Relations + Functions

(1) $y=\frac{1}{x} \rightarrow y=\frac{1}{x}+3$

$y=x^2 \rightarrow y=(x-3)^2$

$y=\sqrt{x} \rightarrow y=\sqrt{x+2}$

$y=5x^2-\sqrt{x} \rightarrow y=5(x-7)^2-\sqrt{x-7}$

$y=\sqrt{2x} \rightarrow y=-\sqrt{2x}$

$y=\frac{1}{\sqrt{x}} \rightarrow y=\frac{1}{\sqrt{-x}}$

$y=\sqrt{2x+3} \rightarrow y=3\sqrt{2x+3}$

$y=\sqrt{5-x} \rightarrow y=\sqrt{5-3x}$

$y=\sqrt{2x}-\frac{1}{x^2} \rightarrow y=\sqrt{-2x}-\frac{1}{x^2}$

$y=3x^2-\sqrt{x} \rightarrow y=-3x^2+\sqrt{x}$

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(2) $y=\frac{3}{x}$

- $y=\frac{3}{x+1}$
- $y=\frac{3}{x+1}+4$
- $y=-\frac{3}{x+1}-4$
- $y=2\left(-\frac{3}{x+1}-4\right)$

$y=-\frac{6}{x+1}-8$

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(3) $y=g(x)$

- $y=g(x)+3$
- $y=g(x-2)+3$
- $y=g(-x-2)+3$
- $y=g\left(-\frac{x}{3}-2\right)+3$
- $y=-g\left(\frac{x}{3}-2\right)-3$

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(4) $y=3\sqrt{x}-x^2$

- $y=3\sqrt{x-1}-(x-1)^2$
- $y=3\sqrt{\frac{x}{2}-1}-\left(\frac{x}{2}-1\right)^2$
- $y=-3\sqrt{\frac{x}{2}-1}-3\left(\frac{x}{2}-1\right)^2$
- $y=-3\sqrt{\frac{x}{2}-1}-3\left(\frac{x}{2}-1\right)^2-3$

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(5) $y=3(x-1)^2+2$ (b)

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(6) $y=-(x-7)^2-3$ (b)

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(7) $y=\sqrt{2-x}$

$y=\sqrt{-(x-2)}$ (d)

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(8) $y=\sqrt{3x-1}+1$

$y=\sqrt{3(x-\frac{1}{3})}+1$ (d)

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(9) $y=\frac{3}{x+1}$ (e)

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(10) $y=2(x+1)^2+1$

- dilate factor 3 from x-axis

$\rightarrow y=6(x+1)^2+3$

- translate 1 unit right

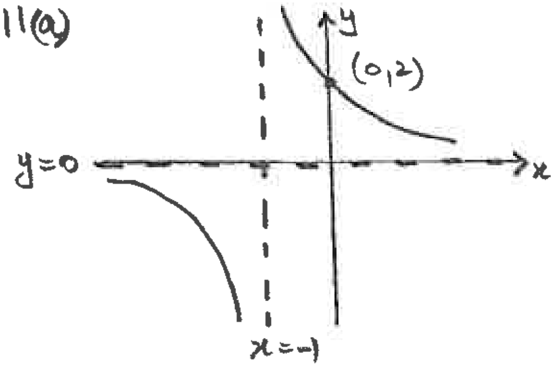
$\rightarrow y=6(x-1+1)^2+3$

$y=6(x)^2+3$

- translate 1 unit down

$\rightarrow y=6x^2+2$ (c)

11(a)

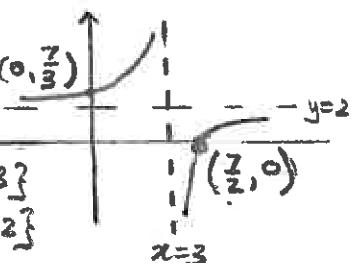


Y int : $x=0$

$y=2$

dom $=R\setminus\{-1\}$ ran $=R\setminus\{0\}$.

(b) $y=-\frac{1}{x-3}+2$



dom $=R\setminus\{3\}$

ran $=R\setminus\{2\}$

Y int ; $x=0$

$y=-\frac{1}{-3}+2=\frac{7}{3}$

X int ; $y=0$

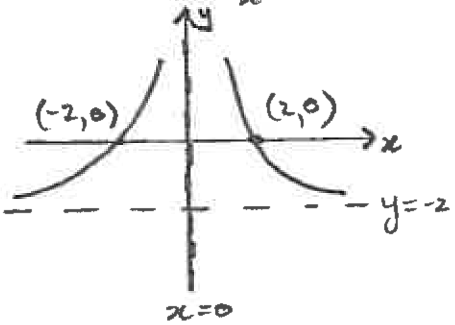
$-\frac{1}{x-3}+2=0$

$\frac{1}{x-3}=2$

$x-3=\frac{1}{2}$

$x=3\frac{1}{2}$

11(c) $y = -2 + \dfrac{8}{x^2}$



x int: $y=0$

$-2 + \dfrac{8}{x^2} = 0$

$\dfrac{8}{x^2} = 2$

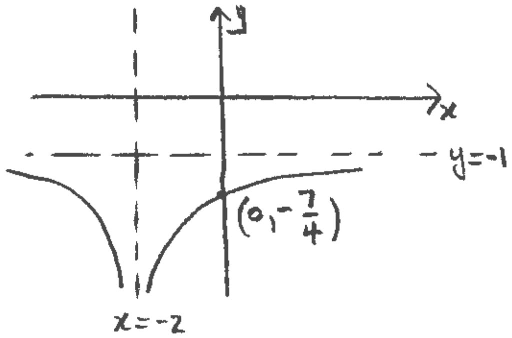
$x^2 = 4$

$x = \pm 2$

dom $= R \setminus \{0\}$

ran $= (-2, \infty)$

11(d) $y = \dfrac{-3}{(x+2)^2} - 1$



Y int: $x=0$

$y = -\dfrac{3}{2^2} - 1$

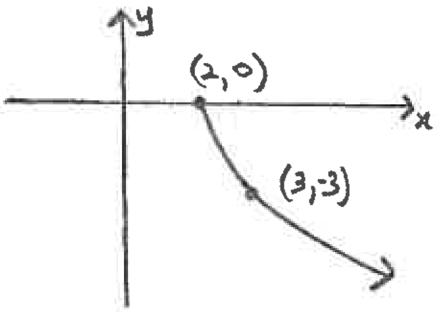
$= -\dfrac{3}{4} - 1$

$= -\dfrac{7}{4}$

dom $= R \setminus \{-2\}$

ran $= (-\infty, -1)$

(e) $y = -3\sqrt{x-2}$



$x = 3$,

$y = -3\sqrt{3-2}$

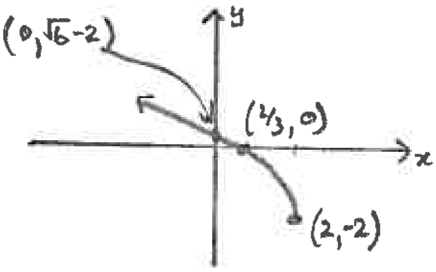
$= -3$

dom $= [2, \infty)$

ran $= (-\infty, 0]$

(f) $y = \sqrt{6-3x} - 2$

$y = \sqrt{-3(x-2)} - 2$



Y int: $x = 0$

$y = \sqrt{6} - 2$

X int: $y = 0$

$\sqrt{6-3x} = 2$

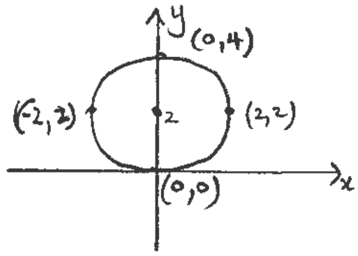
$6 - 3x = 4$

$3x = 2$

$x = \dfrac{2}{3}$

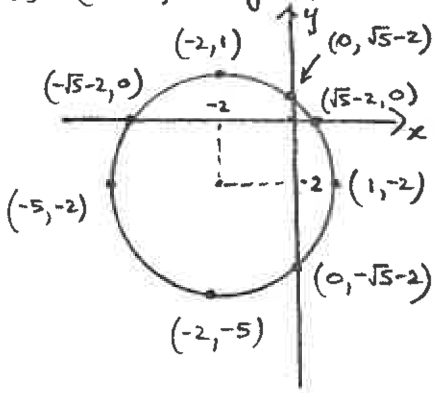
dom $= (-\infty, 2]$ ran $= [-2, \infty)$

(g) $x^2 + (y-2)^2 = 4$



dom $= [-2, 2]$, ran $= [0, 4]$

(h) $(x+2)^2 + (y+2)^2 = 9$



X int: $y = 0$

$(x+2)^2 + 4 = 9$

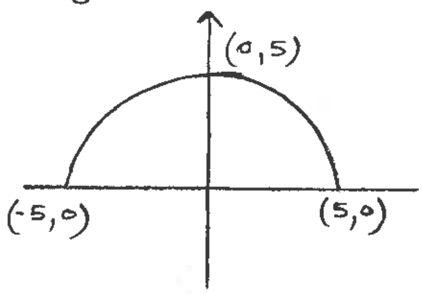
$(x+2)^2 = 5$

$x + 2 = \pm\sqrt{5}$

$x = -2 \pm \sqrt{5}$

dom $= [-5, 1]$ ran $= [-5, 1]$

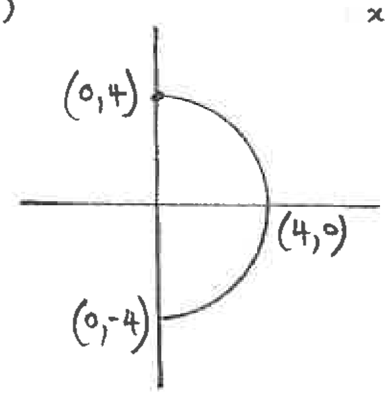
(i) $y = \sqrt{25 - x^2}$



dom $= [-5, 5]$

ran $= [0, 5]$

(j) $x = \sqrt{16 - y^2}$



dom $= [0, 4]$

ran $= [-4, 4]$

(12) $y = \frac{a}{(x-2)^2} + 4$

sub in $(5,2)$

$$2 = \frac{a}{(5-2)^2} + 4$$

$$2 = \frac{a}{9} + 4$$

$$\frac{a}{9} = -2$$

$$a = -18$$

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(13) $y = \frac{a}{x-b} + c$

vert asy $x = -1 \Rightarrow b = -1$

hor asy $y = -2 \Rightarrow c = -2$

$\therefore y = \frac{a}{x+1} - 2$

sub in $(-3,-4)$

$$-4 = \frac{a}{-3+1} - 2$$

$$-4 = \frac{a}{-2} - 2$$

$$-2 = \frac{a}{-2}$$

$$a = 4$$

$a = 4$, $b = -1$, $c = -2$

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(14) Centre $= (3,-1)$, $r = 2$

$$(x-3)^2 + (y+1)^2 = 4$$

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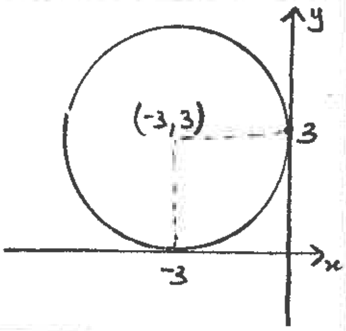
(15)

Centre $= (-3,3)$

radius $= 3$

$$(x+3)^2 + (y-3)^2 = 9$$



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(16) $y = a\sqrt{x+b}$

Sub in $(4,-2)$

$$-2 = a\sqrt{4+b} \quad ①$$

Sub in $(12,-6)$

$$-6 = a\sqrt{12+b} \quad ②$$

② ÷ ① $\quad \frac{-6}{-2} = \frac{a\sqrt{12+b}}{a\sqrt{4+b}}$

$$3 = \frac{\sqrt{12+b}}{\sqrt{4+b}}$$

$$9 = \frac{12+b}{4+b}$$

$$9(4+b) = 12+b$$

$$36 + 9b = 12 + b$$

$$8b = -24$$

$$b = -3$$

Sub into ①

$$-2 = a\sqrt{4-3}$$

$$-2 = a$$

$a = -2$, $b = -3$

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(17) (a) $x^2+y^2-6x-4y+9=0$

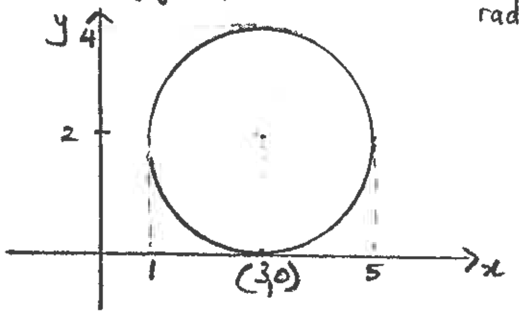
$$x^2-6x+y^2-4y+9=0$$

$$(x-3)^2-9+(y-2)^2-4+9=0$$

$$(x-3)^2+(y-2)^2=4$$

(i) Centre $= (3,2)$
radius $= 2$

(ii)



(iii) dom $= [1,5]$ ran $= [0,4]$

(b) $x^2+y^2+2x+4y=4$

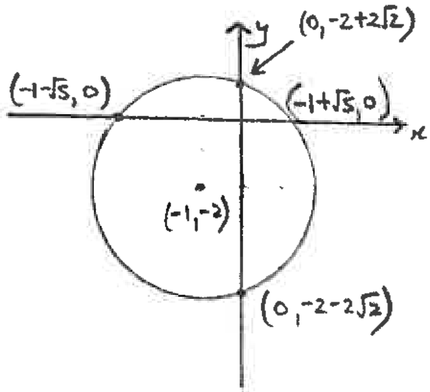
$$x^2+2x+y^2+4y=4$$

$$(x+1)^2-1+(y+2)^2-4=4$$

$$(x+1)^2+(y+2)^2=9$$

(i) Centre $= (-1,-2)$ Radius $= 3$

(ii)



x int: $y=0$

$(x+1)^2+4=9$

$x+1=\pm\sqrt{5}$

$x=-1\pm\sqrt{5}$

y int: $x=0$

$1+(y+2)^2=9$

$y+2=\pm\sqrt{8}$

$y=-2\pm2\sqrt{2}$

(iii) dom $= [-4,2]$

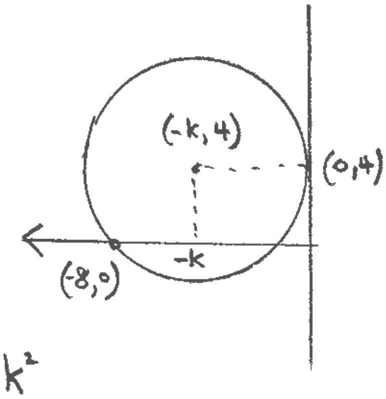
ran $= [-5,1]$

(18)

Let centre $= (-k, 4)$

$\therefore$ Equation is

$(x+k)^2 + (y-4)^2 = k^2$



sub in $(-8,0)$

$(-8+k)^2 + (0-4)^2 = k^2$

$(k-8)^2 + 16 = k^2$

$k^2 - 16k + 64 + 16 = k^2$

$-16k + 80 = 0$

$k = \frac{80}{16} = 5$

$\therefore$ Equation is

$(x+5)^2 + (y-4)^2 = 25$

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(19) (a) $-2 \in Z$ $\boxed{T}$

(b) $0.\dot{9} \in Q'$ $\boxed{F}$

(c) $\frac{2}{3} \in Q$ $\boxed{T}$

(d) $9.2 \in Z$ $\boxed{F}$

(e) $\pi \in R$ $\boxed{T}$

(f) $\sqrt{11} \in Q$ $\boxed{F}$

(g) $Z \cap Q = Z$ $\boxed{T}$

(h) $Q \cup Z = Q$ $\boxed{T}$

(i) $Q \setminus Z = Q'$ $\boxed{F}$

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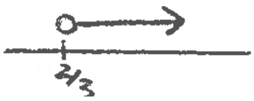
(20) (a) $\{x : 5 \leq x < 6\}$

$x \in [5, 6)$

(b) $\{x : x > \frac{2}{3}\}$

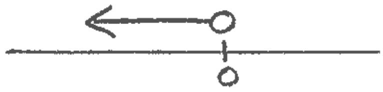
$x \in (\frac{2}{3}, \infty)$



(c) $R^+ \cup \{0\}$

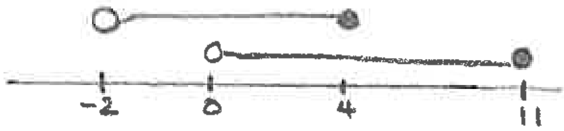
$= [0, \infty)$

(d) $R^- = (-\infty, 0)$

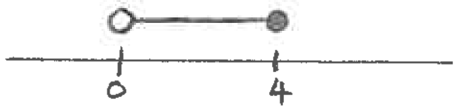


(e) $R \setminus \{-2\}$

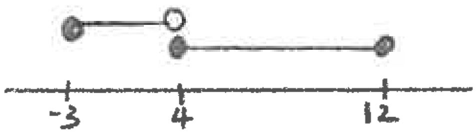
(f) $(-2, 4] \cap (0, 11]$



$= (0, 4]$

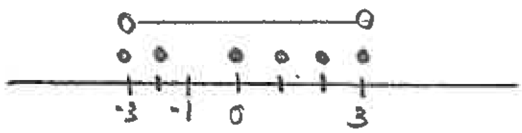


(g) $[-3, 4) \cap [4, 12]$



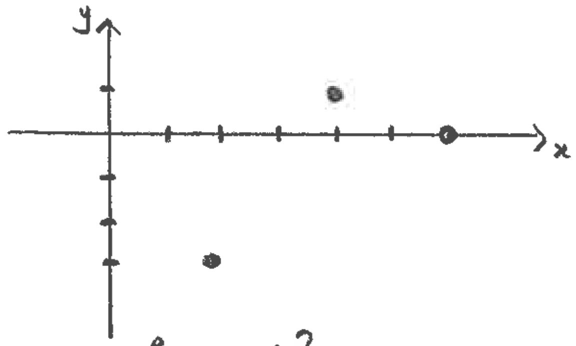
$= \phi$ (empty set)

(h) $Z \setminus \{-1\} \cap (-3, 3)$



$= \{-2, 0, 1, 2\}$

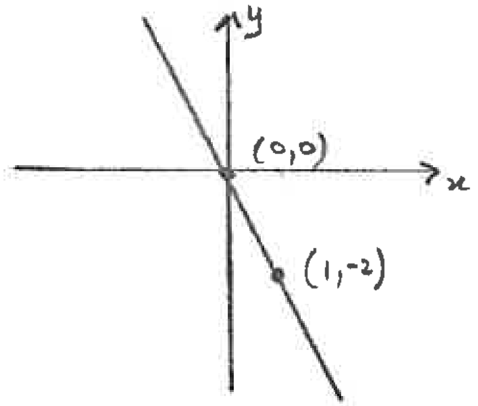
(21) (a) $R = \{(2,-3),(4,1),(6,0)\}$



dom = $\{2,4,6\}$

ran = $\{-3,0,1\}$

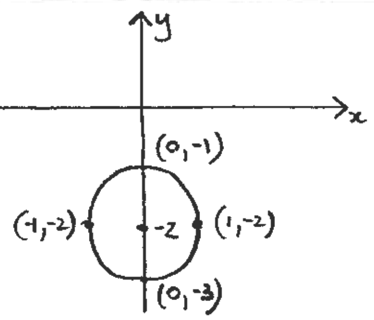
(b) $y = -2x$



dom = R

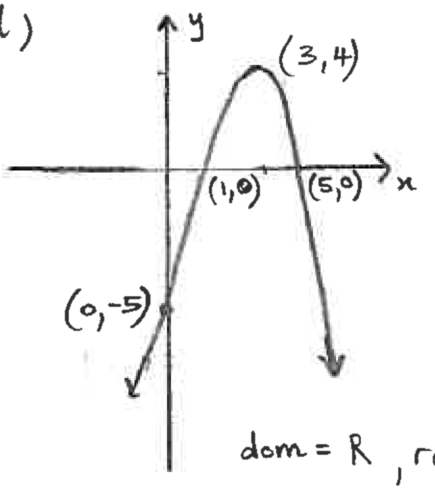
ran = R

(c)



dom = $[-1,1]$ ran = $[-3,-1]$

(d)



Y int : $x=0$

$y = 4-(0-3)^2$

$= -5$

X int : $y=0$

$(x-3)^2 = 4$

$x-3 = \pm 2$

$x = 3 \pm 2$

$= 1, 5$

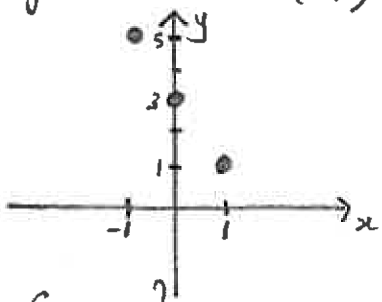
dom = R , ran = $(-\infty, 4]$

22(a) $y = -2x+3$, $x \in \{-1,0,1\}$

$x=-1$, $y=-2(-1)+3 = 5$ $(-1,5)$

$x=0$, $y=3$ $(0,3)$

$x=1$, $y=-2+3=1$ $(1,1)$

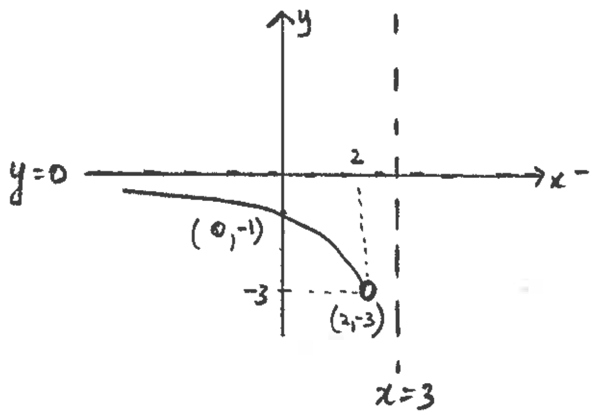


dom = $\{-1,0,1\}$

ran = $\{1,3,5\}$

22(b) $y = \frac{3}{x-3}$, $x \in (-\infty, 2)$

$x=2$, $y = \frac{3}{2-3} = \frac{3}{-1} = -3$ $(2,-3)$ o



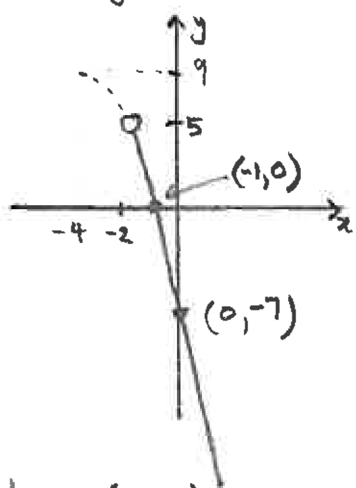
Y int : $x=0$, $y = \frac{3}{-3} = -1$ $(0,-1)$

dom = $(-\infty, 2)$

ran = $(-3, 0)$

(c) $y = -(x+4)^2+9$, $x \in (-2, \infty)$

$x=-2$, $y = -(-2+4)^2+9 = -4+9 = 5$ $(-2,5)$ o



Y int : $x=0$

$y = -16+9 = -7$

X int : $y=0$

$-(x+4)^2+9 = 0$

$-(x+4)^2 = -9$

$(x+4)^2 = 9$

$x+4 = \pm 3$

$x = -4 \pm 3$

$= -1$

dom = $(-2, \infty)$ ran = $(-\infty, 5)$

(23) (a) $y=3$ $\quad$ dom $=\mathbb{R}$

(b) $y=\frac{2}{x+3}$

$x+3\neq 0$

$x\neq -3$

$\therefore$ dom $=\mathbb{R}\setminus\{-3\}$

(c) $y=8-\sqrt{3x-7}$

$3x-7\geqslant 0$

$x\geqslant \frac{7}{3}$

dom $=\left[\frac{7}{3},\infty\right)$

(d) $y=\frac{\pi}{\sqrt{4-x}}$

$4-x>0$

$x<4$

dom $=(-\infty,4)$

(e) $12x^2+7x-12=0$

$$\begin{array}{ccc} 4x & & -3 \\ & \times & \\ 3x & & 4 \end{array}$$

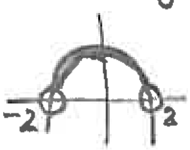
$(4x-3)(3x+4)=0$

$x=\frac{3}{2},\ -\frac{4}{3}$

dom $=\mathbb{R}\setminus\left\{\frac{3}{2},-\frac{4}{3}\right\}$

(f) $4-x^2>0$

Consider $y=4-x^2$



X int:

$4-x^2=0$

$x^2=4$

$x=\pm 2$

$-2<x<2$

dom $=(-2,2)$

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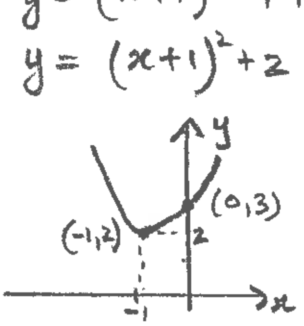
24 (a) $y=x^2+2x+3$

$y=(x+1)^2-1+3$

$y=(x+1)^2+2$



dom $=\mathbb{R}$

ran $=[2,\infty)$

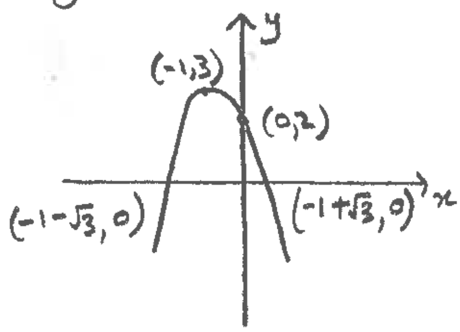
(b) $y=-x^2-2x+2$

$y=-\left[x^2+2x\right]+2$

$y=-\left[(x+1)^2-1\right]+2$

$y=-(x+1)^2+1+2$

$y=-(x+1)^2+3$



Xint: $y=0$

$-(x+1)^2+3=0$

$(x+1)^2=3$

$x+1=\pm\sqrt{3}$

$x=-1\pm\sqrt{3}$

dom $=\mathbb{R}$, ran $=(-\infty,3]$

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25 (a) Not a function

Fails vertical line test

dom $=[0,\infty)$

ran $=\mathbb{R}$

(b) Function

dom $=\mathbb{R}$

ran $=[-5,\infty)$

(c) Function

dom $=(-\infty,6]$

ran $=[-1,\infty)$

(d) Function

dom $=\mathbb{R}$

ran $=\{5\}$

(e) Not a function

Fails vertical line test

at $x=1$

dom $=\{1,2,3\}$ $\quad$ ran $=\{-1,0,1,2\}$

(f) Function

dom $=\{1,2,3,4\}$ $\quad$ ran $=\{-1,0,1\}$

(26) (a) $f: (-\infty, 0) \to R,\ f(x) = 3 - \frac{1}{x}$

(b) $5 - x \geq 0 \Rightarrow x \leq 5$

$g: (-\infty, 5] \to R,\ g(x) = \sqrt{5-x}$

(c) $h: R \to R,\ h(x) = x^2 - 2x$

(d) $f: R \setminus \{-3\} \to R,\ f(x) = \frac{2}{(x+3)^2}$

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(27) $f(x) = 2x^2 - x$

(a) (i) $f(0) = 0$ (y-intercept)

(ii) $f\left(\frac{1}{2}\right) = 2\left(\frac{1}{2}\right)^2 - \frac{1}{2} = 2 \times \frac{1}{4} - \frac{1}{2} = 0$

when $x = \frac{1}{2},\ y = 0$ $\therefore$ This is an x-intercept

(iii) $f(3) = 2(3)^2 - 3 = 15$

when $x = 3,\ y = 15$

(b) (i) $f(-1) = 2(-1)^2 - (-1) = 2 + 1 = 3$

(ii) $f(5a) = 2(5a)^2 - 5a = 50a^2 - 5a$

(iii) $f(2x-1)$

$= 2(2x-1)^2 - (2x-1)$

$= 2(4x^2 - 4x + 1) - 2x + 1$

$= 8x^2 - 8x + 2 - 2x + 1$

$= 8x^2 - 10x + 3$

(c) (i) $f(x) = 3$

$2x^2 - x = 3$

$2x^2 - x - 3 = 0$

$2x \quad -3$

$x \quad 1$

$(2x-3)(x+1) = 0$

$x = \frac{3}{2},\ x = -1$

$\left(\frac{3}{2}, 3\right)$ and $(-1, 3)$ are points on the graph

(ii) $f(x) = 0$

$2x^2 - x = 0$

$x(2x-1) = 0$

$x = 0,\ x = \frac{1}{2}$

$(0,0)$ and $\left(\frac{1}{2}, 0\right)$ are the x-intercepts

(d) $f(x) = -1$

$2x^2 - x = -1$

$2x^2 - x + 1 = 0$

$\Delta = b^2 - 4ac$

$= (-1)^2 - 4(2)(1)$

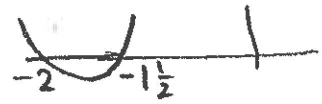
$= 1 - 8$

$= -7$

$\therefore$ No Solution

(e)  $\leftarrow y = f(x)$ (from c ii)

2 units left



x-int $= (-2, 0)\ \left(-\frac{3}{2}, 0\right)$

(f) $h(x) = 2x^2 - x - 2$

x-int: $2x^2 - x - 2 = 0$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$= \frac{+1 \pm \sqrt{1 - 4(2)(-2)}}{4}$

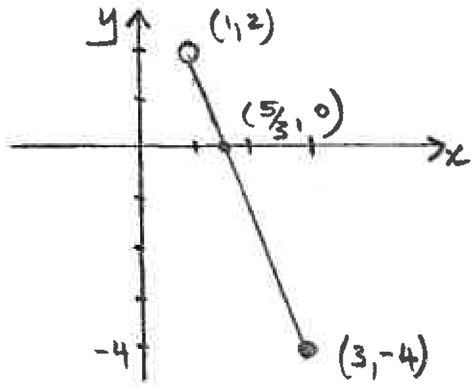
$= \frac{1 \pm \sqrt{17}}{4}$

$\left(\frac{1-\sqrt{17}}{4}, 0\right) \quad \left(\frac{1+\sqrt{17}}{4}, 0\right)$

---

28 (a) $x = 1,\ f(1) = 5 - 3 = 2 \quad (1,2)$ ○

$x = 3,\ f(3) = 5 - 9 = -4 \quad (3,-4)$ ●



X int: $y = 0$

$5 - 3x = 0$

$x = \frac{5}{3}$

dom $= (1, 3]$ ran $= [-4, 2)$

28 (b) $h: [-1,2) \to R,\ h(x) = x^2 - 8x + 17$

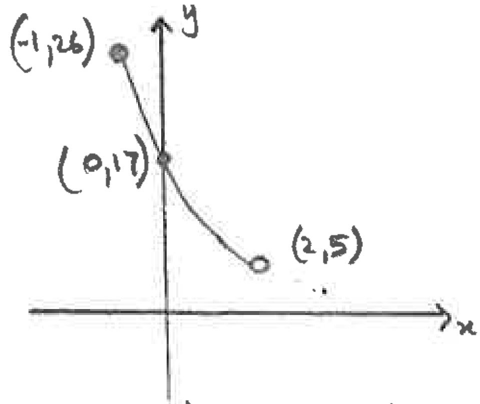
$h(-1) = (-1)^2 - 8(-1) + 17 = 1 + 8 + 17 = 26$ •

$h(2) = 2^2 - 8(2) + 17 = 4 - 16 + 17 = 5$ ∘

$h(x) = (x-4)^2 - 16 + 17$

$h(x) = (x-4)^2 + 1$

$TP = (4,1)$ ← Not in domain



dom $= [-1,2)$ ran $= (5,26]$

28 (c) $g: [-3,1) \to R,\ g(x) = 6 - (x+2)^2$

$g(-3) = 6 - (-3+2)^2 = 6 - (-1)^2 = 5$ $(-3,5)$ •

$g(1) = 6 - (1+2)^2 = 6 - 9 = -3$ $(1,-3)$ ∘

$TP = (-2,6)$

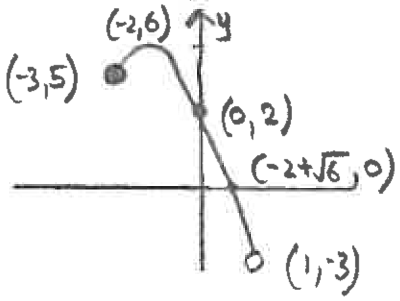
X int: $y = 0$

$6 - (x+2)^2 = 0$

$(x+2)^2 = 6$

$x + 2 = \pm\sqrt{6}$

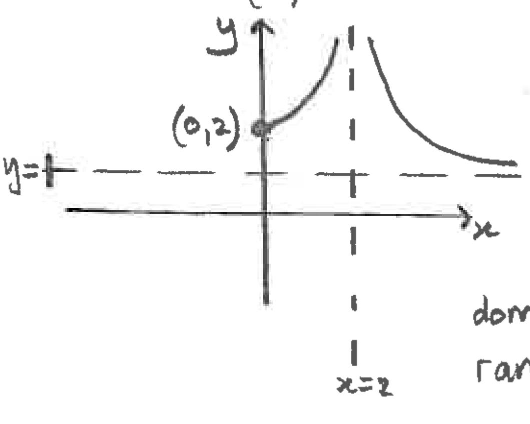
$x = -2 \pm \sqrt{6}$



dom $= [-3,1)$ ran $= (-3,6]$

28 (d) $f: [0,\infty)\setminus\{2\} \to R,\ f(x) = 1 + \dfrac{4}{(x-2)^2}$

$f(0) = 1 + \dfrac{4}{(-2)^2} = 2$ $(0,2)$ •



dom $= [0,\infty)\setminus\{2\}$

ran $= (1,\infty)$

29 (a) $x_{TP} = -\dfrac{b}{2a} = -\dfrac{-8}{4} = 2$

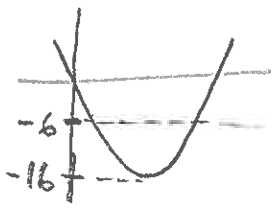
$y_{TP} = f(2) = 2(2)^2 - 8(2)$

$= 8 - 16$

$= -16$

∴ Min value is -16

(b) $f(x) = 2x^2 - 8x$



$k > 10$

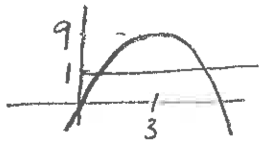
$k \in (10, \infty)$

30 (a) $x_{TP} = -\dfrac{b}{2a} = \dfrac{-6}{-2} = 3$

$f(3) = -3^2 + 6(3) = 9$

Max function value $= 9$

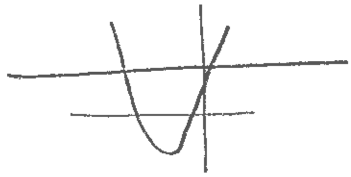
(b)



$k < -8$

$k \in (-\infty, -8)$

31 (a)



Fails horizontal line test

∴ Not 1-1 Function

(b) No - Fails horizontal line test

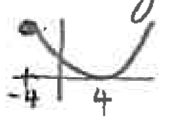
(c) No - Fails horizontal line test

(d) Yes 1-1 Function

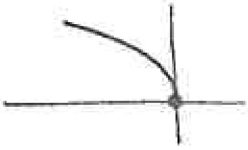
(e) Yes 1-1 Function

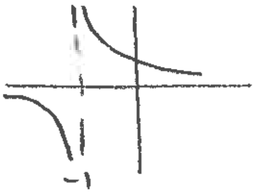
32 (a) Not a function - fails vertical line test through $(3,-1)$ and $(3,2)$

(b)

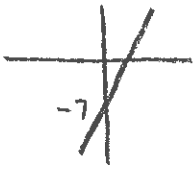


Not 1-1 fails horizontal line test

32 (c)  1-1 Function

(d) 

1-1 Function

(e) 

1-1 Function

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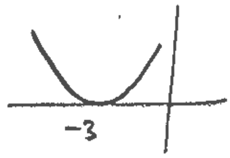
(33) $f(x) = x^2 + 6x + 9$

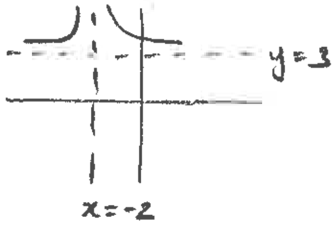
$= (x+3)^2$



Min value of a is -3

---

(34)

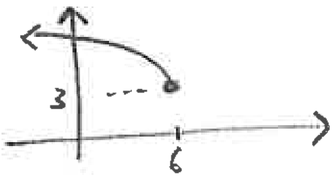


$f_1 : (-2, \infty) \to R,\ f_1(x) = \frac{3}{(x+2)^2} + 3$

$f_2 : (-\infty, -2) \to R,\ f_2(x) = \frac{3}{(x+2)^2} + 3$

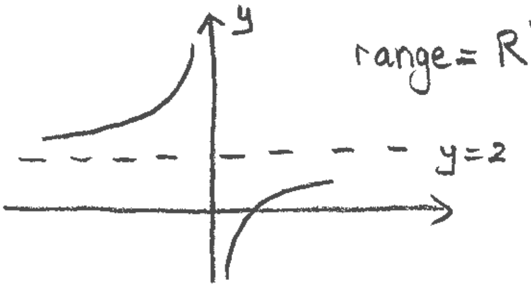
---

(35) (a) $f(x) = \sqrt{-(x-6)} + 3$



ran $= [3, \infty)$

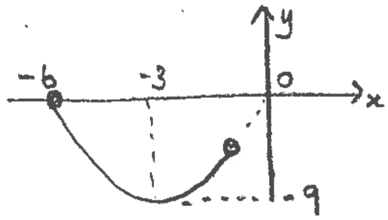
(b) $g(x) = 2 - \frac{5}{x}$



range $= R \setminus \{2\}$

(c) $h : [-6, -2) \to R,\ h(x) = x^2 + 6x$

$h(x) = x(x+6)$



$h(-3) = -3(-3+6) = -9$

range $= [-9, 0]$

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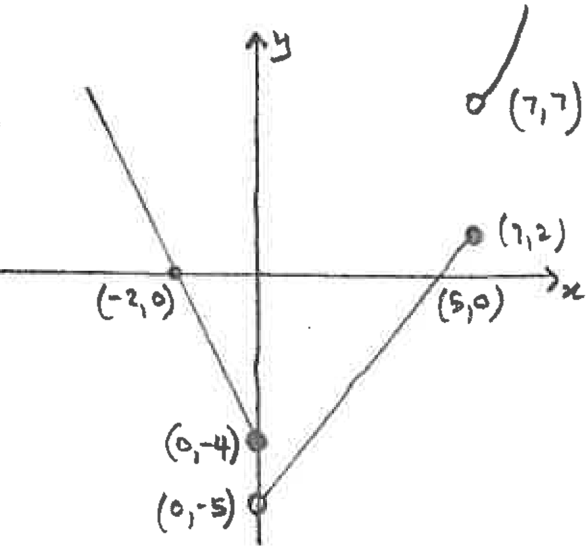
36 (a) (i) $f(-2) = -2(-2) - 4 = 0$

(ii) $f(5) = 5 - 5 = 0$

(b)

(c)



(d) (i) $f(x) = -4 \cdot 5$

Let $x - 5 = -4 \cdot 5$

$x = 0 \cdot 5$

(ii) $f(x) = 12$

Let $-2x - 4 = 12$

$-2x = 16$

$x = -8$

Let $(x-5)^2 + 3 = 12$

$(x-5)^2 = 9$

$x - 5 = \pm 3$

$x = 2, 8$

But $x = 2$ doesn't correspond to this rule $\therefore x = 8$

$x = -8, 8$

(e) $k \in [-4, 2] \cup (7, \infty)$

(37)

$f(x) = x-4$	$f^{-1}(x) = x+4$
$f(x) = \frac{x+5}{3}$	$f^{-1}(x) = 3x-5$

(38) (a) $f(x) = -x+4$

Let $y = -x+4$

$x \leftrightarrow y \quad x = -y+4$

$y = 4-x$

$\therefore f^{-1}(x) = 4-x$

(b) $f = \{(1,3), (2,-1), (3,4)\}$

$f^{-1} = \{(3,1), (-1,2), (4,3)\}$

(c) Let $y = x^2-4$

$x \leftrightarrow y \quad x = y^2-4$

$y^2 = x+4$

$y = \pm\sqrt{x+4}$

Not a function

39 (a) $f = \{(1,-1)\ (3,1), (4,-1)\}$

Not 1-1 Fails horizontal $(1,-1)\ (4,-1)$

$\therefore f^{-1}$ does not exist

(b) $f = \{(0,2)\ (2,-1)\ (3,3)\}$

1-1

$\therefore f^{-1}$ exists

(c) $f(x) = -3x+5$

linear - always 1-1

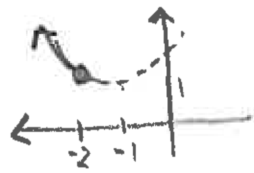
$\therefore f^{-1}$ exists

(d) $f(x) = \sqrt{-x}$

1-1

$\therefore f^{-1}$ exists

(e) $f(x) = (x+1)^2+1$, $x \leq -2$



1-1

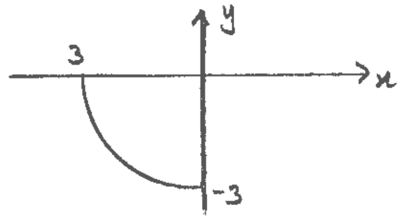
$\therefore f^{-1}$ exists

(f) $y = 2-x^2$

Not 1-1

f^{-1} does not exist

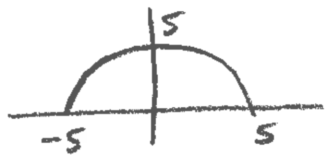
(g) $y = -\sqrt{9-x^2}$



1-1

$\therefore f^{-1}$ exists

(h) $y = \sqrt{25-x^2}$, $-5 \leq x \leq 5$



Not 1-1

$\therefore f^{-1}$ does not exist

40 Let $y = \frac{x+6}{3}$

$x \leftrightarrow y \quad x = \frac{y+6}{3}$

$y = 3x-6$

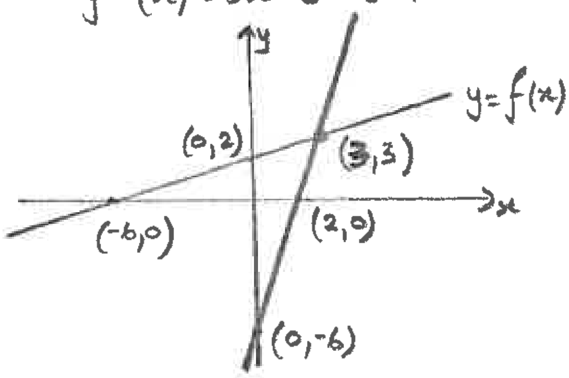
$f^{-1}(x) = 3x-6$

$\frac{x+6}{3} = 3x-6$

$x+6 = 9x-18$

$8x = 24$

$x = 3$



(41) $g: (0,3) \to R,\ g(x) = \sqrt{9-x}$

Let $y = \sqrt{9-x}$

$x \leftrightarrow y: \quad x = \sqrt{9-y}$

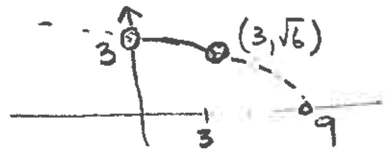
$x^2 = 9-y$

$y = 9-x^2$

$g^{-1}(x) = 9-x^2$

$\text{dom } g^{-1} = \text{ran } g$ — to find ran g sketch

$g(x) = \sqrt{-(x-9)}$



$g(3) = \sqrt{9-3} = \sqrt{6}$

$\text{ran } g = (\sqrt{6}, 3)$

$\therefore \text{dom } g^{-1} = (\sqrt{6}, 3)$

$\therefore g^{-1}: (\sqrt{6},3) \to R,\ g^{-1}(x) = 9-x^2$

$\text{ran } g^{-1} = \text{dom } g = (0,3)$

---

(42) $h: (-\infty, 2] \to R,\ h(x) = (x-5)^2+1$

Let $y = (x-5)^2+1$

$x \leftrightarrow y: \quad x = (y-5)^2+1$

$x-1 = (y-5)^2$

$\therefore y-5 = \pm\sqrt{x-1}$

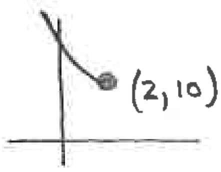
$\therefore y = 5 \pm \sqrt{x-1}$

But $\text{ran } h^{-1} = \text{dom } h = (-\infty, 2]$

$\therefore$ select $-$ from $\pm$

$h^{-1}(x) = 5 - \sqrt{x-1}$

$\text{dom } h^{-1} = \text{ran } h$ — sketch $h(x)$



$h(2) = (2-5)^2+1 = 10$

$\text{ran } h = [10, \infty)$

$\therefore \text{dom } h^{-1} = [10, \infty)$

$h^{-1}: [10,\infty) \to R,\ h^{-1}(x) = (x-5)^2+1$

$\text{ran } h^{-1} = (-\infty, 2]$

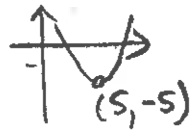
---

(43) $f: [a, 10) \to R,\ f(x) = x^2 - 10x + 20$

(a) $f(x) = (x-5)^2 - 25 + 20$

$= (x-5)^2 - 5$

$a = 5$



(b) Let $y = (x-5)^2 - 5$

swap x and y

$x = (y-5)^2 - 5$

$x+5 = (y-5)^2$

$y-5 = \pm\sqrt{x+5}$

$y = 5 \pm \sqrt{x+5}$

$\text{ran } f^{-1} = [5, 10)$

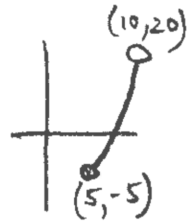
$\therefore f^{-1}(x) = 5 + \sqrt{x+5}$

$\text{dom } f^{-1} = \text{ran } f$ — Sketch f

$f(10) = (10-5)^2 - 5 = 20$

$\text{ran } f = [-5, 20)$

$\therefore \text{dom } f^{-1} = [-5, 20)$



(i) $f^{-1}: [-5, 20) \to R,\ f^{-1}(x) = 5 + \sqrt{x+5}$

(ii) $\text{ran } f^{-1} = [5, 10)$

(44) $f^{-1}(x) = -2$

$\therefore x = f(-2)$

$= (-2)^3 + 6(-2)^2 + 12(-2) + 3$

$= -8 + 24 - 24 + 3$

$= -5$

Solution $x = -5$

(45) $f(x) = \frac{1}{x} - 2$

$\frac{1}{x} = 2$

$x = \frac{1}{2}$

$\frac{1}{-2} - 2$

Let $y = \frac{1}{x} - 2$

$x \leftrightarrow y$: $x = \frac{1}{y} - 2$

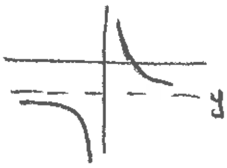
$x + 2 = \frac{1}{y}$

$y = \frac{1}{x+2}$

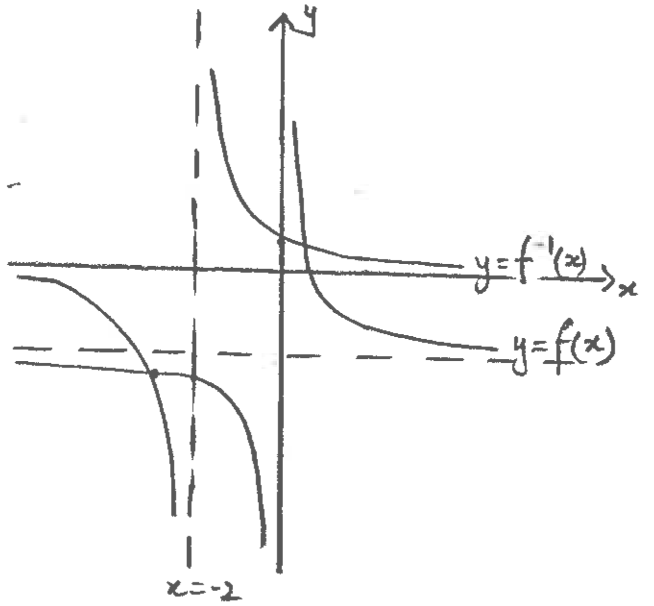
$f^{-1}(x) = \frac{1}{x+2}$

dom f^{-1} = ran f

$= R \backslash \{-2\}$



$f^{-1} : R\backslash\{-2\} \to R$, $f^{-1}(x) = \frac{1}{x+2}$



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(46) $f : (-2, \infty) \to R$, $f(x) = x^2 + kx + 1$

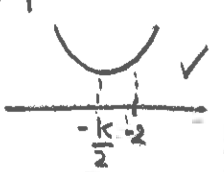
$f(x) = \left(x + \frac{k}{2}\right)^2 - \frac{k^2}{4} + 1$

$\therefore x_{TP} = -\frac{k}{2}$

$\therefore -\frac{k}{2} \leq -2$

$-k \leq -4$

$k \geq 4$



(b)(i) dom f^{-1} = ran f

$f(-2) = (-2)^2 - 2k + 1 = 5 - 2k$

ran $f = (5 - 2k, \infty)$

dom $f^{-1} = (5-2k, \infty)$

Let $y = \left(x + \frac{k}{2}\right)^2 + 1 - \frac{k^2}{4}$

$x \leftrightarrow y$: $x = \left(y + \frac{k}{2}\right)^2 + 1 - \frac{k^2}{4}$

$x - 1 + \frac{k^2}{4} = \left(y + \frac{k}{2}\right)^2$

$y + \frac{k}{2} = \pm\sqrt{x - 1 + \frac{k^2}{4}}$

$y = -\frac{k}{2} \pm \sqrt{x - 1 + \frac{k^2}{4}}$

But ran f^{-1} = dom $f = (-2, \infty)$

$\therefore f^{-1}(x) = -\frac{k}{2} + \sqrt{x - 1 + \frac{k^2}{4}}$

$f^{-1} : (5-2k, \infty) \to R$, $f^{-1}(x) = -\frac{k}{2} + \sqrt{x - 1 + \frac{k^2}{4}}$

---

(47) Let $y = 1 + \frac{1}{x-1}$

$x \leftrightarrow y$: $x = 1 + \frac{1}{y-1}$

$x - 1 = \frac{1}{y-1}$

$\frac{1}{x-1} = y - 1$

$y = 1 + \frac{1}{x-1}$

$f^{-1}(x) = 1 + \frac{1}{x-1}$

$\therefore f^{-1}(x) = f(x)$

---

(48) Let $y = \frac{ax+b}{cx+d}$

$x \leftrightarrow y$: $x = \frac{ay+b}{cy+d}$

$cyx + dx = ay + b$

$cyx - ay = b - dx$

$y(cx - a) = b - dx$

$y = \frac{-dx + b}{cx - a}$

$\therefore f(x)$ will be own inverse if $a = -d$

---

(49) Let $A = (a, b)$ and $B = (b, a)$

$$m_{AB} = \frac{a-b}{b-a}$$
$$= -\frac{(b-a)}{b-a}$$
$$= -1$$

Let M be the midpoint of AB

$$\therefore M = \left(\frac{a+b}{2}, \frac{b+a}{2}\right)$$

Since $m_{AB} = -1$ the gradient of the line perpendicular to AB is $-\frac{1}{-1} = 1$

$\therefore$ Equation of perpendicular bisector of AB is

$$y - y_1 = m(x - x_1)$$
$$y - \frac{a+b}{2} = 1\left(x - \frac{a+b}{2}\right)$$
$$y = x - \frac{a+b}{2} + \frac{a+b}{2}$$
$$y = x$$

So $y = x$ is the perpendicular bisector of AB.

(50) $f(x) = -\frac{1}{x}$

Let $y = -\frac{1}{x}$

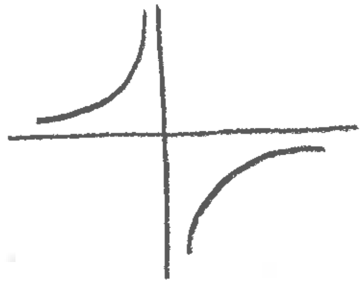
$x \Leftrightarrow y$: $\quad x = -\frac{1}{y}$

$$y = -\frac{1}{x}$$
$$\therefore f^{-1}(x) = -\frac{1}{x}$$

So $f^{-1}(x) = f(x)$ for all x

Hence in this case the graphs of the function and its inverse will intersect at every point on the graph of $y = -\frac{1}{x}$.

In addition to this consider the graph of $y = -\frac{1}{x}$



The graph never intersects the line $y = x$

So all the points of intersection between $y = f(x)$ and $y = f^{-1}(x)$ will not occur on the line $y = x$