Exercises : Coordinate Geometry (Topic 1)

(1) $AB = \sqrt{(y_2 - y_1)^2 + (x_2 - x_1)^2}$

$= \sqrt{(1 - {}^-2)^2 + (6 - {}^-1)^2}$

$= \sqrt{9 + 49}$

$= \sqrt{58}$

(b) $AC = 5$

$\sqrt{(k-1)^2 + (2-6)^2} = 5$

$\sqrt{k^2 - 2k + 1 + 16} = 5$

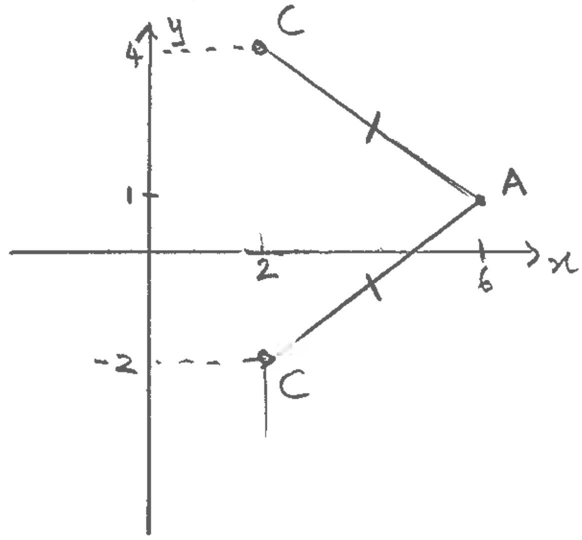
$\sqrt{k^2 - 2k + 17} = 5$

$k^2 - 2k + 17 = 25$

$k^2 - 2k - 8 = 0$

$(k-4)(k+2) = 0$

$k = 4, \quad k = -2$



(2) (a) $AB = \sqrt{(3 - {}^-2)^2 + (-2 - 1)^2}$

$= \sqrt{25 + 9}$

$= \sqrt{34}$

(b) $AC = 4$

$\sqrt{(-2-1)^2 + (1-k)^2} = 4$

$\sqrt{9 + 1 - 2k + k^2} = 4$

$\sqrt{k^2 - 2k + 10} = 4$

$k^2 - 2k + 10 = 16$

$k^2 - 2k - 6 = 0$

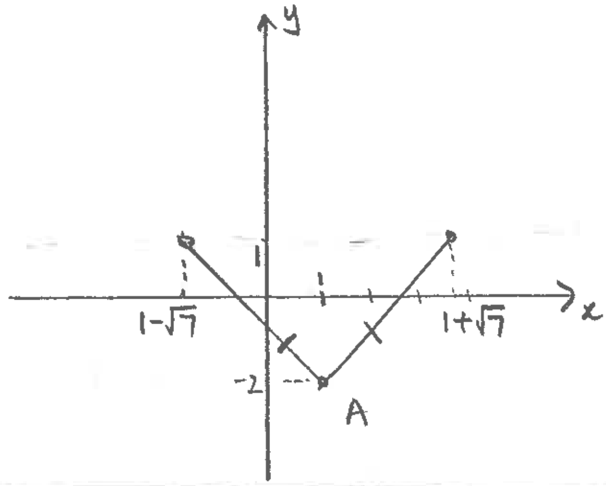
$(k-1)^2 - 1 - 6 = 0$

$(k-1)^2 - 7 = 0$

$(k-1)^2 = 7$

$k - 1 = \pm\sqrt{7}$

$k = 1 \pm \sqrt{7}$



(3) $x_M = \frac{x_1 + x_2}{2} = \frac{2 + {}^-3}{2} = -\frac{1}{2}$

$y_M = \frac{y_1 + y_2}{2} = \frac{5+1}{2} = 3$

$M = \left(-\frac{1}{2}, 3\right)$

(4) $x_M = \frac{x_1 + x_2}{2}$

$\frac{b+2}{2} = 3$

$\therefore b = 2 \times 3 - 2 = 4$

$y_M = \frac{y_1 + y_2}{2}$

$\frac{a+7}{2} = 4$

$a = 2 \times 4 - 7 = 1$

(b) Let $N = (x, y)$ $A(2,1)$ $B(4,7)$

$1:3 \leftarrow 4$ parts

$x = 2 + \frac{1}{4} \times 2 = 2\frac{1}{2} = \frac{5}{2}$

$y = 1 + \frac{3}{4} \times 6 = 1 + \frac{9}{2} = \frac{11}{2}$

$N = \left(\frac{5}{2}, \frac{11}{2}\right)$

5 (a) $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{6-3}{3-2} = 3$

(b) $m = \frac{-1-3}{4-^-2} = \frac{-4}{6} = -\frac{2}{3}$

(c) $m = \frac{a-b}{b-a} = -\frac{(b-a)}{b-a} = -1$

(d) $m = \frac{b^2-a^2}{b-a} = \frac{(b-a)(b+a)}{b-a} = b+a$

(7) $m_{AB} = \frac{1-^-1}{0-^-1} = \frac{2}{1} = 2$

$m_{BC} = \frac{5-1}{2-0} = \frac{4}{2} = 2$

* Note Solution for Q6 is on page 9 after Q23

$\therefore$ A, B and C are collinear

(8) $m_{AB} = \frac{4-1}{-2-2} = \frac{3}{-4} = -\frac{3}{4}$

$m_{BC} = \frac{3a-4}{2a+2}$

$m_{AB} = m_{BC}$ since A, B, C collinear

$\frac{3a-4}{2a+2} = -\frac{3}{4}$

$4(3a-4) = -3(2a+2)$

$12a - 16 = -6a - 6$

$18a = 10$

$a = \frac{10}{18}$

$a = \frac{5}{9}$

(9) (a) $m_{AB} = \frac{-3-3}{3-0} = -2$

$y = mx + c$

$y = -2x + 3$

(b) $m_{AB} = \frac{4-1}{-2-2} = -\frac{3}{4}$

$y - y_1 = m(x - x_1)$

$y - 1 = -\frac{3}{4}(x-2)$

$4(y-1) = -3(x-2)$

$4y - 4 = -3x + 6$

$3x + 4y = 10$

(10) $m_{AB} = \frac{-3-2}{2-^-1} = \frac{-5}{3}$

$y - y_1 = m(x - x_1)$

$y - 2 = -\frac{5}{3}(x+1)$

$3(y-2) = -5(x+1)$

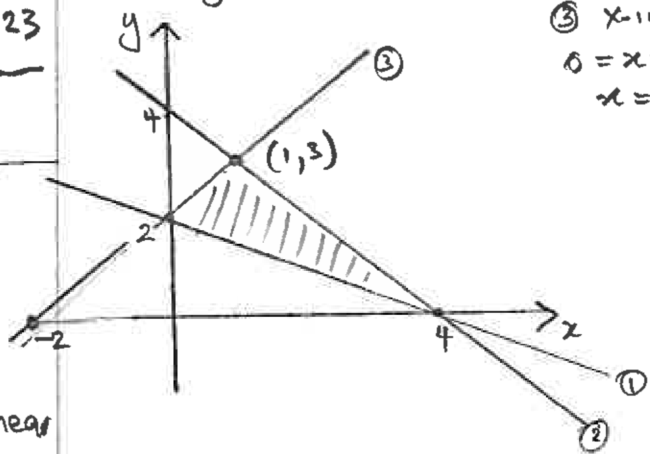
$3y - 6 = -5x - 5$

$5x + 3y = 1$

11) $x + 2y = 4$ —— ①

$x + y = 4$ —— ②

$y = x + 2$ —— ③



③ x-int

$0 = x + 2$

$x = -2$

Find pt of intersection b/w ② + ③

Sub ③ into ②

$x + x + 2 = 4$

$2x + 2 = 4$

$2x = 2$

$x = 1$

$x = 1, y = 3$ (1,3)

Area = A (Large Δ) − A (small)

$= \frac{1}{2} \times 6 \times 3 - \frac{1}{2} \times 6 \times 2$

$= 9 - 6$

$= 3$ units²

(12) $x + 2y = 5$ —— ①

$2x + y = 7$ —— ②

$y = x + 1$ —— ③

Intersection of ① + ③

$x + 2(x+1) = 5$

$x + 2x + 2 = 5$

$3x = 3$

$x = 1$

$y = 1 + 1 = 2$ (1,2)

Intersection ② + ③

$2x + x + 1 = 7$

$3x = 6$

$x = 2$

$\therefore y = 3$

(2,3)

Intersection ① + ②

① × 2

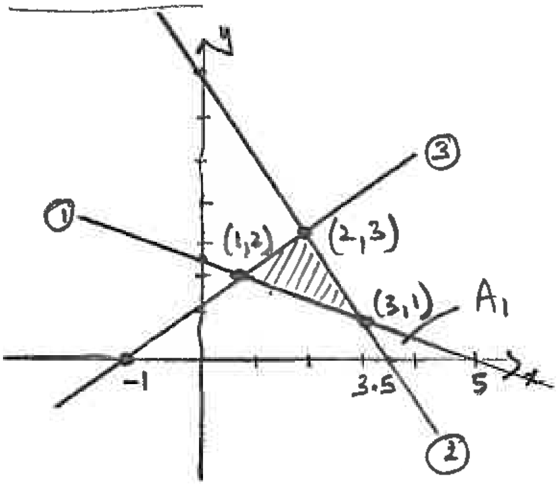
$2x + 4y = 10$ —— ④

$2x + y = 7$ —— ③

4−3 $3y = 3$

$y = 1 \therefore x = 3$ ②

12 continued



X int of ③ : $0 = x + 1$

$\therefore x = -1$

Xint of ② $2x + 0 = 7$

$x = 3.5$

$$\text{Area} = A(\text{Large}\,\Delta) - A(\text{small}\,\Delta) + A_1$$

$$= \tfrac{1}{2} \times 4.5 \times 3 - \tfrac{1}{2} \times 6 \times 2 + \tfrac{1}{2} \times 1.5 \times 1$$

$$= 6.75 - 6 + 0.75$$

$$= 1.5 \text{ units}^2$$

13 (a) $m = 3$ $A(1, -2)$

$y - y_1 = m(x - x_1)$

$y + 2 = 3(x - 1)$

$y = 3x - 3 - 2$

$y = 3x - 5$

(b) $2x - 3y = 5$

$-3y = 5 - 2x$

$y = -\frac{5}{3} + \frac{2}{3}x$

$m = \frac{2}{3}$ $B(-3, 1)$

$y - 1 = \frac{2}{3}(x + 3)$

$3y - 3 = 2x + 6$

$-2x + 3y = 9$

14 (a) $m = -\frac{1}{3}$, $A(-3, 1)$

$y - 1 = -\frac{1}{3}(x + 3)$

$y - 1 = -\frac{1}{3}x - 1$

$y = -\frac{1}{3}x$

(b) $2x + 3y = 5$

$3y = 5 - 2x$

$y = \frac{5}{3} - \frac{2}{3}x$

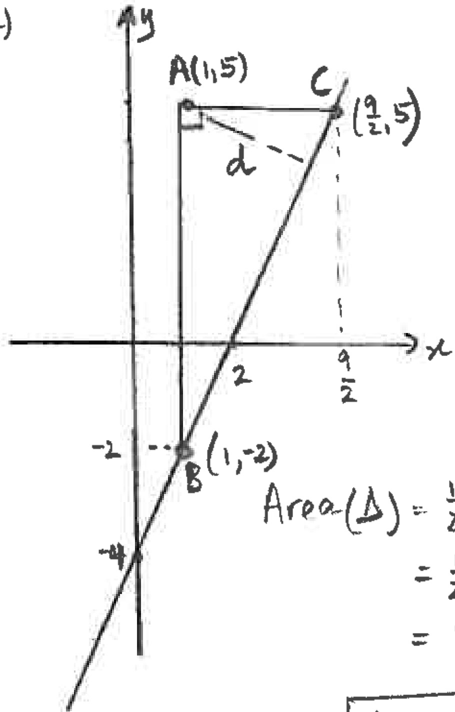
$m_\perp = \frac{3}{2}$ $B(-2, \sqrt{2})$

$y - \sqrt{2} = \frac{3}{2}(x + 2)$

$y - \sqrt{2} = \frac{3}{2}x + 3$

$y = \frac{3}{2}x + 3 + \sqrt{2}$

15 (a)



$y = 2x - 4$

Let $y = 5$

$2x - 4 = 5$

$x = \frac{9}{2}$

Let $x = 1$

$y = 2 - 4$

$= -2$

$\text{Area}(\Delta) = \frac{1}{2} \times b \times h$

$= \frac{1}{2} \times \frac{7}{2} \times 7$

$= \frac{49}{4}$

$BC = \sqrt{(5+2)^2 + \left(\frac{7}{2}\right)^2}$

$= \sqrt{49 + \frac{49}{2}}$

$= \sqrt{\frac{3 \times 49}{2}}$

$= 7\sqrt{\frac{3}{2}}$

$\text{Area} = \frac{1}{2} \times 7\sqrt{\frac{3}{2}} \times d$

$= \frac{7\sqrt{3}\,d}{2\sqrt{2}}$

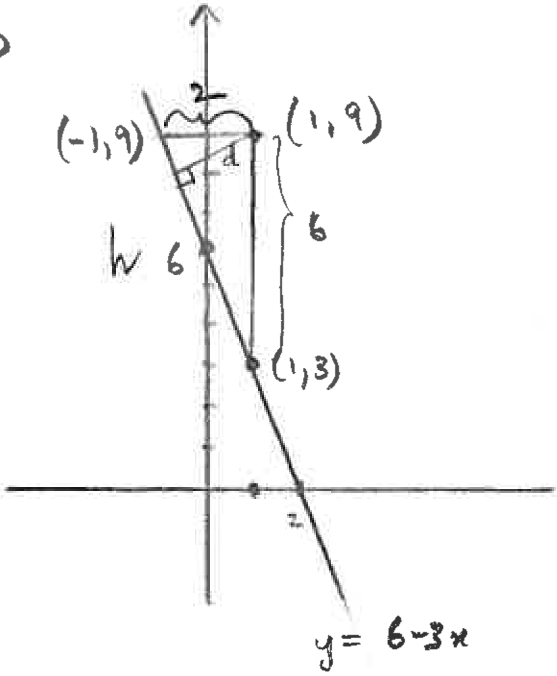
$\therefore \frac{7\sqrt{3}\,d}{2\sqrt{2}} = \frac{49}{4}$

$\frac{\sqrt{3}\,d}{\sqrt{2}} = \frac{7}{2}$

$d = \frac{7\sqrt{2}}{2\sqrt{3}}$

$= \frac{7\sqrt{6}}{6}$

15(b)



$y=9,\quad 6-3x=9$
$-3x=3$
$x=-1\quad(-1,9)$

$x=1,\quad y=6-3=3\quad(1,3)$

$h^2=2^2+6^2$

$h^2=40$

$h=\sqrt{40}$

$\text{Area}(\Delta)=\frac{1}{2}\times2\times6=6$

Also $\text{Area}=\frac{1}{2}\times d\times\sqrt{40}$

$\therefore \frac{1}{2}d\sqrt{40}=6$

$d\sqrt{40}=12$

$d=\frac{12}{\sqrt{40}}=\frac{12}{2\sqrt{10}}=\frac{6}{\sqrt{10}}=\frac{6\sqrt{10}}{10}$

$d=\frac{3\sqrt{10}}{5}$

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16 (a) $m_{AB}=\frac{-4-{}^-2}{5-{}^-1}=\frac{-2}{6}=-\frac{1}{3}$

$y-y_1=m(x-x_1)$

$y+2=-\frac{1}{3}(x+1)$

$y=-\frac{1}{3}x-\frac{1}{3}-2$

$y=-\frac{1}{3}x-\frac{7}{3}$

(b) $M=\left(\frac{-1+5}{2},\frac{-2+-4}{2}\right)=(2,-3)$

$m_1=-\frac{1}{3}$

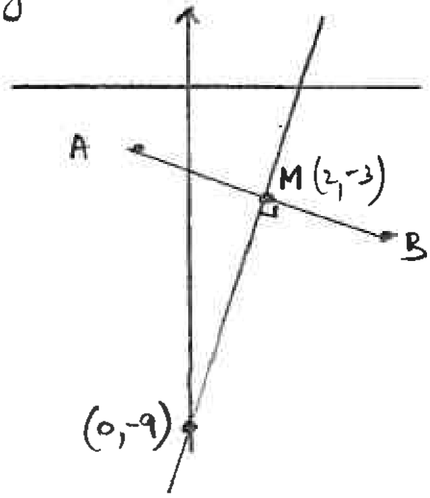
$\therefore m_2=3$

$y-{}^-3=3(x-2)$

$y+3=3x-6$

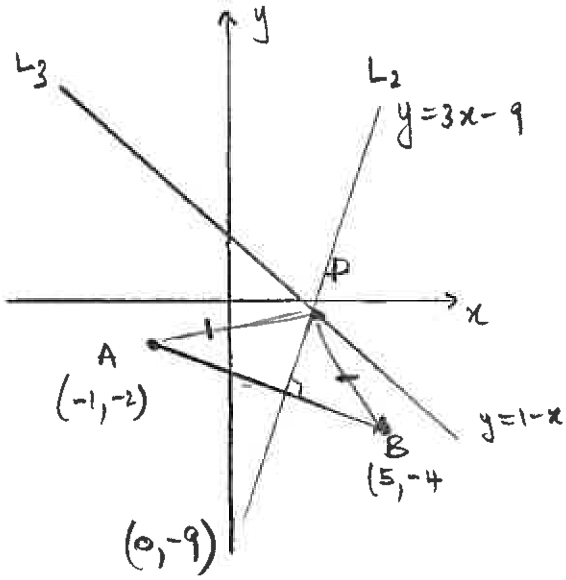
$y=3x-9$

(c)



Answer : $(2,-3)$

(d)



Point of intersection of L_3 and L_2

$y=3x-9$

$y=1-x$

$\therefore 1-x=3x-9$

$4x=10$

$x=\frac{5}{2}$

$x=\frac{5}{2},\quad y=1-\frac{5}{2}=-\frac{3}{2}$

$P\left(\frac{5}{2},-\frac{3}{2}\right)$

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17 $A(1,3)\quad B(-1,-2)$

(a) $m_{AB}=\frac{-2-3}{-1-1}=\frac{-5}{-2}=\frac{5}{2}$

$y-3=\frac{5}{2}(x-1)$

$y=\frac{5}{2}x-\frac{5}{2}+3$

$y=\frac{5}{2}x+\frac{1}{2}$

(b) $m_1 = \frac{5}{2}$

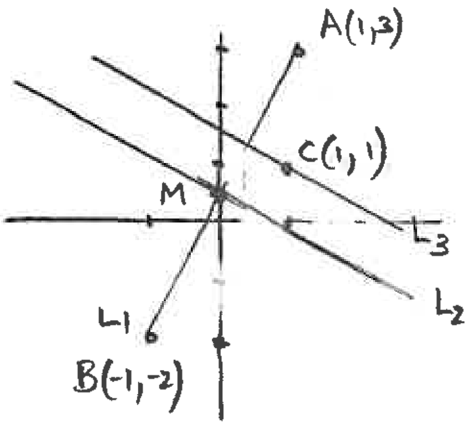
$\therefore m_2 = -\frac{2}{5}$

$M = \left(\frac{1+-1}{2}, \frac{3+-2}{2}\right) = \left(0, \frac{1}{2}\right)$

$y - \frac{1}{2} = -\frac{2}{5}(x - 0)$

$y = -\frac{2}{5}x + \frac{1}{2}$

(c)



L_3: $m = -\frac{2}{5}$ $\quad C(1,1)$

$y - 1 = -\frac{2}{5}(x-1)$

$y = -\frac{2}{5}x + \frac{2}{5} + 1$

$y = -\frac{2}{5}x + \frac{7}{5}$

Pt of INT $L_1 + L_3$

$-\frac{2}{5}x + \frac{7}{5} = \frac{5}{2}x + \frac{1}{2}$

$-4x + 14 = 25x + 5$

$29x = 9$

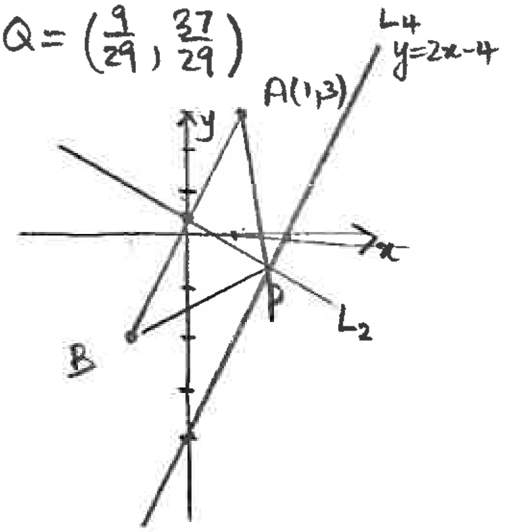
$x = \frac{9}{29}$

$y = -\frac{2}{5}\left(\frac{9}{29}\right) + \frac{7}{5}$

$= \frac{37}{29}$

$Q = \left(\frac{9}{29}, \frac{37}{29}\right)$

(d)



Pt of INT $L_2 + L_4$

$y = 2x - 4$

$y = -\frac{2}{5}x + \frac{1}{2}$

$2x - 4 = -\frac{2}{5}x + \frac{1}{2}$

$20x - 40 = -4x + 5$

$24x = 45$

$x = \frac{45}{24} = \frac{15}{8}$

$y = 2 \times \frac{45}{24} - 4$

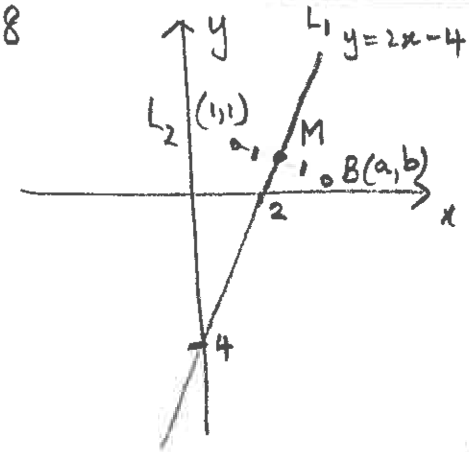
$= \frac{45}{12} - 4$

$= \frac{15}{4} - \frac{16}{4}$

$= -\frac{1}{4}$

$P = \left(\frac{15}{8}, -\frac{1}{4}\right)$

18



$m_1 = 2 \quad \therefore m_2 = -\frac{1}{2}$

L_2: $y - y_1 = m(x - x_1)$

$y - 1 = -\frac{1}{2}(x-1)$

$y = -\frac{1}{2}x + \frac{1}{2} + 1$

$y = -\frac{1}{2}x + \frac{3}{2}$

Point of INT $L_1 + L_2$

$-\frac{1}{2}x + \frac{3}{2} = 2x - 4$

$\frac{5x}{2} = \frac{11}{2}$

$x = \frac{11}{5}$

$y = 2 \times \frac{11}{5} - 4 = \frac{2}{5}$

$M = \left(\frac{11}{5}, \frac{2}{5}\right)$

$\frac{a+1}{2} = \frac{11}{5}$

$5a + 5 = 22$

$5a = 17$

$a = \frac{17}{5}$

$\frac{b+1}{2} = \frac{2}{5}$

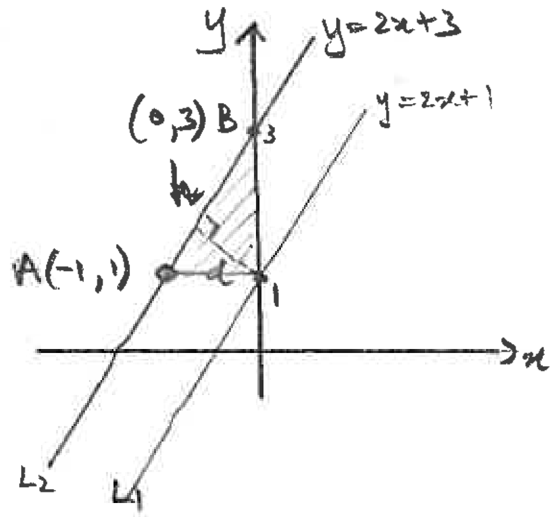
$5b + 5 = 4$

$5b = -1$

$b = -\frac{1}{5}$

$B = \left(\frac{17}{5}, -\frac{1}{5}\right)$

19



L_2 : Let $y=1$ $\therefore$ $1 = 2x+3$

$2x = -2$

$x = -1$

$$A(\Delta) = \frac{1}{2} \times b \times h$$

$$= \frac{1}{2} \times 1 \times 2$$

$$= 1$$

$$h^2 = 1^2 + 2^2$$

$$h^2 = 5$$

$$h = \sqrt{5}$$

$$A(\Delta) = \frac{1}{2} \times h \times d$$

$$= \frac{1}{2} \sqrt{5} \times d$$

$$\therefore \frac{d\sqrt{5}}{2} = 1$$

$$d = \frac{2}{\sqrt{5}}$$

$$d = \frac{2\sqrt{5}}{5}$$

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20 (a) $3x + 2y = 7$ ①

$2x - 3y = -4$ ②

① $2y = -3x + 7$

$y = -\frac{3}{2}x + \frac{7}{2}$

$m_1 = -\frac{3}{2}$, $c_1 = \frac{7}{2}$

② $-3y = -2x - 4$

$y = \frac{2}{3}x + \frac{4}{3}$

$m_2 = \frac{2}{3}$, $c_2 = \frac{4}{3}$

$m_1 \neq m_2 \Rightarrow$ one solution

Point of int : $-\frac{3}{2}x + \frac{7}{2} = \frac{2}{3}x + \frac{4}{3}$

$-9x + 21 = 4x + 8$

$13x = 13$

$x = 1$

$y = -\frac{3}{2} + \frac{7}{2} = \frac{4}{2} = 2$

Solution : $x = 1$, $y = 2$

(b) $3x - 2y = -1$ ①

$-6x + 4y = 2$ ②

① $-2y = -3x - 1$

$y = \frac{3}{2}x + \frac{1}{2}$

$m_1 = \frac{3}{2}$, $c_1 = \frac{1}{2}$

② $4y = 6x + 2$

$2y = 3x + 1$

$y = \frac{3}{2}x + \frac{1}{2}$

$m_2 = \frac{3}{2}$, $c_2 = \frac{1}{2}$

$m_1 = m_2$ and $c_1 = c_2$

$\therefore$ Infinitely many solutions

Let $x = t$, $t \in R$

$\therefore y = \frac{3}{2}t + \frac{1}{2}$

Solution ; $x = t$, $y = \frac{3}{2}t + \frac{1}{2}$

(c) $y = 2x + 3$ ①

$2y - 4x = 1$ ②

① $m_1 = 2$, $c_1 = 3$

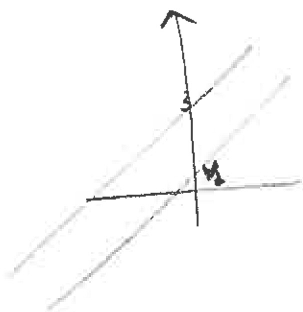
② $2y = 4x + 1$

$y = 2x + \frac{1}{2}$

$m_2 = 2$, $c_2 = \frac{1}{2}$

$m_1 = m_2$ and $c_1 \neq c_2$

$\therefore$ No solution



21 $mx + 3y = m$

$3y = -mx + m$

$y = -\frac{m}{3}x + \frac{m}{3}$

$\therefore m_1 = -\frac{m}{3}, \; c_1 = \frac{m}{3}$

$12x + my = 12$

$my = -12x + 12$

$y = -\frac{12}{m}x + \frac{12}{m}$

$m_2 = \frac{-12}{m}, \; c_2 = \frac{12}{m}$

(i) one solution

$m_1 \neq m_2$

$-\frac{m}{3} \neq -\frac{12}{m}$

$m^2 \neq 36$

$m \neq \pm 6$

$m \in R \setminus \{\pm 6\}$

(ii) No solution

$m_1 = m_2$ and $c_1 \neq c_2$

$m = \pm 6$ and $\frac{m}{3} \neq \frac{12}{m}$

$m^2 \neq 36$

Hence this is impossible
So for any value of m
there will always be
atleast one solution

(iii) infinitely many solutions

$m_1 = m_2$ and $c_1 = c_2$

$m = \pm 6$

(b) One solution.
Assume $m \in R \setminus \{\pm 6\}$
Solve with calculator
$x = 1, \; y = 0$

infinitely many solutions
Let $m = 6$

$6x + 3y = 6$ ①

$12x + 6y = 12$ ②

Note ① + ② are same equation

① ÷ 3 $\quad 2x + y = 2$

Let $x = t \in R$

$2t + y = 2$

$\therefore y = -2t + 2$

$\boxed{x = t, \; y = -2t + 2}$ $t \in R$.

Let $m = -6$

$-6x + 3y = -6$ ①

$12x - 6y = 12$ ②

① + ② same equation

① ÷ 3 $\quad -2x + y = -2$

Let $x = t \in R$

$\therefore -2t + y = -2$

$y = 2t - 2$

$\boxed{x = t, \; y = 2t - 2, \; t \in R}$

(22) $(m+1)x + 3y = 3$ ①

$3y = -(m+1)x + 3$

$y = -\left(\frac{m+1}{3}\right)x + 1$

$m_1 = -\frac{m+1}{3}, \; c_1 = 1$

$2x + my = a$ ②

$my = -2x + a$

$y = \frac{-2}{m}x + \frac{a}{m}$

$m_2 = -\frac{2}{m}, \; c_2 = \frac{a}{m}$

(a) (i) one solution

$m_1 \neq m_2$

$-\dfrac{m+1}{3} \neq -\dfrac{2}{m}$

$\dfrac{m+1}{3} \neq \dfrac{2}{m}$

$m^2 + m \neq 6$

$m^2 + m - 6 \neq 0$

$(m+3)(m-2) \neq 0$

$\therefore m \neq -3, 2$

$m \in R \setminus \{-3, 2\}$

(ii) no solution

$m_1 = m_2$ and $c_1 \neq c_2$

$m = -3, 2$ and $1 \neq \dfrac{a}{m}$

$m \neq a$

$m = -3, 2$ and $m \neq a$

(iii) infinitely many solutions

$m_1 = m_2$ and $c_1 = c_2$

$\therefore$ $m = -3, 2$ and $a = m$

(b) (i) one solution

From calculator

$x = \dfrac{-3(a-m)}{m^2+m-6}$, $y = \dfrac{am+a+6}{m^2+m-6}$

(iii) infinitely many

Let $m = a = -3$

$-2x + 3y = 3$ ①

$2x - 3y = -3$ ②

① + ② identical equations

① $3y = 2x + 3$

Let $x = t \in R$, $y = \dfrac{2t+3}{3}$

Let $m = a = 2$

$\left.\begin{array}{ll} 3x + 3y = 3 & \text{— ①} \\ 2x + 2y = 2 & \text{②} \end{array}\right\}$ identical

$\therefore x + y = 1$

Let $x = t \in R$, $y = 1 - t$

$x = t$, $y = 1 - t$, $t \in R$

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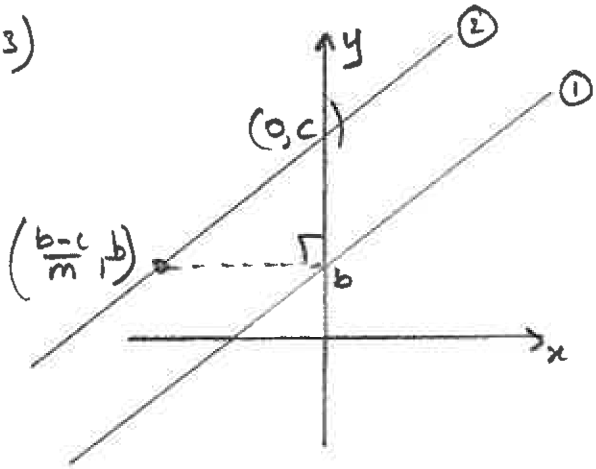
(23)

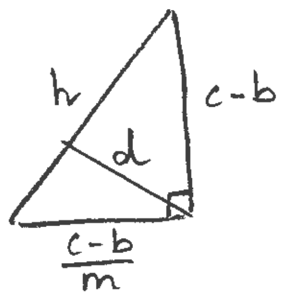


$y = mx + b$ ①

$y = mx + c$ ②

② Let $y = b$, $b = mx + c$

$\therefore x = \dfrac{b-c}{m}$



$h^2 = \left(\dfrac{c-b}{m}\right)^2 + (c-b)^2$

$h^2 = \dfrac{(c-b)^2}{m^2} + (c-b)^2$

$= \dfrac{(c-b)^2 + m^2(c-b)^2}{m^2}$

$= \dfrac{(c-b)^2(1+m^2)}{m^2}$

$\therefore h = \dfrac{(c-b)\sqrt{1+m^2}}{m}$

$\text{Area}(\Delta) = \frac{1}{2} \times \left(\dfrac{c-b}{m}\right) \times (c-b)$

$= \dfrac{(c-b)^2}{2m}$

OR

$$\text{Area} = \frac{1}{2} h d$$

$$= \frac{1}{2}\,\frac{(c-b)\sqrt{1+m^2}}{m} \times d$$

$$\therefore \frac{(c-b)^2}{2m} = \frac{d(c-b)\sqrt{1+m^2}}{2m}$$

$$\therefore d = \frac{(c-b)^2}{2m} \times \frac{2m}{(c-b)\sqrt{1+m^2}}$$

$$= \frac{(c-b)}{\sqrt{1+m^2}}$$

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(6) This question is out of order because it was accidentally not included between (5) and (7)

$A(-1,2)$, $B(a,b)$, $C(2,-1)$

$m_{AB} = 4$

$$\frac{b-2}{a+1} = 4$$

$b-2 = 4a+4$

$4a - b = -6$ ①

$m_{BC} = -2$

$$\frac{b+1}{a-2} = -2$$

$b+1 = -2a+4$

$2a+b = 3$ ②

① + ② $6a = -3$

$a = -\frac{1}{2}$

sub $a = -\frac{1}{2}$ into ②

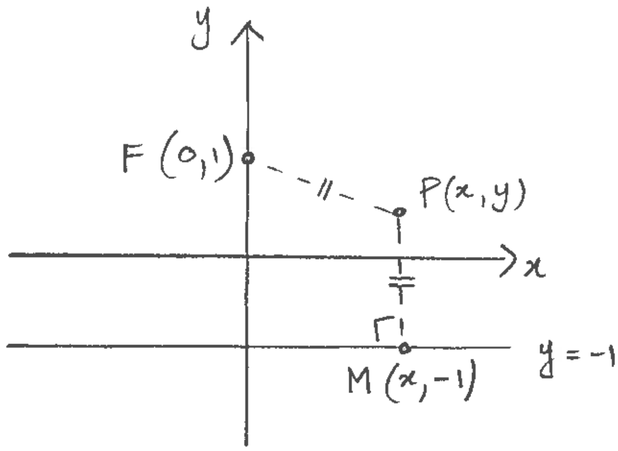
$2 \times -\frac{1}{2} + b = 3$

$-1 + b = 3$

$b = 4$

So $a = -\frac{1}{2}$, $b = 4$

(24)



$$PF = \sqrt{(y-1)^2 + (x-0)^2}$$

$$= \sqrt{(y-1)^2 + x^2}$$

$PM = y+1$

$PF = PM$

$$\therefore \sqrt{(y-1)^2 + x^2} = y+1$$

$$\therefore (y-1)^2 + x^2 = (y+1)^2$$

$$\cancel{y^2} - 2y + 1 + x^2 = \cancel{y^2} + 2y + 1$$

$4y = x^2$

$$y = \frac{x^2}{4}$$

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(25) $OA^2 = 1^2 + m^2$

(a) $OB^2 = 1^2 + n^2$

$\therefore OA^2 + OB^2 = 2 + m^2 + n^2$

Now $AB = \sqrt{(m-n)^2 + (1-1)^2}$

$= \sqrt{(m-n)^2}$

$\therefore AB^2 = (m-n)^2$

Since $\angle AOB = 90^\circ$ then

$AB^2 = OA^2 + OB^2$

$(m-n)^2 = 2 + m^2 + n^2$

$m^2 - 2mn + n^2 = 2 + m^2 + n^2$

$\therefore -2mn = 2$

$mn = -1$

(b) $AB^2 = OA^2 + OB^2 - 2OA \times OB \cos\theta$

$\therefore \cos\theta = \dfrac{OA^2 + OB^2 - AB^2}{2 \times OA \times OB}$

$= \dfrac{2 + m^2 + n^2 - (m-n)^2}{2\sqrt{1+m^2}\sqrt{1+n^2}}$

$= \dfrac{2 + 2mn}{2\sqrt{1+m^2}\sqrt{1+n^2}}$

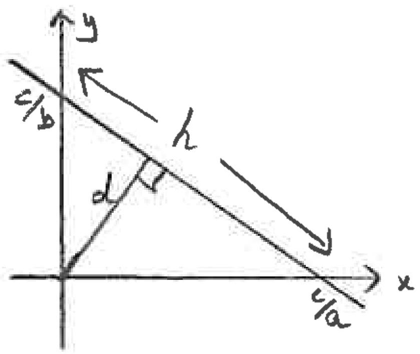
$= \dfrac{2-2}{2\sqrt{1+m^2}\sqrt{1+n^2}}$ if $mn = -1$

$= 0$

$\therefore \theta = 90^\circ$

where $\theta = \angle AOB$

(26)



$h^2 = \left(\frac{c}{b}\right)^2 + \left(\frac{c}{a}\right)^2$

$h^2 = \dfrac{c^2}{b^2} + \dfrac{c^2}{a^2}$

$= \dfrac{c^2a^2 + c^2b^2}{a^2b^2}$

$= \dfrac{c^2(a^2+b^2)}{a^2b^2}$

$\Rightarrow h = \dfrac{c\sqrt{a^2+b^2}}{ab}$

Area $(\triangle) = \frac{1}{2} \times \frac{c}{a} \times \frac{c}{b} = \dfrac{c^2}{2ab}$

Also Area$(\triangle) = \frac{1}{2}hd$

$= \dfrac{1}{2}\,\dfrac{c\sqrt{a^2+b^2}}{ab} \times d$

$\therefore \dfrac{c^2}{2ab} = \dfrac{cd\sqrt{a^2+b^2}}{2ab}$

$c = d\sqrt{a^2+b^2}$

$d = \dfrac{c}{\sqrt{a^2+b^2}}$

27 (a) $M = \left(\dfrac{x_1+x_2}{2}, \dfrac{y_1+y_2}{2}\right)$

$N = \left(\dfrac{x_1+x_3}{2}, \dfrac{y_1+y_3}{2}\right)$

(b) $m_{BC} = \dfrac{y_3 - y_2}{x_3 - x_2}$

$m_{MN} = \dfrac{\frac{y_1+y_3}{2} - \frac{y_1+y_2}{2}}{\frac{x_1+x_3}{2} - \frac{x_1+x_2}{2}}$

$= \dfrac{\frac{y_1+y_3-y_1-y_2}{2}}{\frac{x_1+x_3-x_1-x_2}{2}}$

$= \dfrac{\frac{y_3-y_2}{2}}{\frac{x_3-x_2}{2}}$

$= \dfrac{y_3-y_2}{2} \times \dfrac{2}{x_3-x_2}$

$= \dfrac{y_3-y_2}{x_3-x_2}$

$= m_{BC}$

$\therefore$ MN is parallel to BC