EXPONENTIALS + LOGARITHMS SOLUTIONS

Q1)

A) $2 \times 3^x = 18$

$3^x = 9$

$3^x = 3^2$

$\underline{x = 2}$

B) $4 \times \left(\frac{1}{3}\right)^x = 36$

$\left(\frac{1}{3}\right)^x = 9$

$\frac{1}{3^x} = 3^2$

$3^{-x} = 3^2$

$\underline{x = -2}$

C) $3^{x-2} = 9^{x-3}$

$3^{x-2} = (3^2)^{x-3}$

$3^{x-2} = 3^{2x-6}$

$x - 2 = 2x - 6$

$4 = x$

$\underline{x = 4}$

D) $25^{x-2} = \sqrt{5}$

$(5^2)^{x-2} = 5^{1/2}$

$5^{2x-4} = 5^{1/2}$

$2x - 4 = \frac{1}{2}$

$4x - 8 = 1$

$4x = 9$

$\underline{x = \frac{9}{4}}$

E) $8^{1-2x} = 4^{x-3}\sqrt{8}$

$(2^3)^{1-2x} = (2^2)^{x-3}(2^3)^{1/2}$

$2^{3-6x} = 2^{2x-6} \cdot 2^{3/2}$

$2^{3-6x} = 2^{2x-6+3/2}$

$3 - 6x = 2x - 6 + \frac{3}{2}$

$\times 2)$ $6 - 12x = 4x - 12 + 3$

$6 - 12x = 4x - 9$

$15 = 16x$

$\underline{x = \frac{15}{16}}$

F) $16^x - 5 \times 8^x = 0$

$(2^4)^x - 5 \cdot (2^3)^x = 0$

$2^{4x} - 5 \cdot 2^{3x} = 0$

$2^{4x} = 5 \cdot 2^{3x}$

$\frac{2^{4x}}{2^{3x}} = 5$

$2^x = 5$

$\underline{x = \log_2 5}$

G) $10^{2x} - 11 \times 10^x + 10 = 0$

$(10^x)^2 - 11 \times (10^x) + 10 = 0$

$(10^x - 1)(10^x - 10) = 0$

$10^x = 1$ or $10^x = 10$

$x = 0$ or $x = 1$

$\underline{x = 0, 1}$

H) $4\times 2^{2x}+8=33\times 2^{x}$

$4\times(2^{x})^{2}-33\times 2^{x}+8=0$

Let $y=2^{x}$

$4y^{2}-33y+8=0$

$(4y-1)(y-8)=0$

$4y \quad -1$
$y \quad -8$

$4y=1$ or $y=8$

$y=\frac{1}{4}$ or $y=8$

$2^{x}=\frac{1}{4}$ or $2^{x}=8$

$\underline{x=-2,3}$

(i) $(3^{x})^{2}-6\times 3^{x}-27=0$

$(3^{x}-9)(3^{x}+3)=0$

$3^{x}=9$ or $\underbrace{3^{x}=-3}_{\text{NO SOL}^{\underline{n}}}$

$\underline{x=2}$

Q2) $(2^{x})^{2}+5\cdot 2^{x}+k=0$

$2^{x}=\frac{-5\pm\sqrt{25-4(1)(k)}}{2}$

NOTE $2^{x}=\frac{-5-\sqrt{25-4k}}{2}$ HAS NO SOL$^{\underline{n}}$

CONSIDER $2^{x}=\underbrace{\frac{-5+\sqrt{25-4k}}{2}}_{\text{NEEDS TO BE POSITIVE}}$

$\frac{-5+\sqrt{25-4k}}{2}>0$

×2) $-5+\sqrt{25-4k}>0$

$\sqrt{25-4k}>5$

$25-4k>25$

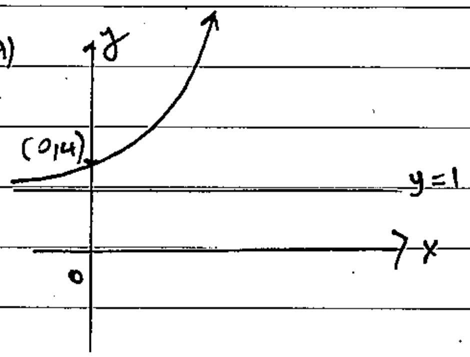
$-4k>0$

$k<0$

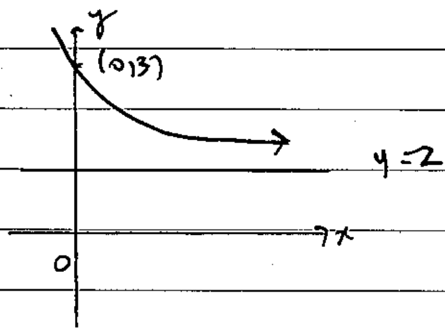
HAS ONE SOL$^{\underline{n}}$ IFF $k<0$.

Q3) $y=3^{x}+1$

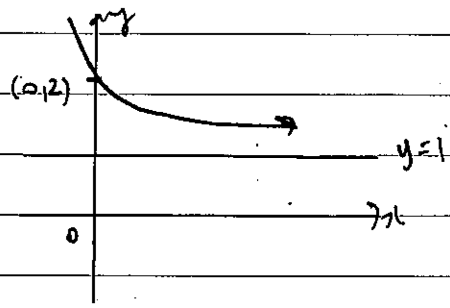
A)



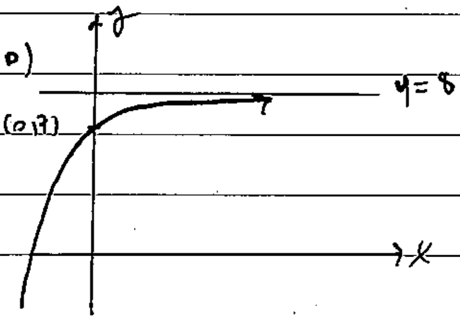
B) $y=(\frac{1}{2})^{x}+2$ (NOTE $\frac{1}{2}<1$)



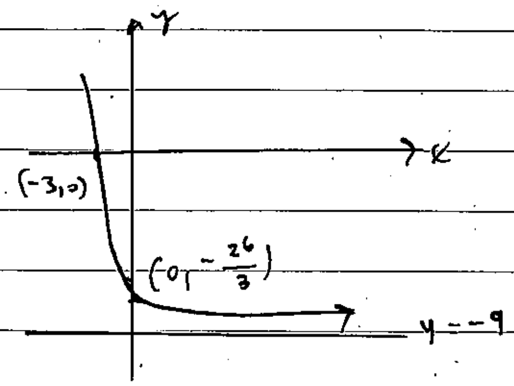
c) $y=10^{-x}+1$



D)



E) $y = \left(\frac{1}{3}\right)^{x+1} - 9$



WHEN $y=0$ $\quad 0 = \left(\frac{1}{3}\right)^{x+1} - 9$

$\left(\frac{1}{3}\right)^{x+1} = 9$

$(3^{-1})^{x+1} = 3^2$

$3^{-x-1} = 3^2$

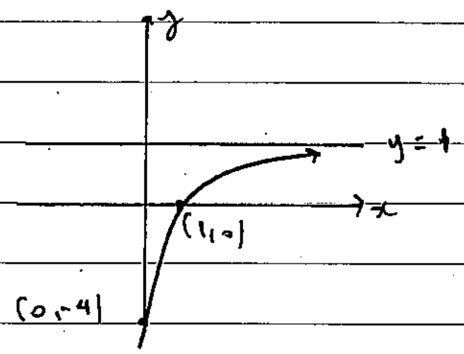
$-x-1 = 2$

$-x = 3$

$x = -3$

F) $y = -5^{-x+1} + 1$

$= -5^{-(x-1)} + 1$



WHEN $x=0$, $y = -5^1 + 1 = -4$

$y=0 \quad 0 = -5^{-x+1} + 1$

$5^{-x+1} = 1$

$-x+1 = 0$

$x = 1$

Q4) A) $\log_2 35$

$= 5$

B) $\log_3 27$

$= \log_3 3^3$

$= 3$

C) $\log_{10} \frac{1}{10,000}$

$= \log_{10} 10^{-4}$

$= -4$

D) $\log_2 \sqrt{8}$

$= \log_2 (2^3)^{1/2}$

$= \log_2 2^{2/2}$

$= 3/2$

E) $\log_4 8$

$= \log_4 2^3$

$= \log_4 (\sqrt{4})^3$

$= \log_4 4^{3/2}$

$= 3/2$

Q5) (A) $\log_b x = m \therefore x = b^m$

$\log_b y = n \therefore y = b^n$

$x \cdot y = b^m \cdot b^n$

$= b^{m+n}$

$\log_b (x \cdot y) = m+n$

$= \log_b x + \log_b y$

B) $\log_b x = m \therefore x = b^m$

$x = b^m$

$\Rightarrow x^d = (b^m)^d$

$\Rightarrow x^d = b^{md}$

$\log_b x^d = md$

$= d \log_b x$

Q6) A) $\log_2 28 - \frac{1}{2}\log_2 49$

$= \log_2 28 - \log_2 \sqrt{49}$

$= \log_2 28 - \log_2 7$

$= \log_2 \frac{28}{7}$

$= \log_2 4$

$= \log_2 2^2$

$= \underline{2}$

B) $2\log_3 x - 2\log_3 3x + \log_3 6x$

$= \log_3 x^2 - \log_3 (3x)^2 + \log_3 6x$

$= \log_3 x^2 - \log_3 9x^2 + \log_3 6x$

$= \log_3 \frac{6x^3}{9x^2}$

$= \underline{\log_3 \left(\frac{2x}{3}\right)}$

c) $3\log_5 x - \frac{1}{2}\log_5 4x^8 + \log_5(10x)$

$= \log_5 x^3 - \log_5 (4x^8)^{1/2} + \log_5(10x)$

$= \log_5 x^3 - \log_5 2x^4 + \log_5(10x)$

$= \log_5 \left(\frac{10x^4}{2x^4}\right)$

$= \log_5 5$

$= \underline{1}$

D) $3\log_2 x + \frac{1}{2}\log_2 (16y^4) - 2\log_2(2xy) + 2$

$= \log_2 x^3 + \log_2 (16y^4)^{1/2} - \log_2 (2xy)^2 + 2$

$= \log_2 x^3 + \log_2 4y^2 - \log_2 4x^2y^2 + \log_2 4$

$= \log_2 \left(\frac{16x^3 \cancel{y^2}}{4x^2 \cancel{y^2}}\right)$

$= \underline{\log_2 (4x)}$

Q7) A) $\log_2 a = c \quad \log_2 b = d$

$\log_2 (ab) = \log_2 a + \log_2 b$

$= \underline{c + d.}$

B) $\log_2 \left(\frac{a}{b}\right) = \log_2 a - \log_2 b$

$= \underline{c - d}$

c) $\log_2 (8a^3 b)$

$= \log_2 8 + \log_2 a^3 + \log_2 b$

$= \log_2 2^3 + 3\log_2 a + \log_2 b$

$= \underline{3 + 3c + d}$

D) $\log_2 \frac{16a}{\sqrt{b}}$

$= \log_2 16 + \log_2 a - \log_2 b^{1/2}$

$= \log_2 2^4 + \log_2 a - \frac{1}{2}\log_2 b$

$= \underline{4 + c - \frac{1}{2}d}$

(E) $\log_2\left(\frac{8b^2}{\sqrt[3]{a}}\right)$

$= \log_2 8 + \log_2 b^2 - \log_2 a^{1/3}$

$= \log_2 2^3 + 2\log_2 b - \frac{1}{3}\log_2 a$

$= \underline{3 + 2d - \frac{1}{3}c}$

(F) $\log_2\sqrt{\frac{4a^3}{b^5}}$

$= \frac{1}{2}\log_2\left(\frac{4a^3}{b^5}\right)$

$= \frac{1}{2}\left(\log_2 4 + \log_2 a^3 - \log_2 b^5\right)$

$= \frac{1}{2}\left(\log_2 2^2 + 3\log_2 a - 5\log_2 b\right)$

$= \underline{\frac{1}{2}\left(2 + 3c - 5d\right)}$

Q8) $3^{2x-1} = 4^{3-x}$

$\log_3 3^{2x-1} = \log_3 4^{3-x}$

$(2x-1)\log_3 3 = (3-x)\log_3 4$

$2x-1 = (3-x)\log_3 4$

$2x-1 = 3\log_3 4 - x\log_3 4$

$2x + x\log_3 4 = 3\log_3 4 + 1$

$x(2 + \log_3 4) = 3\log_3 4 + 1$

$x = \frac{3\log_3 4 + 1}{2 + \log_3 4}$

$= \frac{\log_3 4^3 + \log_3 3}{\log_3 9 + \log_3 4}$

$= \frac{\log_3(3 \times 64)}{\log_3 36}$

$= \underline{\frac{\log_3 192}{\log_3 36}}$

(Q9) (A) $b = c\log_{10} N$

$\frac{b}{c} = \log_{10} N$

$N = 10^{b/c}$

(B) $d = 3 + \log_2 N$

$d - 3 = \log_2 N$

$N = 2^{d-3}$

(C) $\log_3 N = 2\log_3 d + 2$

$\log_3 N = \log_3 d^2 + 2\log_3 3$

$\log_3 N = \log_3 d^2 + \log_3 9$

$\log_3 N = \log_3(9d^2)$

$N = 9d^2$

D) $\log_{10} P = \frac{1}{3}\log_{10} N$

$\log_{10} P = \log_{10} N^{1/3}$

$P = N^{1/3}$

$P^3 = N$

$N = P^3$

Q10) (A) $2\log_3 x = 4$

$\log_3 x = 2$

$\underline{x = 3^2 = 9}$ only

(B) $\log_3 x^2 = 4$

$x^2 = 3^4 = 81$

$x = \pm\sqrt{81}$

$\underline{x = 9, -9}$

(C) $\log_2 (x-3) = 4$

$x - 3 = 2^4 = 16$

$\underline{x = 19}$

D) $\log_3 \left(\frac{x+1}{x-1}\right) = 2$

$\frac{x+1}{x-1} = 9$

$x + 1 = 9x - 9$

$10 = 8x$

$x = \frac{10}{8} = \frac{5}{4}$

(E) $\log_{10} x + \log_{10}(x+1) = \log_{10} 30$

$\log_{10} x(x+1) = \log_{10} 30$

$x(x+1) = 30$

$x^2 + x - 30 = 0$

$(x+6)(x-5) = 0$

$x = 5, -6$

<u>NOTE</u> $\log_{10} x$ DEFINED FOR $x > 0$

SO REJECT $x = -6$

$\therefore \underline{x = 5}$

F) $\log_5 x = \log_5 8 - \log_5 (6-x)$

$\log_5 x = \log_5 \left(\frac{8}{6-x}\right)$

$x = \frac{8}{6-x}$

$6x - x^2 = 8$

$x^2 - 6x + 8 = 0$

$(x-4)(x-2) = 0$

$\underline{x = 2, 4}$

(G) $\log_4 \frac{x+1}{x-1} = \frac{1}{2}$

$\frac{x+1}{x-1} = 4^{1/2} = 2$

$x + 1 = 2x - 2$

$3 = x$

$\underline{x = 3}$

(H) $\log_8 (x^2+7)^{1/4} = \frac{1}{3}$

$\frac{1}{4} \log_8 (x^2+7) = \frac{4}{3}$

$\log_8 (x^2+7) = \frac{4}{3}$

$x^2 + 7 = 8^{4/3}$

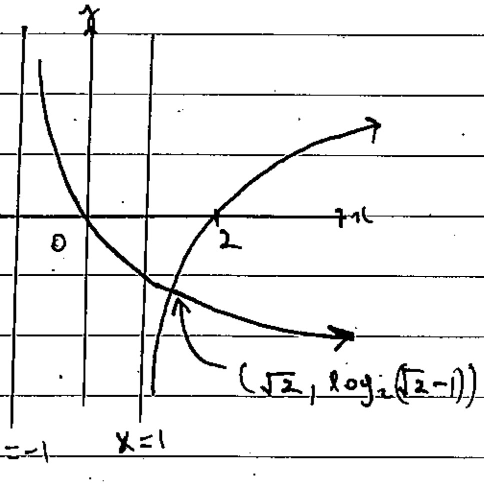
$x^2 + 7 = (2^3)^{4/3}$

$x^2 + 7 = 16$

$x^2 = 9$

$\underline{x = \pm 3}$

Q11) $y = -\log_2(x+1)$ ① $y = \log_2(x-1)$ ②.



① = ② $-\log_2(x+1) = \log_2(x-1)$

$0 = \log_2(x-1) + \log_2(x+1)$

$0 = \log_2(x^2-1)$

$x^2 - 1 = 2^0 = 1$

$x^2 = 2$

$x = \pm\sqrt{2}$ $(x > 0)$

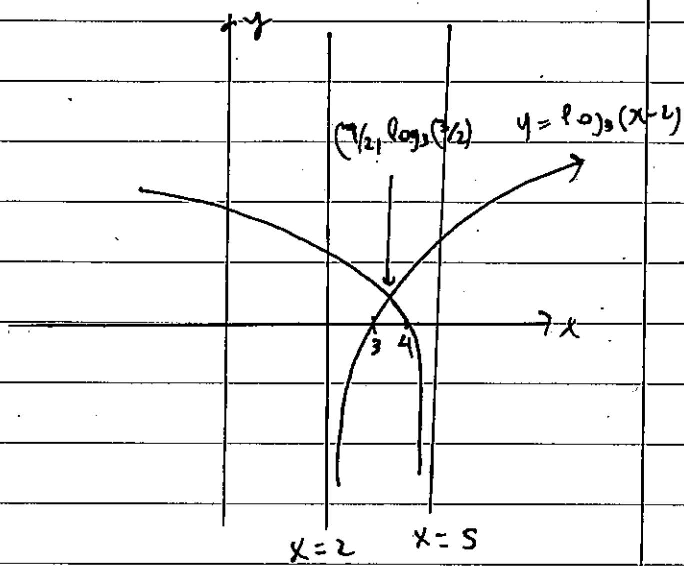
$\therefore x = \sqrt{2}$

#7

(Q12) $y = \log_3(x-2)$ ①

$y = \log_3(5-x)$

$= \log_3(-(x-5))$ ②



BY SYMMETRY, THE GRAPHS INTERSECT @ $x = 3.5$. TO CHECK THIS,

① = ② $\log_3(x-2) = \log_3(5-x)$

$\Rightarrow x-2 = 5-x$

$\Rightarrow 2x = 7$

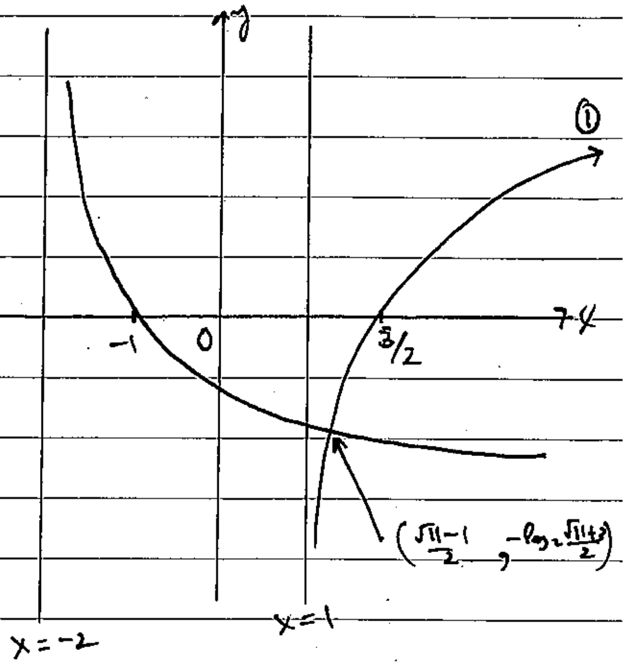
$\Rightarrow x = 7/2$

$\Rightarrow y = \log_3(7/2-2)$

$= \log_3(3/2)$

(Q13) $y = -\log_2(x+2)$ ①

$y = \log_2(x-1)+1$ ②



① = ② $\log_2(x-1)+1 = -\log_2(x+2)$

$\log_2(x-1)+\log_2(x+2) = -1$

$\log_2((x-1)(x+2)) = -1$

$(x-1)(x+2) = 2^{-1}$

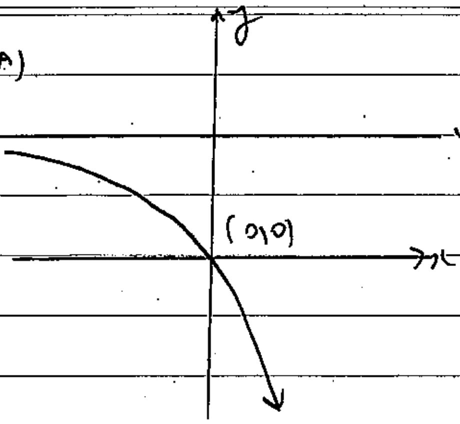
$x^2+x-2 = \frac{1}{2}$

$2x^2+2x-4 = 1$

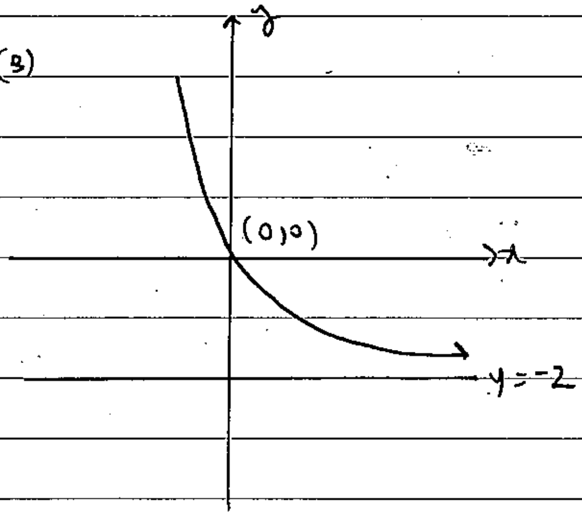
$2x^2+2x-5 = 0$ (QUAD-FORM)

$x = \frac{\sqrt{11}-1}{2}$ $y = -\log_2\left(\frac{\sqrt{11}+3}{2}\right)$

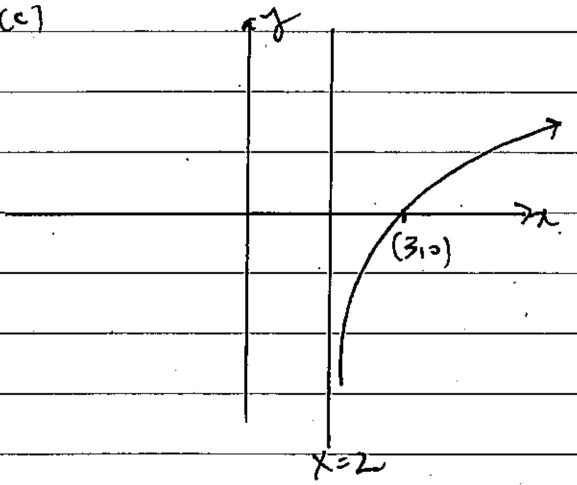
Q14) (A)



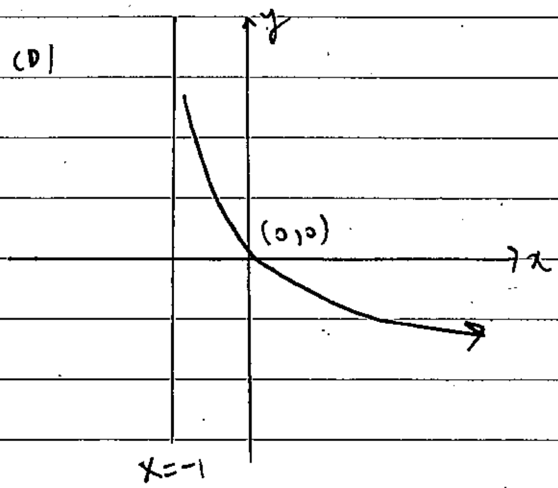
(B)



(C)



(D)



Q15) $f(x) = 2^{x+1} - 1$

$y = 0 \Rightarrow 0 = 2^{x+1} - 1$

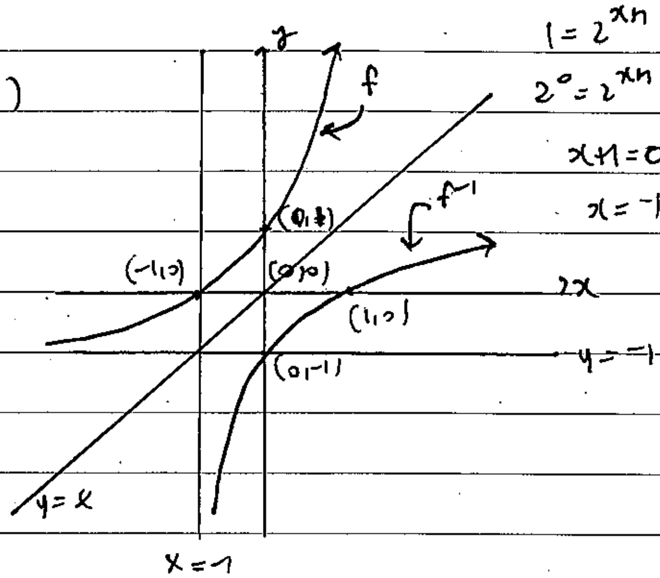
$1 = 2^{x+1}$

$2^0 = 2^{x+1}$

$x + 1 = 0$

$x = -1$

(A)



FOR RULE $x \leftrightarrow y$

$x = 2^{y+1} - 1$

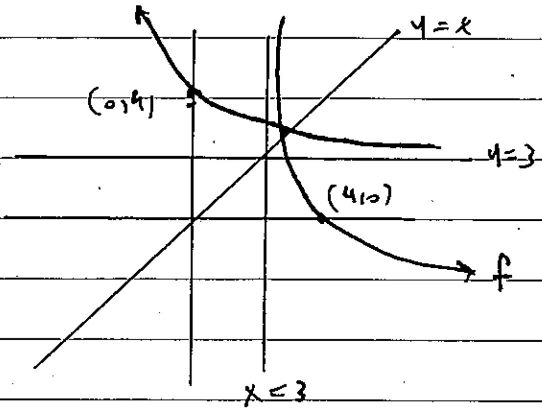
$x + 1 = 2^{y+1}$

$\log_2(x+1) = y + 1$

$y = \log_2(x+1) - 1$

$f^{-1}: (-1, \infty) \to \mathbb{R}$, $f^{-1}(x) = \log_2(x+1) - 1$

(B) $f(x) = -\log_e(x-3)$



FOR INVERSE $x \leftrightarrow y$ $x = -\log_e(y-3)$

$e^{-x} = y - 3$

$y = e^{-x} + 3$

$f^{-1}: \mathbb{R} \to \mathbb{R}$, $f^{-1}(x) = e^{-x} + 3$

(Q16) (A) $2 \times 3^{2a} + 2 = 5$

$2 \times 3^{2a} = 3$

$3^{2a} = \frac{3}{2}$

$2a = \log_3\left(\frac{3}{2}\right)$

$a = \frac{1}{2}\log_3\left(\frac{3}{2}\right)$

(B) $3 \times \left(\frac{1}{2}\right)^{2b} + 7 = 11$

$3 \times \left(\frac{1}{2}\right)^{2b} = 4$

$\left(\frac{1}{2}\right)^{2b} = \frac{4}{3}$

$2^{-2b} = \frac{4}{3}$

$\log_2\left(\frac{4}{3}\right) = -2b$

$b = -\frac{1}{2}\log_2\left(\frac{4}{3}\right)$

(C) $b \times 10^{4a} = 6$ ①

$b \times 10^{2a} = 3$ ②

① ÷ ② $10^{2a} = 2$

$2a = \log_{10} 2$

$a = \frac{1}{2}\log_{10} 2$

$\therefore b \times 10^{\log_{10} 2} = 3$

$b \times 2 = 3$

$b = 3/2$

$\begin{cases} a = \frac{1}{2}\log_{10} 2 \\ b = \frac{3}{2} \end{cases}$

(D) $b \times 2^{2a} = 4$ ①

$b \times 2^{-2a} = 2$ ②

① ÷ ② $2^{4a} = 2^1$

$4a = 1$

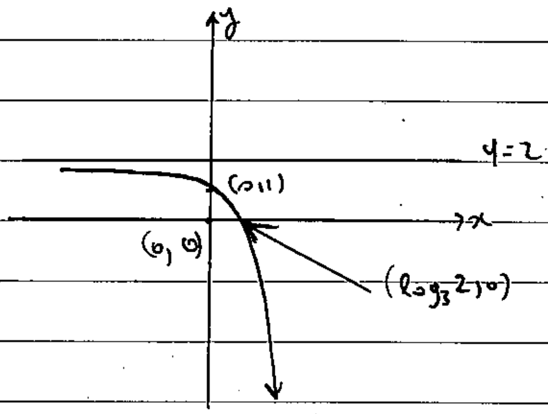
$a = \frac{1}{4}$

$\therefore b \times 2^{\frac{1}{2}} = 4$

$b = \frac{4}{\sqrt{2}} = \frac{4\sqrt{2}}{2} = 2\sqrt{2}$

$\begin{cases} a = \frac{1}{4} \\ b = 2\sqrt{2} \end{cases}$

(Q17) $y = -3^x + 2$



When $y = 0$ $\quad 0 = -3^x + 2$

$3^x = 2$

$x = \log_3 2$

(18) $3 = a \times 3^{2b}$ ①

$9 = a \times 3^{6b}$ ②

② ÷ ① $3^1 = 3^{4b}$

$4b = 1,\ b = \frac{1}{4}$

$3 = a \times 3^{\frac{1}{2}}$

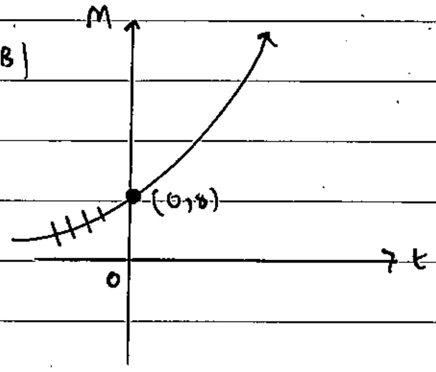
$a = \frac{3}{\sqrt{3}} = \frac{3\sqrt{3}}{3} = \sqrt{3}$

$\begin{cases} a = \sqrt{3} \\ b = \frac{1}{4} \end{cases}$

$\therefore y = \sqrt{3} \times 3^{\frac{x}{4}}$

(Q19) (A) $t = 0 \Rightarrow M = 8 \times 2^{0} = \underline{8}$ milligrams

(B)



(C) $16 = 8 \times 2^{0.2t}$

$2^{1} = 2^{0.2t}$

$\times 5)$ $0.2t = 1$

$\underline{t = 5 \text{ hrs}}$

(D) $M = 8 \times 2^{0.2 \times 10}$

$= 8 \times 2^{2}$

$= 8 \times 4$

$= \underline{32 \text{ milligrams}}$

(E) $\dfrac{\Delta M}{\Delta t} = \dfrac{M(10) - M(5)}{10 - 5}$

$= \dfrac{32 - 16}{5}$

$= \underline{\dfrac{16}{5} \text{ mg/hr.}}$

(F) $12 = 8 \times 2^{0.2t}$

$\frac{3}{2} = 2^{0.2t}$

$\log_2\left(\frac{3}{2}\right) = 0.2t$

$\times 5)$ $\underline{t = 5 \log_2\left(\frac{3}{2}\right) \text{ hrs}}$

(Q20) $M(t) = 3 \times 2^{-0.001t}$

(A) $M(0) = 3 \times 2^{0} = 3$ grams

(B) $\frac{3}{2} = 3 \times 2^{-0.001t}$

$\frac{1}{2} = 2^{-0.001t}$

$2^{-1} = 2^{-0.001t}$

$-1 = -0.001t$

$\times -1000)$ $1000 = t$

$\therefore$ $t = 1{,}000$ years

(C) $M(200) = 3 \times 2^{-0.001 \times 200}$

$= 3 \times 2^{-0.2}$

% REMAINING $= \dfrac{3 \times 2^{-0.2}}{3} \times 100\%$

$\approx 87.05\%$

(D) % REMAINING $= \dfrac{M(t)}{3} \times 100\%$

$\therefore$ $P\% = \dfrac{\cancel{3} \times 2^{-0.001t}}{\cancel{3}} \times 100\%$

$\dfrac{P\%}{100\%} = 2^{-0.001t}$

$\log_2\left(\frac{P}{100}\right) = -0.001t$

$\times -1000)$ $t = -1000 \log_2\left(\frac{P}{100}\right)$

OR $t = 1000 \log_2\left(\frac{100}{P}\right)$

Q21) LET n = # OF ROLLS

$$Pr(\geq \text{ONE } 6) \geq 0.9$$
$$1 - Pr(\text{NO SIXES}) \geq 0.9$$
$$1 - \left(\tfrac{5}{6}\right)^n \geq 0.9$$
$$\left(\tfrac{5}{6}\right)^n \leq 0.1$$
$$n \geq 12.63$$

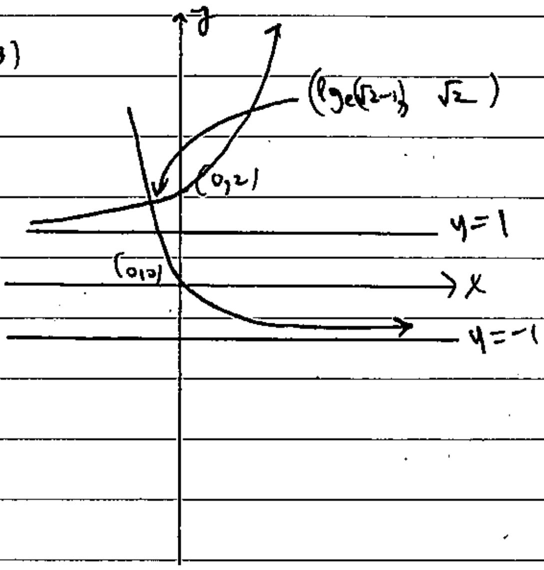
I.E $\underline{n \geq 13 \text{ ROLLS}}$

Q22) n = # OF TOSSES

$$Pr(\geq \text{ONE TAIL}) \geq 0.99$$
$$1 - Pr(\text{NO TAILS}) \geq 0.99$$
$$1 - Pr(\text{ALL HEADS}) \geq 0.99$$
$$1 - \left(\tfrac{1}{2}\right)^n \geq 0.99$$
$$\left(\tfrac{1}{2}\right)^n \leq 0.01$$
$$n \geq 6.64$$

I.E $\underline{n \geq 7 \text{ TOSSES}}$

Q23)



$y = e^{-x} - 1$ ①

$y = e^{x} + 1$ ②

① = ② $\quad e^{-x} - 1 = e^{x} + 1$

$\times e^{x}) \quad e^{0} - e^{x} = e^{2x} + e^{x}$

$$e^{2x} + 2e^{x} - 1 = 0$$
$$(e^{x})^2 + 2(e^{x}) - 1 = 0$$
$$e^{x} = \frac{-2 \pm \sqrt{4 - 4(1)(-1)}}{2}$$
$$= \frac{-2 \pm \sqrt{8}}{2}$$
$$= \frac{-2 \pm 2\sqrt{2}}{2} = -1 + \sqrt{2} \quad (e^{x} > 0)$$

$$x = \log_e(\sqrt{2} - 1)$$
$$y = e^{\log_e(\sqrt{2}-1)} + 1$$
$$= \sqrt{2} - 1 + 1$$
$$= \sqrt{2}$$

$$\text{POI} = (\log_e(\sqrt{2}-1), \sqrt{2})$$

(PTO)

(Q24) ① $y = e^{2x}$

② $y = e^{x} + 1$

① = ② $\quad e^{2x} = e^{x} + 1$

$$(e^{x})^2 - (e^{x}) - 1 = 0$$

$$e^{x} = \frac{1 \pm \sqrt{1 - 4(1)(-1)}}{2}$$

$$= \frac{1 \pm \sqrt{5}}{2}$$

$$= \frac{1 + \sqrt{5}}{2} \quad (e^{x} > 0)$$

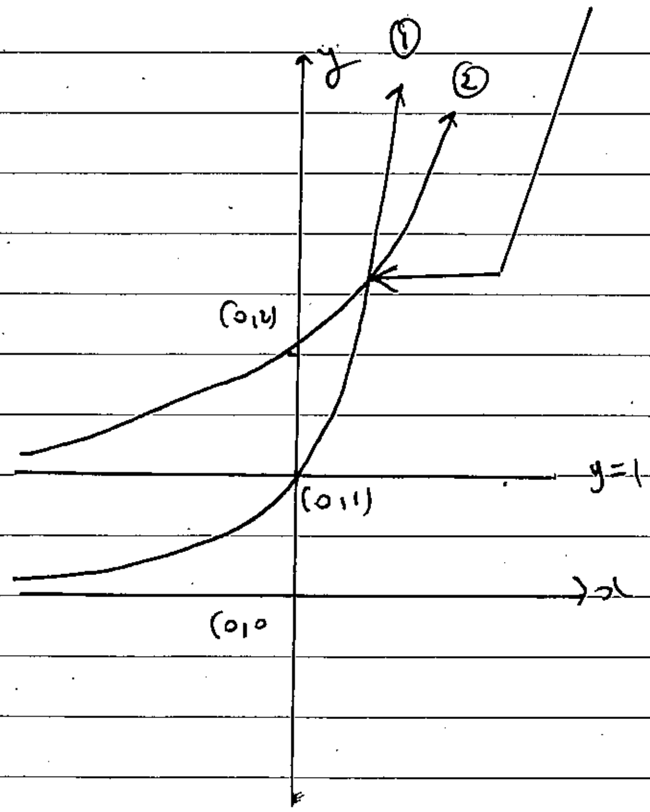
$$x = \log_e\left(\frac{1+\sqrt{5}}{2}\right)$$

$$\therefore y = e^{\log_e\left(\frac{1+\sqrt{5}}{2}\right)} + 1$$

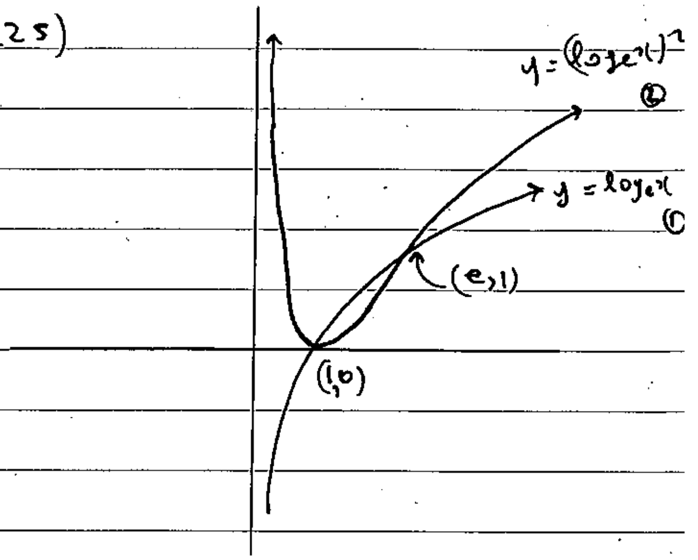
$$= \frac{1+\sqrt{5}}{2} + 1$$

$$= \frac{3+\sqrt{5}}{2}$$

POI $= \left(\log_e\left(\frac{1+\sqrt{5}}{2}\right), \frac{3+\sqrt{5}}{2}\right)$



(Q25)



① = ② $\quad \log_e x = (\log_e x)^2$

$$(\log_e x)^2 - \log_e x = 0$$

$$\log_e x\,(\log_e x - 1) = 0$$

$\log_e x = 0$ or $\log_e x = 1$

$x = e^0$ OR $x = e^1$

$= 1 \qquad = e$

$\Rightarrow y = 0 \qquad \Rightarrow y = \log_e e = 1$

Q26) $(x^2 + 3x + 3)^{x^2 - 5x + 6} = 1$

$\Leftrightarrow$ $x^2 + 3x + 3 = 1$ ①

OR $x^2 - 5x + 6 = 0$ ②

③ OR $x^2 + 3x + 3 = -1$ AND $x^2 - 5x + 6$ IS EVEN

CASE ① $\quad x^2 + 3x + 3 = 1$

$$x^2 + 3x + 2 = 0$$

$$(x+1)(x+2) = 0$$

$$x = -1, -2$$

CASE ② $\quad x^2 - 5x + 6 = 0$

$$(x-3)(x-2) = 0$$

$$x = 2, 3$$

CASE ③ $\quad x^2 + 3x + 4 = 0$

NO REAL SOLUTION

$\therefore x = -1, -2, 2, 3$

(Q27) (A) LET $x = \log_b a \quad y = \log_b c$

$$b^x = a \qquad b^y = c$$

THEN $\log_c a = \log_c b^x$

$$\log_c a = x \log_c b$$

$$\log_c a = \log_b a \log_c b$$

$$\therefore \log_b a = \frac{\log_c a}{\log_c b}$$

(B) $\log_2 5 \log_5 2$

$$= \log_5 5$$

$$\underline{= 1}$$

(C) $x^{\log_b y}$

$$= x^{\frac{\log_b y \log_b x}{\log_b x}}$$

$$= \left(x^{\frac{\log_b y}{\log_b x}} \right)^{\log_b x}$$

$$= \left(x^{\log_x y} \right)^{\log_b x}$$

$$= y^{\log_b x}$$

(Q28) SUPPOSE SUCH NUMBERS EXIST. THEN:

$$\log_2 3 = \frac{m}{n}$$

$$2^{\frac{m}{n}} = 3$$

$$2^m = 3^n \quad \text{CONTRADICTION!}$$

LHS IS EVEN

RHS IS ODD

Q29) WE FIRST PROVE THAT

$$\sqrt{xy} \le \frac{x+y}{2} \quad \text{IF } x, y \ge 0$$

PROOF

$$\frac{x+y}{2} - \sqrt{xy}$$

$$= \frac{x + y - 2\sqrt{xy}}{2}$$

$$= \frac{x - 2\sqrt{x}\sqrt{y} + y}{2}$$

$$= \frac{(\sqrt{x})^2 - 2\sqrt{x}\sqrt{y} + (\sqrt{y})^2}{2}$$

$$= \frac{(\sqrt{x} - \sqrt{y})^2}{2} \ge 0$$

NOW,

$$\frac{\log_b x + \log_b y}{2}$$

$$= \frac{\log_b (xy)}{2}$$

$$= \log_b \sqrt{xy}$$

$$\le \log_b \left(\frac{x+y}{2} \right)$$

Q30) $f(x) = \dfrac{e^{+x} - e^{-x}}{2}$

(A) $f(0) = \dfrac{e^{0} - e^{-0}}{2} = \dfrac{1-1}{2} = 0$

(B) $f(-x) = \dfrac{e^{-x} - e^{-(-x)}}{2}$

$= \dfrac{e^{-x} - e^{x}}{2}$

$= -\left(\dfrac{e^{x} - e^{-x}}{2}\right)$

$= -f(x)$

AS $f(-x) = -f(x)$, f IS ODD

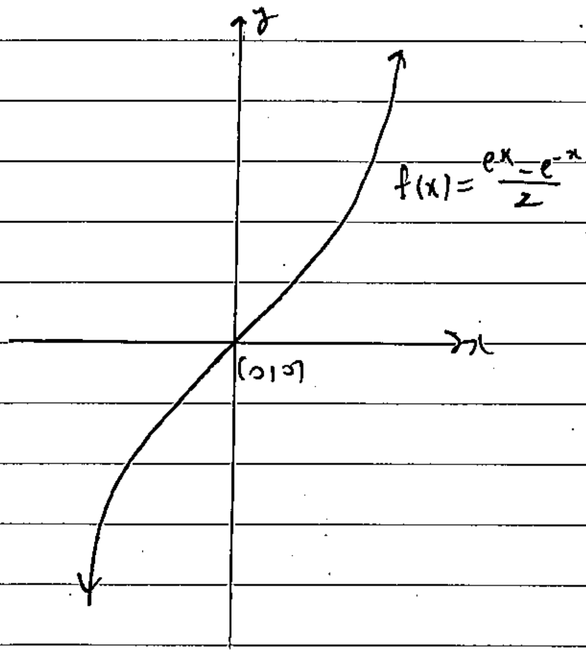
(C) AS $x \to \infty$ $f(x) = \dfrac{e^{x} - \overset{\nearrow 0}{e^{-x}}}{2}$

$\to \dfrac{e^{x}}{2} \to \infty$

(D) AS $x \to -\infty$ $f(x) = \dfrac{\overset{\nearrow 0}{e^{x}} - e^{-x}}{2}$

$\to -\dfrac{e^{-x}}{2} \to -\infty$

(E)



(F) SWITCH $x \Leftrightarrow y$

$x = \dfrac{e^{y} - e^{-y}}{2}$

$2x = e^{y} - e^{-y}$

$\times e^{y}$) $(2x)e^{y} = e^{2y} - 1$

$e^{2y} - (2x)e^{y} - 1 = 0$

$(e^{y})^2 - 2x(e^{y}) - 1 = 0$

$e^{y} = \left(2x \pm \sqrt{4x^2 - 4(1)(-1)}\right)/2$

$= \left(2x \pm \sqrt{4x^2+4}\right)/2$

$= \dfrac{2x \pm 2\sqrt{x^2+1}}{2}$

$= \dfrac{2x + 2\sqrt{x^2+1}}{2}$ (AS $e^{y} > 0$)

$= x + \sqrt{x^2+1}$

$\therefore y = \ln(x + \sqrt{x^2+1})$

$f^{-1} : \mathbb{R} \to \mathbb{R}$ $f^{-1}(x) = \ln(x + \sqrt{x^2+1})$

Q31) SUPPOSE $a \geq b$

$f(a) - f(b)$

$= \log_c a - \log_c b$

$= \log_c\left(\frac{a}{b}\right)$

≥ 0 AS $\frac{a}{b} \geq 1$

$\therefore f(a) \geq f(b)$