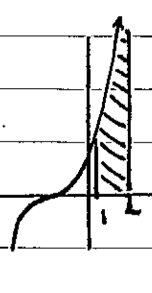
$A = \int_1^3 x^2 + x\ dx$

$= \left[\frac{x^3}{3} + \frac{x^2}{2}\right]_1^3$

$= \left(9 + \frac{9}{2}\right) - \left(\frac{1}{3} + \frac{1}{2}\right)$

$= \frac{38}{3}$

(b) $y = (2x+1)^3$



$A = \int_1^2 (2x+1)^3\ dx$

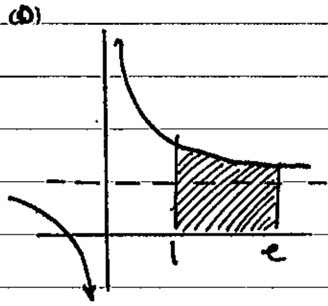
$= \left[\frac{(2x+1)^4}{4\cdot 2}\right]_1^2$

$= \left[\frac{(2x+1)^4}{8}\right]_1^2$

$= \frac{5^4}{8} - \frac{3^4}{8}$

$= 68$

(D)



$A = \int_1^e \frac{1}{x} + 1\ dx$

$= [\log_e x + x]_1^e$

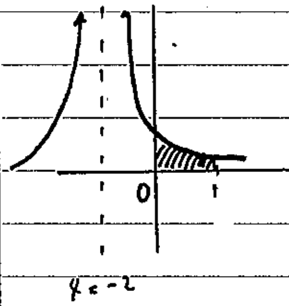
$= (\log_e e + e) - (\log_e 1 + 1)$

$= (1+e) - (0+1)$

$= 1 + e - 1$

$= e$

(C)



$A = \int_0^1 \frac{9}{(x+2)^2}\ dx$

$= \int_0^1 9(x+2)^{-2}\ dx$

$= \left[\frac{9(x+2)^{-1}}{-1}\right]_0^1$

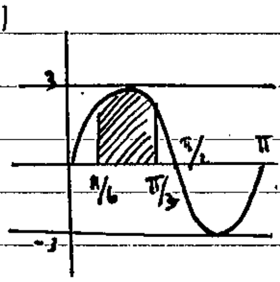
$= \left[\frac{-9}{x+2}\right]_0^1$

$= -\frac{9}{3} + \frac{9}{2}$

$= -3 + \frac{9}{2}$

$= \frac{3}{2}$

(E)



$A = \int_{\pi/6}^{\pi/3} 3\sin(2x)\ dx$

$= \left[-\frac{3}{2}\cos 2x\right]_{\pi/6}^{\pi/3}$

$= -\frac{3}{2}\cos\frac{2\pi}{3} + \frac{3}{2}\cos\frac{\pi}{3}$

$= -\frac{3}{2}\cdot\left(-\frac{1}{2}\right) + \frac{3}{2}\cdot\frac{1}{2}$

$= \frac{3}{2}$

(F) $A = \int_0^1 2e^{2x} + x\ dx$

$= \left[e^{2x} + x^2/2\right]_0^1$

$= \left(e^2 + \frac{1}{2}\right) - (e^0 + 0)$

$= e^2 + \frac{1}{2} - 1$

$= e^2 - \frac{1}{2}$

(6) $A = \int_4^{12} (2x+1)^{1/2}\, dx$

$= \left[\frac{(2x+1)^{3/2}}{3/2 \cdot 2} \right]_4^{12}$

$= \left[\frac{1}{3} (2x+1)^{3/2} \right]_4^{12}$

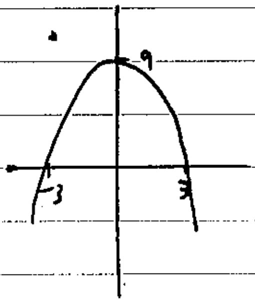
$= \frac{1}{3} (25)^{3/2} - \frac{2}{3} (9)^{3/2}$

$= \frac{1}{3} \cdot 5^3 - \frac{2}{3} \cdot 3^3$

$= \frac{1}{3} (125 - 27)$

$= \frac{98}{3}$

(Q9)



$A = \int_{-3}^{3} 9 - x^2\, dx$

$= 2\int_0^3 9 - x^2\, dx$

$= 2 \left[9x - x^3/3 \right]_0^3$

$= 2\left(\left(9 \cdot 3 - 27/3 \right) - 0 \right)$

$= 2 \, (27 - 9)$

$= 36$.

(Q10) $y = 2e^{-x} - 1$ WHEN $y = 0$

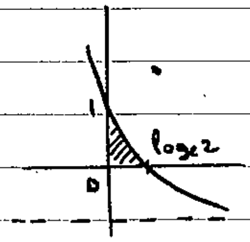
$0 = 2e^{-x} - 1$

$e^{-x} = \frac{1}{2}$

$-x = \log_e \frac{1}{2}$

$x = -\log_e \frac{1}{2}$

$= \log_e 2$



$A = \int_0^{\log_e 2} 2e^{-x} - 1\, dx$

$= \left[-2e^{-x} - x \right]_0^{\log_e 2}$

$= \left(-2e^{-\log_e 2} - \log_e 2 \right) - \left(-2 \cdot e^0 - 0 \right)$

$= \left(-2 \cdot \frac{1}{2} - \log_e 2 \right) + 2$

$= 1 - \log_e 2$

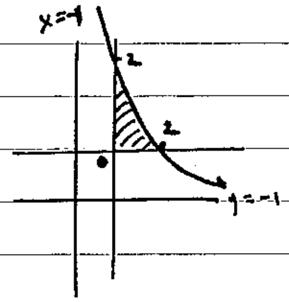
(Q11) $y = \frac{3}{x+1} - 1$ $x = 0,\ y = 2$

$y = 0,\ 0 = \frac{3}{x+1} - 1$

$1 = \frac{3}{x+1}$

$x + 1 = 3$

$x = 2$



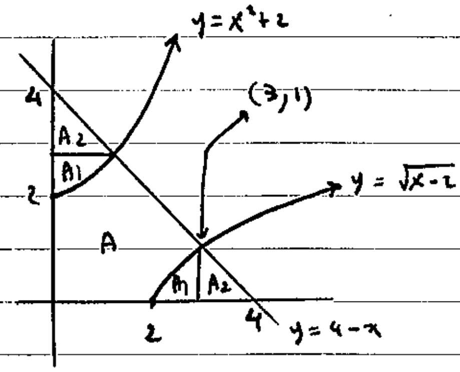
$A = \int_0^2 \frac{3}{x+1} - 1\, dx$

$= \left[3\log_e(x+1) - x \right]_0^2$

$= \left(3\log_e(3) - 2 \right) - \left(3\log_e 1 - 0 \right)$

$= 3\log_e 3 - 2$

(Q12)



LET $\sqrt{x-2} = 4 - x$

$x - 2 = (4-x)^2$

$x - 2 = 16 - 8x + x^2$

$x^2 - 9x + 18 = 0$

$(x-3)(x-6) = 0$

$\underline{x = 3},\ x = 6$

$A_1 = \int_2^3 \sqrt{x-2}\, dx$

$= \int_2^3 (x-2)^{1/2}$

$= \left[\frac{2}{3} (x-2)^{3/2} \right]_2^3$

$= \frac{2}{3} (1)^{3/2} - \frac{2}{3} (0)^{3/2}$

$= \frac{2}{3}$

$A = \frac{1}{2} \cdot 4 \cdot 4 - 2A_1 - 2A_2$

$= 8 - 2 \cdot \frac{2}{3} - 2 \cdot \frac{1}{2} \cdot 1 \cdot 1$

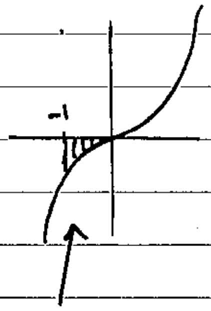
$= \frac{17}{3}$

(Q13) $\int_{-1}^{0} x^3 \, dx$

$= \left[\frac{x^4}{4} \right]_{-1}^{0}$

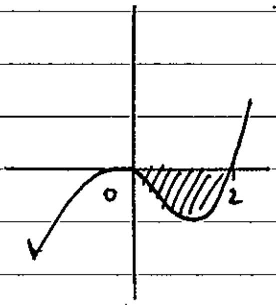
$= 0 - \frac{1}{4}$

$= -\frac{1}{4}$



GRAPH IS BELOW x-AXIS

(Q14)



$A = -\int_0^2 x^2(x-2) \, dx$

$= -\int_0^2 x^3 - 2x^2 \, dx$

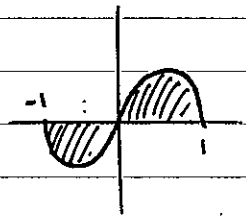
$= -\left[\frac{x^4}{4} - \frac{2x^3}{3} \right]_0^2$

$= -\left[\left(\frac{2^4}{4} - \frac{2 \cdot 2^3}{3} \right) - 0 \right]$

$= -\left[4 - \frac{16}{3} \right]$

$= \frac{4}{3}$

(Q15)



$\int_{-1}^{1} \sin(\pi x) \, dx$

$= \left[-\frac{1}{\pi} \cos(\pi x) \right]_{-1}^{1}$

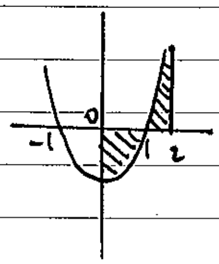
$= \left(-\frac{1}{\pi} \cos \pi \right) + \frac{1}{\pi} \cos(-\pi)$

$= \frac{1}{\pi} - \frac{1}{\pi}$

$= 0$ AREA ABOVE CANCELS AREA BELOW.

(Q16) $y = 3x^2 - 3$

$= 3(x-1)(x+1)$



$A = -\int_0^1 3x^2 - 3 \, dx + \int_1^2 3x^2 - 3 \, dx$

$= -\left[x^3 - 3x \right]_0^1 + \left[x^3 - 3x \right]_1^2$

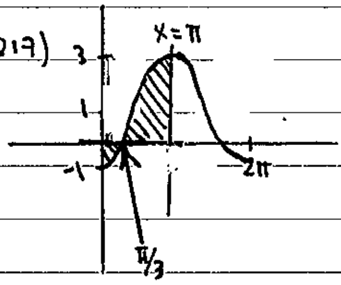
$= -\left[(1-3) - (0) \right] + \left[(8-6) - (1-3) \right]$

$= -(-2) + [2+2]$

$= 2 + 2 + 2$

$= 6$

(Q17)



$y = 1 - 2\cos x$

IF $y = 0$

$0 = 1 - 2\cos x$

$\cos x = \frac{1}{2}$

$x = \pi/3$

$A = -\int_0^{\pi/3} 1 - 2\cos x \, dx + \int_{\pi/3}^{\pi} 1 - \cos x \, dx$

$= -\left[x - 2\sin x \right]_0^{\pi/3} + \left[x - 2\sin x \right]_{\pi/3}^{\pi}$

$= -\left[\pi/3 - 2\sin \pi/3 \right] + (\pi - 2\sin \pi) - (\pi/3 - 2\sin \pi/3)$

$= -\pi/3 + 2\sin \pi/3 + \pi - \pi/3 + 2\sin \pi/3$

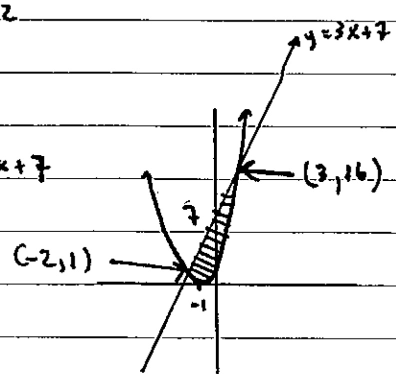
$= \pi/3 + 4\sin \pi/3$

$= \pi/3 + 4\sqrt{3}/2$

$= \pi/3 + 2\sqrt{3}.$

(Q18) $y = x^2 + 2x + 1$ $y = 3x + 7$

$= (x+1)^2$



POINT OF INT:

$x^2 + 2x + 1 = 3x + 7$

$x^2 - x - 6 = 0$

$(x-3)(x+2) = 0$

$x = -2, 3$

$A = \int_{-2}^{3} 3x + 7 - (x^2 + 2x + 1) \, dx$

$= \int_{-2}^{3} -x^2 + x + 6 \, dx$

$= \left[-x^3/3 + x^2/2 + 6x \right]_{-2}^{3}$

$= \left(-3^3/3 + 3^2/2 + 6 \cdot 3 \right) - \left(-(-2)^3/3 + (-2)^2/2 + 6(-2) \right)$

$= \frac{125}{6}$

(Q19) $y = e^{2x} + 2$

$y = 3e^{x}$

PT. OF INT: $e^{2x} + 2 = 3e^{x}$

$y^2 + 2 = 3y \quad (y = e^{x})$

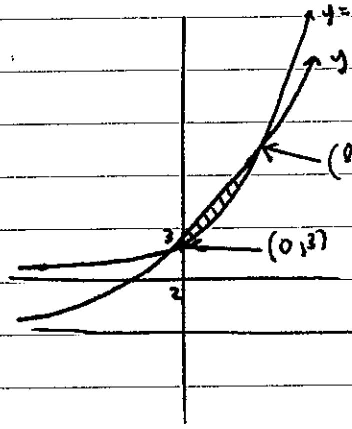
$y^2 - 3y + 2 = 0$

$(y-1)(y-2) = 0$

$y = 1$ or $y = 2$

$e^{x} = 1$ or $e^{x} = 2$

$x = 0$ or $x = \log_e 2$



$A = \int_0^{\log_e 2} 3e^{x} - (e^{2x} + 2)\, dx$

$= \left[3e^{x} - \frac{1}{2}e^{2x} - 2x\right]_0^{\log_e 2}$

$= \left(6 - \frac{1}{2}e^{2\log_e 2} - 2\log_e 2\right) - \left(3e^0 - \frac{1}{2}e^0 - 0\right)$

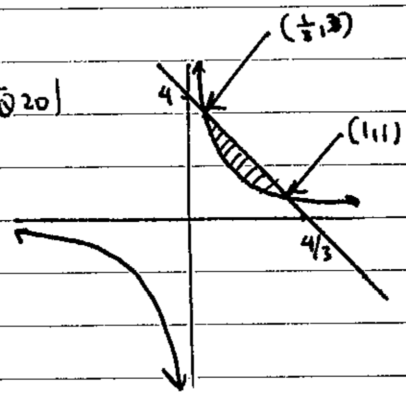
$= \left(6 - \frac{1}{2}e^{\log_e 4} - 2\log_e 2\right) - \left(3 - \frac{1}{2}\right)$

$= 6 - \frac{1}{2} \times 4 - 2\log_e 2 - 3 + \frac{1}{2}$

$= 6 - 2 - \log_e 4 - 3 + \frac{1}{2}$

$= \frac{3}{2} - \log_e 4.$

(Q20)



$y = \frac{1}{x}$, $y = 4 - 3x$

PT. OF INT: $\frac{1}{x} = 4 - 3x$

$\times x) \quad 1 = 4x - 3x^2$

$3x^2 - 4x + 1 = 0$

$(3x-1)(x-1) = 0$

$x = 1/3, 1$

$\Rightarrow y = 3, 1$

AREA $= \int_{1/3}^{1} 4 - 3x - \frac{1}{x}\, dx$

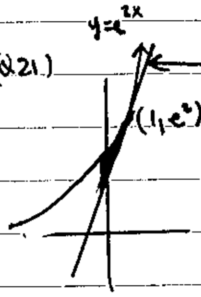
$= \left[4x - \frac{3x^2}{2} - \log_e x\right]_{1/3}^{1}$

$= \left(4 - \frac{3}{2} - \log_e 1\right) - \left(\frac{4}{3} - \frac{3(1/3)^2}{2} - \log_e \frac{1}{3}\right)$

$= \frac{5}{2} - \frac{4}{3} + \frac{1}{6} + \log_e \frac{1}{3}$

$= \frac{4}{3} - \log_e 3$

(Q21)



$\frac{dy}{dx} = 2e^{2x}$

@ $x = 1$ $\frac{dy}{dx} = 2e^2 = m$

$y - y_1 = m(x - x_1)$

$y - e^2 = 2e^2(x-1)$

$y - e^2 = 2e^2 x - 2e^2$

$y = 2e^2 \cdot x - e^2$

$A = \int_0^1 e^{2x} - (2e^2 x - e^2)\, dx$

$= \left[\frac{1}{2}e^{2x} - e^2 x^2 + e^2 x\right]_0^1$

$= \left(\frac{1}{2}e^2 - e^2 \cdot 1^2 + e^2\right)$

$- \left(\frac{1}{2}e^0 - 0 + 0\right)$

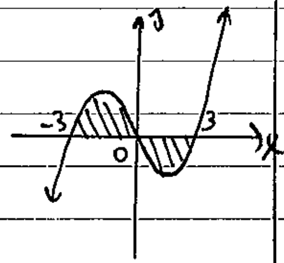
$= \frac{1}{2}e^2 - \cancel{e^2} + \cancel{e^2} - \frac{1}{2}$

$= \frac{1}{2}(e^2 - 1)$

(Q22) $y = x^3 - 9x$

$= x(x^2 - 9)$

$= x(x-3)(x+3)$



$A = -2\int_0^3 x^3 - 9x \, dx$

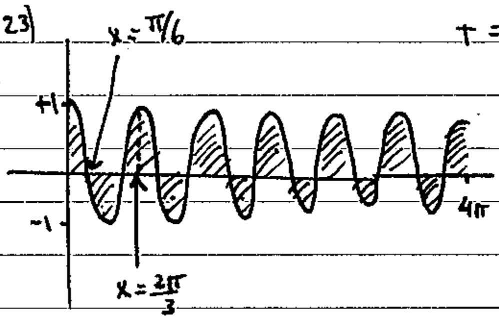
$= -2\left[\frac{x^4}{4} - \frac{9x^2}{2}\right]_0^3$

$= -2\left[\frac{3^4}{4} - \frac{9 \cdot 3^2}{2} - 0\right]$

$= -2\left(\frac{81}{4} - \frac{81}{2}\right)$

$= \frac{81}{2}$

(Q23) $T = \frac{2\pi}{3}$



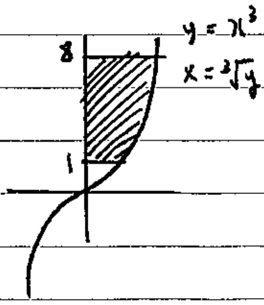
$A = 24\int_0^{\pi/6} \cos 3x \, dx$

$= 24\left[\frac{1}{3}\sin 3x\right]_0^{\pi/6}$

$= 24\left[\frac{1}{3}\sin\frac{\pi}{2} - 0\right]$

$= 8$

(Q24)



$A = \int_1^8 \sqrt[3]{y} \, dy$

$= \int_1^8 y^{1/3} \, dy$

$= \left[\frac{y^{4/3}}{4/3}\right]_1^8$

$= \left[\frac{3}{4}y^{4/3}\right]_1^8$

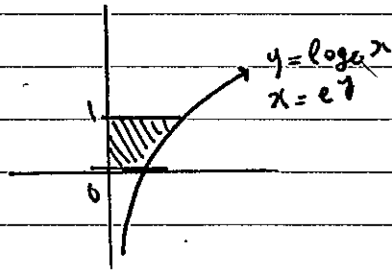
$= \frac{3}{4} \cdot 8^{4/3} - \frac{3}{4} \cdot 1^{4/3}$

$= \frac{3}{4} \cdot 2^4 - \frac{3}{4}$

$= 12 - \frac{3}{4}$

$= \frac{45}{4}$

(Q25)



$A = \int_0^1 e^y \, dy$

$= [e^y]_0^1$

$= e^1 - e^0$

$= e - 1$

(Q26) $\bar{f} = \frac{1}{2-1}\int_1^2 x^2 \, dx$

$= \int_1^2 x^2 \, dx$

$= [x^3/3]_1^2$

$= 8/3 - 1/3$

$= 7/3$

(Q27) $\bar{f} = \frac{1}{\pi - 0}\int_0^\pi 2 + \sin x \, dx$

$= \frac{1}{\pi}\left[2x - \cos x\right]_0^\pi$

$= \frac{1}{\pi}\left[(2\pi - \cos\pi) - (0 - \cos 0)\right]$

$= \frac{1}{\pi}(2\pi + 2)$

$= \frac{2\pi + 2}{\pi}$

(Q28) $10 = \frac{1}{a}\int_0^a x^2 \, dx$

$10a = [x^3/3]_0^a$

$10a = a^3/3$

$30a = a^3$

$a^3 - 30a = 0$

$a(a^2 - 30) = 0$

$a(a - \sqrt{30})(a + \sqrt{30}) = 0$

$\therefore a = 0, -\sqrt{30}, \sqrt{30}$

$\therefore \underline{a = \sqrt{30}}$, As $a > 0$

(Q29) $2 = \frac{1}{1-0}\int_0^1 ae^x \, dx$

$2 = \int_0^1 ae^x \, dx$

$2 = [ae^x]_0^1$

$2 = a\cdot e^1 - a\cdot e^0$

$2 = a(e-1)$

$\therefore a = \frac{2}{e-1}$

(Q30) $f(x) = x \sin x$

$f'(x) = x\cos x + \sin x$

$\therefore x\cos x = f'(x) - \sin x$

$\therefore \int_0^{\pi/2} x\cos x \, dx$

$= \int_0^{\pi/2} f'(x) - \sin x \, dx$

$= [f(x) + \cos x]_0^{\pi/2}$

$= [x\sin x + \cos x]_0^{\pi/2}$

$= (\pi/2 \sin \pi/2 + \cos \pi/2) - (0 \sin 0 + \cos 0)$

$= \pi/2 - 1$

$= 1/2(\pi - 2)$

(Q31) $f(x) = x\log_e x$

$f'(x) = x\cdot\frac{1}{x} + \log_e x$

$= 1 + \log_e x$

$\therefore \log_e x = f'(x) - 1$

$\therefore \int_2^3 \log_e x \, dx$

$= \int_2^3 f'(x) - 1 \, dx$

$= [x\log_e x - x]_2^3$

$= (3\log_e 3 - 3) - (2\log_e 2 - 2)$

$= 3\log_e 3 - 3 - 2\log_e 2 + 4$

$= \log_e 27 - \log_e 4 + 1$

$= \log_e(27/4) + 1.$

(Q32) $\int_0^1 e^{ax} \, dx = 2$

$[\frac{1}{a}e^{ax}]_0^1 = 2$

$\frac{1}{a}e^a - \frac{1}{a}e^0 = 2$

$\frac{1}{a}e^a - \frac{1}{a} = 2$

$\times a)$ $e^a - 1 = 2a$

SOLVE WITH CALC:

$a \approx 1.26$

(Q33) $\int_1^a \frac{1}{x} \, dx = 2$

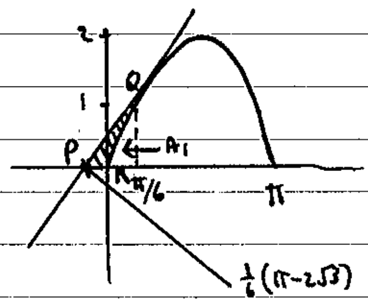
$(\log_e x)_1^a = 2$

$\log_e a - \log_e 1 = 2$

$\log_e a = 2$

$a = e^2$

(Q34)



AREA ΔPQR = ?

$y = 2\sin x$

$\frac{dy}{dx} = 2\cos x$

@ $x = \pi/6$ $\frac{dy}{dx} = 2\cos\pi/6 = \sqrt{3} = m$

$y - y_1 = m(x - x_1)$ ← TANGENT LINE EQ^{n}

$y - 1 = \sqrt{3}(x - \pi/6)$

WHEN $y = 0$

$-1 = \sqrt{3}(x - \pi/6)$

$-\frac{1}{\sqrt{3}} = x - \pi/6$

$x = \frac{\pi}{6} - \frac{1}{\sqrt{3}}$

$= \frac{\pi}{6} - \frac{\sqrt{3}}{3}$

$= \frac{\pi}{6} - \frac{2\sqrt{3}}{6}$

$= \frac{1}{6}(\pi - 2\sqrt{3})$

AREA (ΔPQR)

$= \frac{1}{2}\cdot(\cancel{\frac{\pi}{6}} - \cancel{\frac{\pi}{6}} + \frac{2\sqrt{3}}{6})\cdot 1$

$= \frac{\sqrt{3}}{6}$

$\therefore$ REQ AREA

$= \frac{\sqrt{3}}{6} - \int_0^{\pi/6} 2\sin x \, dx$

$= \frac{\sqrt{3}}{6} - [-2\cos x]_0^{\pi/6}$

$= \frac{\sqrt{3}}{6} + [2\cos x]_0^{\pi/6}$

$= \frac{\sqrt{3}}{6} + 2\cos\pi/6 - 2\cos 0$

$= \frac{\sqrt{3}}{6} + \sqrt{3} - 2$

$= \frac{7\sqrt{3}}{6} - 2$

(Q35) $\int_a^b \frac{1}{x}\, dx = 1$

$[\log_e x]_a^b = 1$

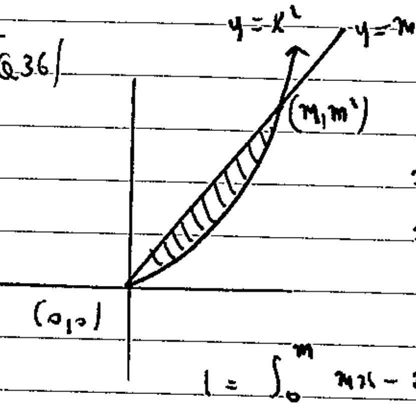
$\log_e b - \log_e a = 1$

$\log_e \frac{b}{a} = 1$

$\frac{b}{a} = e^1$

$\underline{b = ea.}$

(Q36)



$x^2 = mx$

$x^2 - mx = 0$

$x(x-m) = 0$

$x = 0,\ x = m$

$1 = \int_0^m mx - x^2\, dx$

$= \left[\frac{mx^2}{2} - \frac{x^3}{3}\right]_0^m$

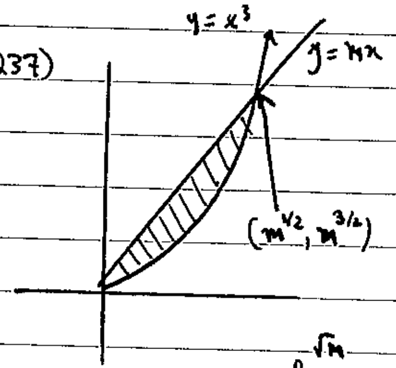
$= \frac{m^3}{2} - \frac{m^3}{3} - 0$

$= \frac{m^3}{6}$

$\therefore\ m^3 = 6$

$\therefore\ \underline{m = \sqrt[3]{6}.}$

(Q37)



$x^3 = mx$

$x^3 - mx = 0$

$x(x^2 - m) = 0$

$x = \sqrt{m}$

$1 = \int_0^{\sqrt{m}} mx - x^3\, dx$

$= \left[\frac{mx^2}{2} - \frac{x^4}{4}\right]_0^{\sqrt{m}}$

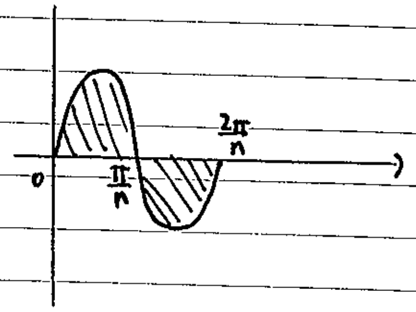
$= \frac{m^2}{2} - \frac{m^2}{4}$

$= \frac{m^2}{4}$

$\therefore\ m^2 = 4$

$\underline{m = 2}$

(Q38)



PERIOD $= \frac{2\pi}{n}$ $\therefore$ # OF CYCLES $= n$

$A = 2 \times n \times \int_0^{\pi/n} \sin(nx)\, dx$

$= 2n\left[-\frac{1}{n}\cos(nx)\right]_0^{\pi/n}$

$= 2n\left[-\frac{1}{n}\cos\pi + \frac{1}{n}\cos 0\right]$

$= 2n\left(\frac{1}{n} + \frac{1}{n}\right)$

$= 2n \cdot \frac{2}{n}$

$= \underline{4}$

(Q39) $f(x) = ax+1$ $\quad x \leftrightarrow y$

$x = ay + 1$

$y = \frac{x-1}{a}$

$\therefore f^{-1}(x) = \frac{x-1}{a}$

$\therefore \int_0^1 (ax+1)(x-1)/a\, dx = 1$

$\times a)$ $\int_0^1 (ax+1)(x-1)\, dx = a$

$\int_0^1 ax^2 + (1-a)x - 1\, dx = a$

$\left[\frac{ax^3}{3} + \frac{(1-a)x^2}{2} - x\right]_0^1 = a$

$\frac{a}{3} + \frac{1-a}{2} - 1 = a$

$\times 6)$ $2a + 3(1-a) - 6 = 6a$

$2a + 3 - 3a - 6 = 6a$

$-a - 3 = 6a$

$-3 = 7a$

$\underline{a = -\frac{3}{7}}$

(Q40) (A) $f(x) = e^{x}\cos x$

$f'(x) = e^{x}(-\sin x) + \cos x \cdot e^{x}$

$= e^{x}\cos x - e^{x}\sin x$

$= f(x) - g(x)$ ①

(B) $g(x) = e^{x}\sin x$

$g'(x) = e^{x}\cos x + \sin x \cdot e^{x}$

$= e^{x}\cos x + e^{x}\sin x$

$= f(x) + g(x)$ ②

(C) ① + ② $f'(x) + g'(x) = 2f(x)$

$\therefore f(x) = \frac{1}{2}\left(f'(x) + g'(x)\right)$

(D) $\int e^{x}\cos x \, dx$

$= \int f(x)\, dx$

$= \int \frac{1}{2}\left(f'(x) + g'(x)\right) dx$

$= \frac{1}{2}f(x) + \frac{1}{2}g(x) + c$

$= \frac{1}{2}e^{x}(\cos x + \sin x) + c$

(E) ② − ① $g'(x) - f'(x) = 2g(x)$

$\therefore g(x) = \frac{1}{2}\left(g'(x) - f'(x)\right)$

$\therefore \int e^{x}\sin x \, dx$

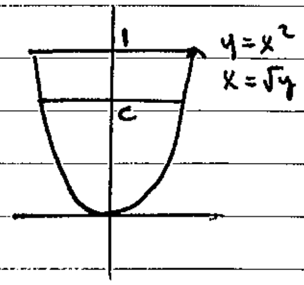
$= \int g(x)\, dx$

$= \int \frac{1}{2}\left(g'(x) - f'(x)\right) dx$

$= \frac{1}{2}\left(g(x) - f(x)\right) + c$

$= \frac{1}{2}e^{x}(\sin x - \cos x) + c$

(Q41)



TOTAL AREA $= 2\int_0^1 \sqrt{y}\, dy$

$= 2\int_0^1 y^{1/2}\, dy$

$= 2\left[\frac{y^{3/2}}{3/2}\right]_0^1$

$= 2\left[\frac{2}{3}y^{3/2}\right]_0^1$

$= 4/3$

HALF AREA $= 2/3$

$\therefore \quad 2/3 = 2\int_0^c \sqrt{y}\, dy$

$2/3 = 2\left[\frac{2}{3}y^{3/2}\right]_0^c$

$\div 2) \quad \frac{1}{3} = \left[\frac{2}{3}y^{3/2}\right]_0^c$

$\frac{1}{3} = \frac{2}{3}c^{3/2} - 0$

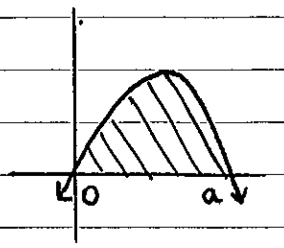
$\times 3) \quad 1 = 2c^{3/2}$

$c^{3/2} = \frac{1}{2}$

$c = \left(\frac{1}{2}\right)^{2/3}$

$= \frac{1}{\sqrt[3]{4}}$

(Q42)



$$36 = \int_0^a ax - x^2\,dx$$
$$36 = \left[\frac{ax^2}{2} - \frac{x^3}{3} \right]_0^a$$
$$36 = \frac{a^3}{2} - \frac{a^3}{3}$$
$$36 = \frac{a^3}{6}$$
$$a^3 = 6^3$$
$$\therefore \underline{a = 6}$$

(Q43) $\int_{-1}^{1} (x^2+ax+b)(cx+d)\,dx = 0$

$\downarrow$ EXPAND + INTEGRATE TO OBTAIN :

$$\frac{2ac + 6bd + 2d}{3} = 0$$

$$\therefore \quad 2ac + 6bd + 2d = 0$$

MUST BE ZERO FOR EVERY CHOICE OF c AND d.

$\therefore$ MUST HAVE $a = 0$

$\therefore \quad 6bd + 2d = 0$

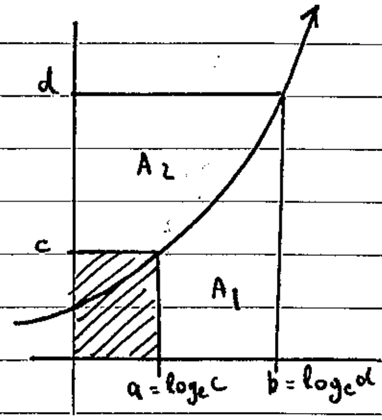
$\div 2d)$ $\quad 3b + 1 = 0$

$$b = -\frac{1}{3}$$

$$\therefore \begin{cases} a = 0 \\ b = -\frac{1}{3} \end{cases}$$

(Q44) (A) $A_1 = \int_a^b e^x\,dx$
$$= \left[e^x \right]_a^b$$
$$= e^b - e^a$$

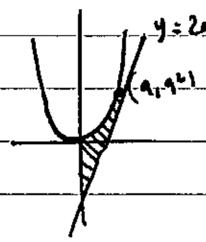
(B)



A_2 = TOTAL AREA $-$ A_1 $-$ RED AREA
$$= db - (e^b - e^a) - ca$$
$$= d\log_e d - (e^{\log_e d} - e^{\log_e c}) - c\log_e c$$
$$= (d\log_e d - c\log_e c) - (d - c)$$

$$\therefore \int_c^d \log_e y\,dy = (d\log_e d - c\log_e c) - (d - c)$$

(Q45)



$$\therefore q = \int_0^a x^2 - (2ax - a^2)\,dx$$
$$q = \left[\frac{x^3}{3} - ax^2 + a^2 x \right]_0^a$$
$$q = \frac{a^3}{3} - a^3 + a^3$$
$$q = \frac{a^3}{3}$$

$\frac{dy}{dx} = 2x$

@ $x = a$, $m = \frac{dy}{dx} = 2a$

$\therefore$ TANGENT LINE :

$y - a^2 = 2a(x - a)$

$y - a^2 = 2ax - 2a^2$

$y = 2ax - a^2$

$\therefore \quad a^3 = 27$

$\therefore \quad a = 3$