APPLICATIONS OF DIFFERENTIATION

Q1 A) $y = 3x^2 - 3x + 1$

$\frac{dy}{dx} = 6x - 3$

@ $x=1$ $\frac{dy}{dx} = m = 6-3 = 3$, $y = 3-3+1 = 1$

$y - y_1 = m(x - x_1)$

$y - 1 = 3(x-1)$

$y = 3x + 2$

B) $y = \sin 2x$

$\frac{dy}{dx} = 2\cos 2x$

@ $x = \pi/6$ $\frac{dy}{dx} = m = 2\cos \pi/3 = 2 \times \frac{1}{2} = 1$

$y = \sin \pi/3 = \frac{\sqrt{3}}{2}$

$y - y_1 = m(x - x_1)$

$y - \sqrt{3}/2 = 1(x - \pi/6)$

$y = x + \frac{\sqrt{3}}{2} - \frac{\pi}{6}$

C) $y = \cos 3x$

$\frac{dy}{dx} = -3\sin 3x$

@ $x = -\pi/4$ $\frac{dy}{dx} = -3\sin\left(\frac{-3\pi}{4}\right) = -\frac{3}{\sqrt{2}}$

$y = \cos\left(-\frac{3\pi}{4}\right) = -\frac{3}{\sqrt{2}}$

$y - y_1 = m(x - x_1)$

$y + \frac{3}{\sqrt{2}} = -\frac{3}{\sqrt{2}}(x + \frac{\pi}{4})$

$y = -\frac{3x}{\sqrt{2}} - \frac{3\pi}{4\sqrt{2}} - \frac{3}{\sqrt{2}}$

D) $y = e^{3x}$ $\frac{dy}{dx} = 3e^{3x}$

@ $x = \frac{1}{3}$, $y = e$ $\frac{dy}{dx} = 3e$

$y - y_1 = m(x - x_1)$

$y - e = 3e(x - \frac{1}{3})$

$y = 3ex - 2e$

E) $y = \log_e(2x-1)$

$\frac{dy}{dx} = \frac{2}{2x-1}$

@ $x = 1$ $y = \log_e(2-1) = \log_e(1) = 0$

$\frac{dy}{dx} = \frac{2}{2-1} = 2$

$y - y_1 = m(x - x_1)$

$y = 2(x-1) = 2x - 2$

F) $y = x\cos x$

$\frac{dy}{dx} = -x\sin x + \cos x$

@ $x = \pi/6$ $y = \pi/6 \cos \pi/6 = \frac{\pi}{6}\frac{\sqrt{3}}{2} = \frac{\sqrt{3}\pi}{12}$

$\frac{dy}{dx} = -\pi/6 \sin \pi/6 + \cos \pi/6$

$= -\pi/6 \cdot 1/2 + \sqrt{3}/2$

$= -\frac{\pi}{12} + \frac{\sqrt{3}}{2}$

$y - y_1 = m(x - x_1)$

$y - \frac{\sqrt{3}\pi}{12} = \left(\frac{\sqrt{3}}{2} - \frac{\pi}{12}\right)(x - \pi/6)$

$\therefore y = \left(\frac{\sqrt{3}}{2} - \frac{\pi}{12}\right)x - \frac{\pi^2}{72}$

G) $y = (\sin x)^{1/2}$

$\frac{dy}{dx} = \frac{1}{2}(\sin x)^{-1/2}\cos x$

$= \frac{\cos x}{2\sqrt{\sin x}}$

@ $x = \pi/4$ $y = \sqrt{\sin \pi/4} = \left(\frac{1}{\sqrt{2}}\right)^{1/2} = \frac{1}{\sqrt[4]{2}}$

$\frac{dy}{dx} = \frac{1/\sqrt{2}}{2 \cdot \frac{1}{\sqrt[4]{2}}}$

$= \frac{1/2^{1/2}}{2^{3/4}}$

$= \frac{1}{2^{5/4}}$

$y - y_1 = m(x - x_1)$

$y - \frac{1}{\sqrt[4]{2}} = \frac{1}{2^{5/4}}(x - \pi/4)$

$y = \frac{1}{2^{5/4}}(x - \pi/4) + \frac{1}{\sqrt[4]{2}}$

H) $y = \frac{x+1}{x^2+1}$

$\frac{dy}{dx} = \frac{(x^2+1)\cdot 1 - (x+1)2x}{(x^2+1)^2}$

$= \frac{x^2+1-2x^2-2x}{(x^2+1)^2}$

$= \frac{1-x^2-2x}{(x^2+1)^2}$

@ $x = 1$ $y = \frac{1+1}{1+1} = 1$, $\frac{dy}{dx} = \frac{1-1-2}{(1+1)^2} = -\frac{1}{2}$

$y - y_1 = m(x - x_1)$

$y - 1 = -\frac{1}{2}(x-1)$

$y = -\frac{1}{2}x + \frac{3}{2}$

Q2) $y = a\sqrt{x} + \dfrac{b}{\sqrt{x}}$

$$\frac{dy}{dx} = \frac{a}{2\sqrt{x}} + \frac{b}{2x^{3/2}}$$

TANGENT : $x + y = 1$

$\Rightarrow y = -x + 1$

$\Rightarrow m = -1$ WHEN $x = 4$

AND $y = -4 + 1 = -3$ WHEN $x = 4$

$y = -3$, $x = 4$:

$-3 = a\sqrt{4} + \dfrac{b}{\sqrt{4}}$

$-3 = 2a + \dfrac{b}{2}$

$-6 = 4a + b$

$b = -6 - 4a$ ①

$m = -1$, $x = 4$:

$-1 = \dfrac{a}{2\sqrt{4}} - \dfrac{b}{2 \cdot 4^{3/2}}$

$-1 = \dfrac{a}{4} - \dfrac{b}{2 \cdot 8}$

$-1 = \dfrac{a}{4} - \dfrac{b}{16}$

$\times 16)$ $-16 = 4a - b$

$b = 4a + 16$ ②

① = ② $-6 - 4a = 4a + 16$

$8a = -22$

$a = -\dfrac{22}{8} = -\dfrac{11}{4}$

$b = 4(-\frac{11}{4}) + 16 = 5$

$$\begin{cases} a = \dfrac{-11}{4} \\ b = 5 \end{cases}$$

Q3) $y = e^{3x}$

$\dfrac{dy}{dx} = 3e^{3x}$

@ $x = a$, $y = e^{3a}$ AND $\dfrac{dy}{dx} = 3e^{3a}$

$y - y_1 = m(x - x_1)$

$y - e^{3a} = 3e^{3a}(x - a)$

WHEN $x = 0$, $y = 0$

$-\cancel{e^{3a}} = 3\cancel{e^{3a}}(-a)$

$-1 = -3a$

$a = \frac{1}{3}$.

$\therefore$ POINT $= (\frac{1}{3}, e)$

Q4) $y = \log_e 2x$

$\dfrac{dy}{dx} = \dfrac{1}{x}$

@ $x = a$ $y = \log_e 2a$ AND $\dfrac{dy}{dx} = \dfrac{1}{a}$

$y - y_1 = m(x - x_1)$

$y - \log_e(2a) = \frac{1}{a}(x - a)$

WHEN $x = 0$, $y = 0$

$-\log_e(2a) = -1$

$\log_e(2a) = 1$

$2a = e$

$a = e/2$

$\therefore y = \log_e(e) = 1$

$\therefore$ POINT $= (e/2, 1)$

Q5) (A) $y = 3x^2 - 3x + 1$

$\frac{dy}{dx} = 6x - 3$

WHEN $x = 1$, $y = 3 - 3 + 1 = 1$

$x = 1$, $\frac{dy}{dx} = 6 - 3 = 3$ $\quad m_N = -\frac{1}{3}$

$y - y_1 = m(x - x_1)$

$y - 1 = -\frac{1}{3}(x - 1)$

$y = -\frac{1}{3}x + 4/3$

(B) $y = \sin 2x + e^{2x}$

$\frac{dy}{dx} = 2\cos 2x + 2e^{2x}$

WHEN $x = 0$, $y = \sin 0 + e^0 = 1$

$x = 0$, $\frac{dy}{dx} = 2\cos 0 + 2e^0 = 2 + 2 = 4$ $\quad m_N = -\frac{1}{4}$

$y - y_1 = m(x - x_1)$

$y - 1 = -\frac{1}{4}(x - 0)$

$y = -\frac{1}{4}x + 1$

(C) $y = \log_e x$

$\frac{dy}{dx} = \frac{1}{x}$

WHEN $x = e^2$, $y = \log_e e^2 = 2$

$x = e^2$, $\frac{dy}{dx} = \frac{1}{e^2}$ $\quad m_N = -e^2$

$y - 2 = -e^2(x - e^2)$

$y - 2 = -e^2x + e^4$

$y = -e^2x + (e^4 + 2)$

(D) $y = x\sin x$

$\frac{dy}{dx} = x\cos x + \sin x$

WHEN $x = \pi/2$ $\quad y = \frac{\pi}{2}\sin\frac{\pi}{2} = \frac{\pi}{2}$

$x = \pi/2$, $\frac{dy}{dx} = \frac{\pi}{2}\cos\frac{\pi}{2} + \sin\frac{\pi}{2} = 1$ $\quad m_N = -1$

$y - y_1 = m(x - x_1)$

$y - \frac{\pi}{2} = -1(x - \frac{\pi}{2})$

$y = -x + \pi$.

(E) $y = \frac{1}{\sin x} = (\sin x)^{-1}$

$\frac{dy}{dx} = -(\sin x)^{-2}\cos x$

$= -\frac{\cos x}{\sin^2 x}$

WHEN $x = \frac{\pi}{4}$, $y = \frac{1}{\sin \pi/4} = \frac{1}{1/\sqrt{2}} = \sqrt{2}$

$x = \pi/4$, $\frac{dy}{dx} = -\frac{\cos \pi/4}{\sin^2 \pi/4} = -\frac{1/\sqrt{2}}{1/2} = -\frac{2}{\sqrt{2}}$

$\therefore m_N = +\frac{\sqrt{2}}{2}$

$y - y_1 = m(x - x_1)$

$y - \sqrt{2} = \frac{\sqrt{2}}{2}(x - \frac{\pi}{4})$

$y = \frac{\sqrt{2}}{2}x - \frac{\pi\sqrt{2}}{8} + \sqrt{2}$

(F) $y = \frac{1-x}{1+x}$

$\frac{dy}{dx} = \frac{(1+x)(-1) - (1-x)(1)}{(1+x)^2} = \frac{-2}{(1+x)^2}$

$x = 1$, $y = \frac{1-1}{1+1} = 0$

$x = 1$, $\frac{dy}{dx} = -\frac{2}{(1+1)^2} = -\frac{1}{2}$ $\therefore m_N = 2$

$y - y_1 = m(x - x_1)$

$y - 0 = 2(x - 1)$

$y = 2x - 2$.

(Q6) $y = x^2 + 2$

$\frac{dy}{dx} = 2x$

WHEN $x = a$, $y = a^2 + 2$

$\frac{dy}{dx} = 2a$ $\therefore m_N = -\frac{1}{2a}$

NORMAL : $y - y_1 = m(x - x_1)$

$y - a^2 - 2 = -\frac{1}{2a}(x - a)$

WHEN $x = 1$, $y = 0$

$-a^2 - 2 = -\frac{1}{2a}(1 - a)$

$a^2 + 2 = \frac{1}{2a}(1 - a)$

$2a(a^2 + 2) = 1 - a$

$2a^3 + 4a = 1 - a$

$2a^3 + 5a - 1 = 0$

USING CALCULATOR, $a \approx 0.1969$

(Q7) $f(x) = \sin^2 x + \sin x = 0$

$\sin x(\sin x + 1) = 0$

$\Rightarrow \sin x = 0$ or $\sin x = -1$

$\Rightarrow x = 0, \pi, 2\pi \qquad x = \frac{3\pi}{2}$

$f'(x) = 2\sin x\cos x + \cos x = 0$

$\cos x(2\sin x + 1) = 0$

$\cos x = 0 \qquad \sin x = -\frac{1}{2}$

$x = \frac{\pi}{2} \qquad x = \frac{7\pi}{6}, \frac{11\pi}{6}$

WHEN $x = \frac{\pi}{2}$ $f(\pi/2) = \sin^2 \pi/2 + \sin \pi/2 = 2$

$x = \frac{7\pi}{6}$ $f(7\pi/6) = \sin^2 \frac{7\pi}{6} + \sin \frac{7\pi}{6}$

$= \frac{1}{4} + \frac{1}{2}$

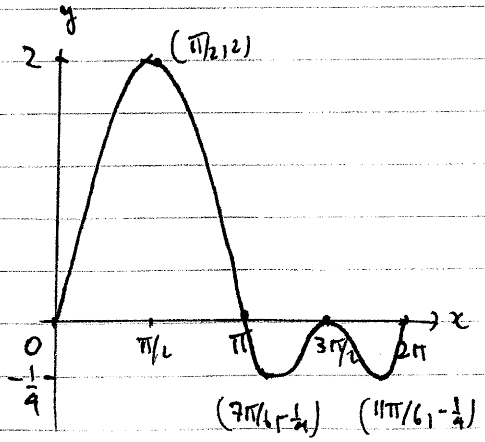
$= \cancel{=} \frac{3}{4}$

$f(\frac{11\pi}{6}) = \sin^2 \frac{11\pi}{6} + \sin \frac{11\pi}{6}$

$= \frac{1}{4} - \frac{1}{2}$

$= -\frac{1}{4}$

TP $= (\frac{7\pi}{6}, -\frac{1}{4}), (\frac{11\pi}{6}, -\frac{1}{4}), (\frac{\pi}{2}, 2)$



(Q8) $f: (0, \infty) \to \mathbb{R}$ $f(x) = 2x\log_e x$

$f(x) = 2x\log_e x = 0$

$\Rightarrow x = 0$ or $\log_e x = 0$

$\Rightarrow \cancel{x=0}$ or $x = e^0 = 1$

NOT DEFINED

$f'(x) = 2x \cdot \frac{1}{x} + \log_e x \cdot 2$

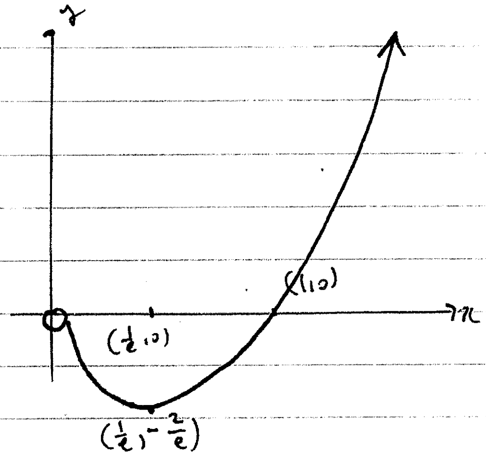
$= 2 + 2\log_e x = 0$

$\log_e x = -1$

$x = e^{-1} = \frac{1}{e}$

$f(\frac{1}{e}) = 2 \cdot \frac{1}{e}\log_e \frac{1}{e}$

$= -\frac{2}{e}$



(Q9) $f : \mathbb{R} \to \mathbb{R}$ $f(x) = x^2 e^x$

ROOTS : $f(x) = 0$

$x^2 e^x = 0$

$\Rightarrow x = 0$

STAT. PTS : $f'(x) = x^2 e^x + e^x (2x)$

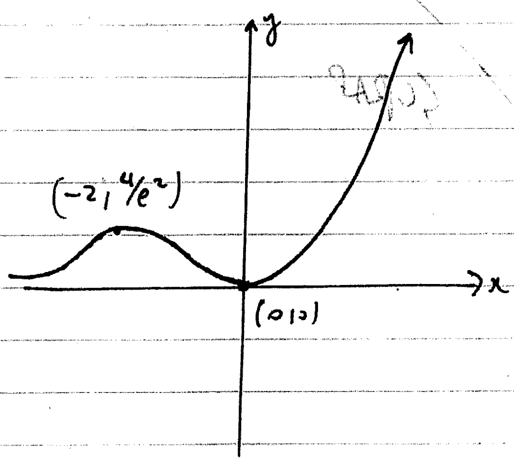
$= (x^2 + 2x) e^x = 0$

$\Rightarrow x(x+2) e^x = 0$

$\Rightarrow x = 0, -2$

$f(0) = 0$ TP$(0,0)$

$f(-2) = 4e^{-2}$ TP $(-2, 4/e^2)$



(Q10) $f(x) = (x+2)^3 (x-2)^4$

ROOTS : $(x+2)^3 (x-2)^4 = 0$

$\Rightarrow x = -2, 2.$

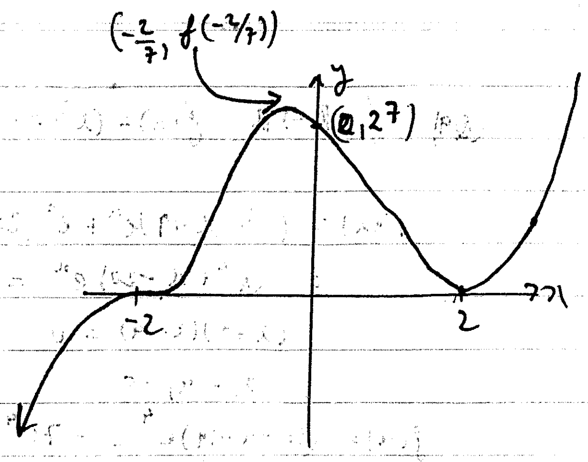
STAT PTS : $f'(x) = (x+2)^3 4(x-2)^3 + (x-2)^4 \cdot 3(x+2)^2$

$= (x+2)^2 (x-2)^3 (4(x+2) + 3(x-2))$

$= (x+2)^2 (x-2)^3 (7x+2) = 0$

$\Rightarrow x = \pm 2, -\frac{2}{7}$

NOTE $f(0) = 2^3 \cdot (-2)^4 = 2^7$



(Q11) $f(x) = xe^x$

$f'(x) = xe^x + e^x$

$= (x+1)e^x$

f INC $(x+1)e^x > 0$

$x+1 > 0$

$x > -1$

f DEC : $(x+1)e^x < 0$

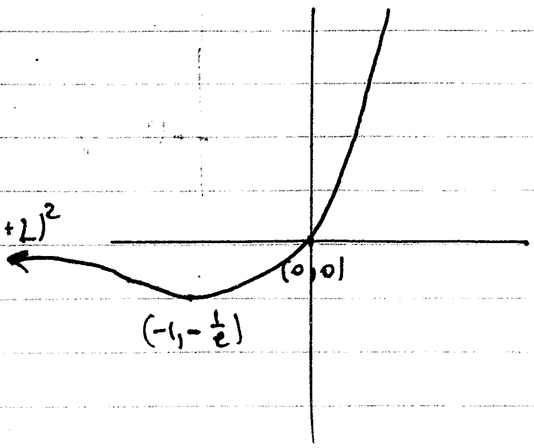
$x+1 < x$

$x < -1$

f STAT. $f'(x) = 0$

$\Rightarrow x = -1$ $f(-1) = -e^{-1} = -\frac{1}{e}$

ROOTS $f(x) = 0 \Rightarrow x = 0$



(Q12) $f(x) = x + \frac{1}{x} = x + x^{-1}$

$f'(x) = 1 - x^{-2} = 1 - \frac{1}{x^2}$

STAT. PTS $f'(x) = 0$

$1 - \frac{1}{x^2} = 0$

$x^2 = 1$

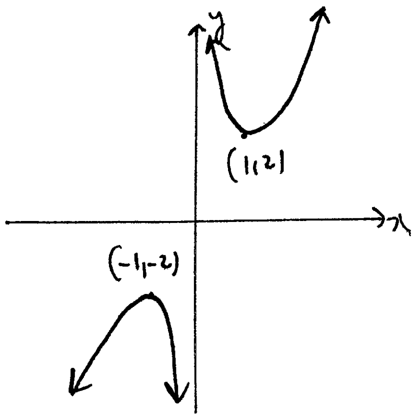
$x = \pm 1$

$f(1) = 1 + \frac{1}{1} = 2$

$f(-1) = -1 - 1 = -2$

S.t. POINTS $(1,2)$, $(-1,-2)$

x	-2	-1	$-\frac{1}{2}$	$\frac{1}{2}$	1	2
$f'(x)$	$\frac{3}{4}$	0	-3	-3	0	$\frac{3}{4}$
	/	—	\	\	—	/
		MAX			MIN	



(Q13) $f(x) = 2x^3 - 3x^2 + 18x + 1$

$f'(x) = 6x^2 - 6x + 18$

$\Delta = b^2 - 4ac = 36 - 4(6)(18) < 0$

$\therefore f'(x) > 0$ FOR ALL x

$\therefore f$ IS STRICTLY INCREASING.

(Q14) $f(x) = (ax^2+1)e^{2x}$

$f'(x) = (ax^2+1)2e^{2x} + e^{2x}(2ax)$

$= (2ax^2 + 2ax + 2)e^{2x}$

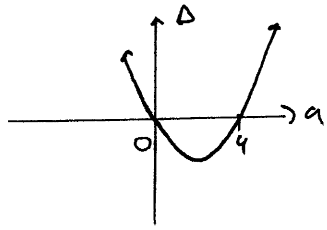
REQUIRE $2ax^2 + 2ax + 2 > 0$

FOR ALL x.

$\Delta = (2a)^2 - 4(2a)(2)$

$= 4a^2 - 16a$

$= 4a(a-4)$



REQUIRE $\Delta < 0$

i.e. $0 < a < 4$

(Q15) $f(x) = kx^3 + x^2 + x + 1$

$f'(x) = 3kx^2 + 2x + 1$

REQUIRE $\Delta \leq 0$

$\Delta = b^2 - 4ac$

$= 4 - 4(3k)(1)$

$= 4 - 12k \leq 0$

$\therefore 12k \geq 4 \Rightarrow k \geq \frac{1}{3}$

(Q16) $V(t) = 100t^2 - 10t^3$

(A) $V'(t) = 20\cancel{0}\,t - 3\cancel{0}t^2 = 12\cancel{0}$

$3t^2 - 20t + 12 = 0$

$(t-6)(3t-2) = 0$

$\therefore \; t = 6,\ 2/3$

(B) $V'(2) = 200 \cdot 2 - 30 \cdot 2^2$

$= 400 - 30 \cdot 4$

$= 400 - 120$

$= 280$ L/min

(C) MAX VOLUME : $V'(t) = 0$

$\Rightarrow \; 200t - 30t^2 = 0$

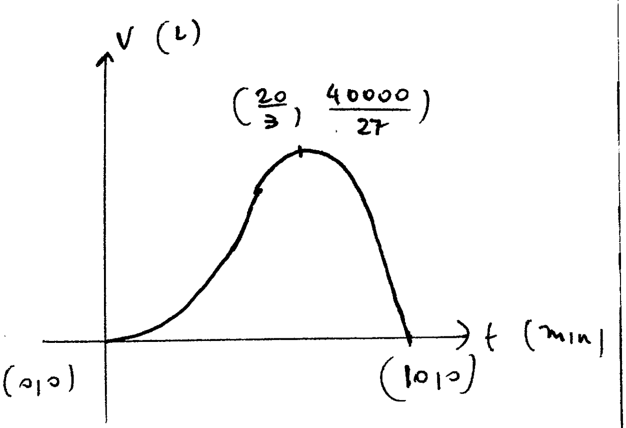
$20t - 3t^2 = 0$

$t(20 - 3t) = 0$

$\Rightarrow \; t = 0,\ \frac{20}{3}$ min

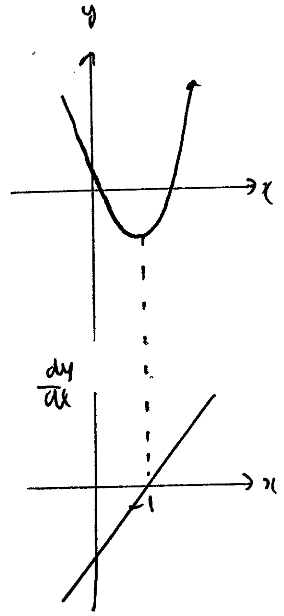
MIN MAX

(D)

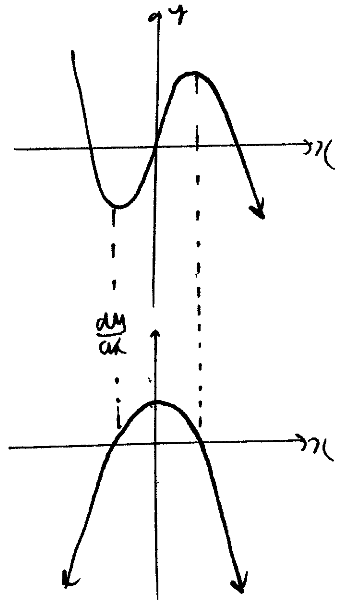


Q17)

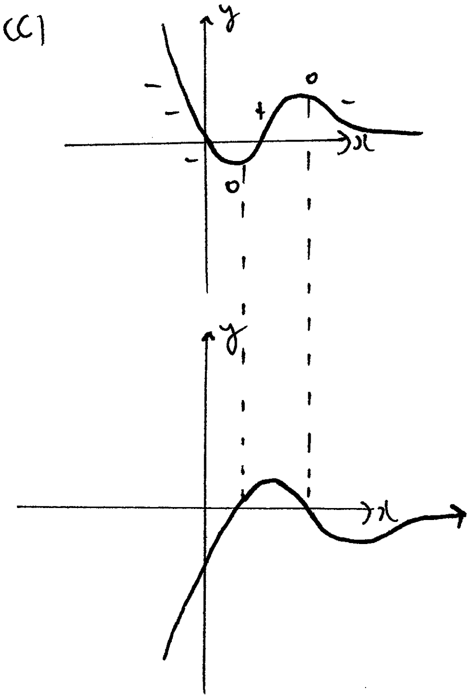
(A)



(B)



(C)



(Q18) (A) $f : [-1,5] \to \mathbb{R}$

$f(x) = x^2(x-3)$

$f(-1) = 1(1-3) = -2$

$f(5) = 25(5-3) = 50$

$f'(x) = 3x^2 - 6x$

ST. PTS $\quad f'(x) = 0$

$3x(x-2) = 0$

$x = 0, 2$

$f(0) = 0, \quad f(2) = 4(2-3) = -4$

CANDIDATE POINTS $\longrightarrow$ $(-1,-2)$

$(5, 50) \longleftarrow$ ABS - MAX

$(2,-4) \longleftarrow$ ABS. MIN

$(0,0)$

(B) $f(x) = x^3(x-1)^2 \quad x \in [-2,2]$

$f(2) = 8(2-1)^2 = 8$

$f(-2) = -8(-2-1)^2 = -72$

$\cancel{f(x) = x^3(x^2-2x+1)}$

$\cancel{= x}$

$f'(x) = x^3 \cdot 2(x-1) + (x-1)^2 3x^2$

$= x^2(x-1)(2x + 3(x-1))$

$= x^2(x-1)(5x-3)$

ST. PTS. $\quad f'(x) = 0$

$x = 0, 1, \frac{3}{5}$

$f(0) = 0, \quad f(1) = 0 \quad f(\frac{3}{5}) = \frac{108}{3125}$

CANDIDATE POINTS

$\hookrightarrow (2,8) \longleftarrow$ ABS MAX

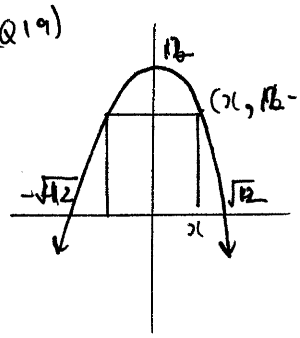
$(-2,-72) \longleftarrow$ ABS MIN

$(0,0)$

$(1,0)$

$(\frac{3}{5}, \frac{108}{3125})$

(Q19)



$A = 2x(12-x^2)$

$= 24x - 2x^3$

$\frac{dA}{dx} = 24 - 6x^2 = 0$

$6x^2 = 24$

$x^2 = 4$

$x = 2 \quad (x > 0)$

$\therefore A = 2 \cdot 2(12 - 2^2)$

$= 4 \cdot (8)$

$= 32$

(Q20) SEPARATION : $f(x) = 3 + \cos x - 1 - \sin x$

$f'(x) = -\sin x - \cos x = 0$

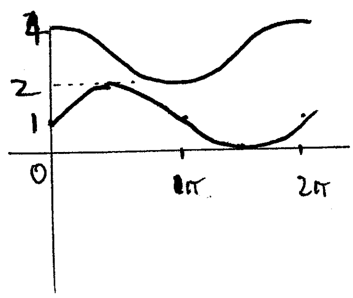
$\tan x = -1$

$x = \frac{3\pi}{4}, \frac{7\pi}{4}$

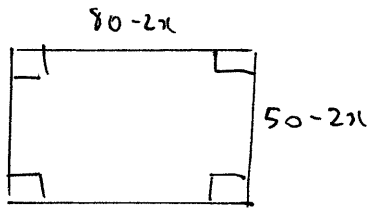
MIN $\quad$ MAX

$f(\frac{3\pi}{4}) = 3 + \cos(\frac{3\pi}{4}) - 1 - \sin(\frac{3\pi}{4}) = 2 - \sqrt{2}$ (MIN

$f(\frac{7\pi}{4}) = 3 + \cos(\frac{7\pi}{4}) - 1 - \sin(\frac{7\pi}{4}) = 2 + \sqrt{2}$ (MAX



(Q21)



$$V = x(80-2x)(50-2x)$$

$$= 4x(40-x)(25-x)$$

$$= 4x(x^2 - 65x + 1000)$$

$$= 4(x^3 - 65x^2 + 1000x)$$

$$\frac{dV}{dx} = 4(3x^2 - 130x + 1{,}000)$$

$$= 4(x-10)(3x-100)$$

$$x = 10, \quad \cancel{x = \frac{100}{3}}$$

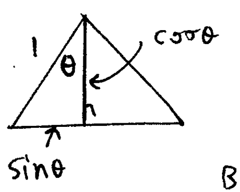
SINCE $0 \le x \le 25$

$$V(10) = 10 \cdot (80-20)(50-20)$$

$$= 10 \cdot 60 \cdot 30$$

$$= 18{,}000 \text{ cm}^3$$

(Q22) TRIG SOLUTION



$$A = \frac{BH}{2}$$

$$= \frac{2\sin\theta\cos\theta}{2}$$

$$= \sin\theta\cos\theta$$

$$0 \le \theta \le \frac{\pi}{2}$$

$$\frac{dA}{d\theta} = \sin\theta(-\sin\theta) + \cos\theta(\cos\theta)$$

$$= -\sin^2\theta + \cos^2\theta$$

$$= -\sin^2\theta + 1 - \sin^2\theta$$

$$= 1 - 2\sin^2\theta$$

ST. PTS $\frac{dA}{d\theta} = 0$

$$\therefore \sin^2\theta = \frac{1}{2}$$

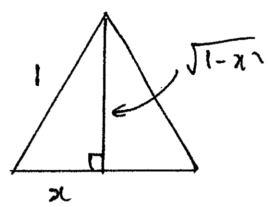
$$\sin\theta = \frac{1}{\sqrt{2}} \quad (0 \le \theta \le \pi/2)$$

$$\therefore \theta = \frac{\pi}{4}.$$

$$\therefore A = \sin\frac{\pi}{4}\cos\frac{\pi}{4}$$

$$= \frac{1}{\sqrt{2}} \cdot \frac{1}{\sqrt{2}} = \frac{1}{2}$$

NO TRIG SOLN $\quad 0 \le x \le 2$



$$A = \frac{BH}{2}$$

$$= \frac{2x\sqrt{1-x^2}}{2}$$

$$= x\sqrt{1-x^2}$$

$$= \sqrt{x^2 - x^4}$$

$$\frac{dA}{dx} = \frac{1}{2}(x^2-x^4)^{-\frac{1}{2}}(2x - 4x^3)$$

$$= \frac{2x - 4x^3}{2\sqrt{x^2-x^4}}$$

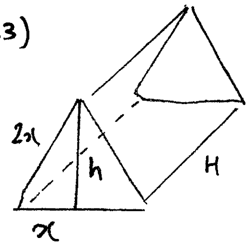
$$= \frac{x - 2x^3}{\sqrt{x^2-x^4}} = 0$$

$$\therefore x - 2x^3 = 0$$

$$x(1-2x^2) = 0 \implies x = \frac{1}{\sqrt{2}} \; (x_0$$

$$\therefore A = \frac{1}{\sqrt{2}} \cdot \frac{1}{\sqrt{2}} = \frac{1}{2}$$

(Q23)



LET $2x$ = BASE LENGTH.

$h^2 = \sqrt{4x^2 - x^2}$

$= \sqrt{3}x$

$12x + 3H = 12$

$4x + H = 4$

$H = 4 - 4x$

$A = \frac{2xh}{2} = xh$

$\therefore V = xhH$

$= x\sqrt{3}x(4 - 4x)$

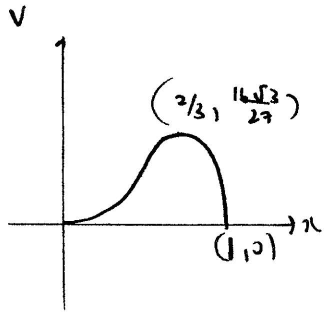
$= 4\sqrt{3}x^2(1-x)$

$= 4\sqrt{3}(x^2 - x^3)$

$\frac{dV}{dx} = 4\sqrt{3}(2x - 3x^2)$

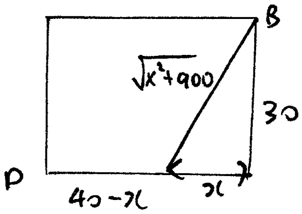
$= 4\sqrt{3}x(2 - 3x) = 0$ FOR ST. PTS

$\therefore x = 0, \frac{2}{3}$



WHEN $x = \frac{2}{3}$ $V = \frac{16\sqrt{3}}{27}$

(Q24)



$T = \frac{D_1}{S_1} + \frac{D_2}{S_2}$

$= \frac{40-x}{10} + \frac{\sqrt{x^2+900}}{2}$

$\frac{dT}{dx} = -\frac{1}{10} + \frac{2x}{4\sqrt{x^2+900}} = 0$ (FOR ST. PTS)

$\frac{2x}{4\sqrt{x^2+900}} = +\frac{1}{10}$

$20x = 4\sqrt{x^2+900}$

$5x = \sqrt{x^2+900}$

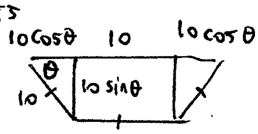
$25x^2 = x^2 + 900$

$24x^2 = 900$

$x = \sqrt{\frac{900}{24}} = \frac{30}{2\sqrt{6}} = \frac{15}{\sqrt{6}}$

$\therefore$ WHEN $x = \frac{15}{\sqrt{6}}$, $\underline{T = 4+6\sqrt{6}}$ sec.

(Q25)



$A = \left(\frac{a+b}{2}\right)h$

$= \left(\frac{20+20\cos\theta}{2}\right)10\sin\theta$

$= 100(1+\cos\theta)\sin\theta$, $0 \le \theta \le 2\pi/3$

$\frac{dA}{d\theta} = 100(1+\cos\theta)\cos\theta + 100\sin\theta(-\sin\theta)$

$= 100(\cos\theta + \cos^2\theta - \sin^2\theta)$

$= 100(\cos\theta + \cos^2\theta - (1-\cos^2\theta))$

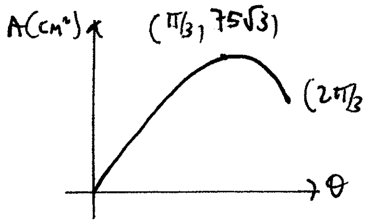
$= 100(2\cos^2\theta + \cos\theta - 1)$ $\begin{matrix} 2x & -1 \\ x & 1 \end{matrix}$

$= 100(2\cos\theta - 1)(\cos\theta + 1)$

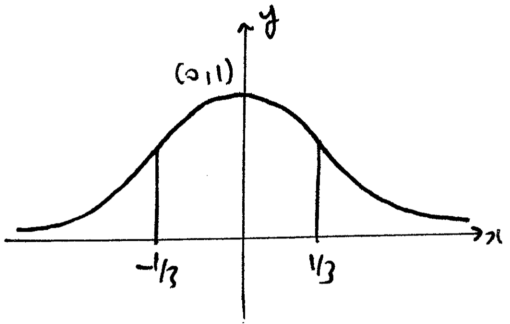
$\cos\theta = \frac{1}{2}$ or $\cos\theta = -1$

$\theta = \frac{\pi}{3}$

$\cancel{\theta = \pi}$ REJECT AS $\theta < 2\pi/3$



Q26) (A)



(B) $\frac{dy}{dx} = \frac{(1+x^2)0 - (2x)}{(1+x^2)^2}$

$= \frac{-2x}{(1+x^2)^2}$

WHEN $x = a$ $\quad m = \frac{-2a}{(1+a^2)^2}$

(C) AIM : MAXIMISE m.

$\frac{dm}{da} = \frac{(1+a^2)^2(-2) + 2a(2)(1+a^2)2a}{(1+a^2)^4}$

$= \frac{-2(1+a^2)^2 + 8a^2(1+a^2)}{(1+a^2)^4}$

$= \frac{2(1+a^2)(4a^2-(1+a^2))}{(1+a^2)^4}$

$= \frac{2(4a^2-1-a^2)}{(1+a^2)^3}$

$= \frac{6a^2-2}{(1+a^2)^3}$

MAX WHEN $\frac{dm}{da} = 0$

I.E. $6a^2 - 2 = 0$

$a^2 = \frac{1}{3}$

$a = -\frac{1}{3}, \frac{1}{3}$

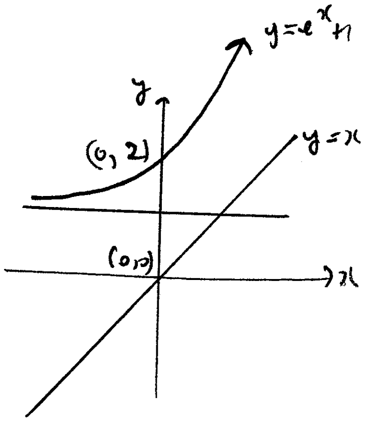
MAX m $\qquad$ MIN m

Q27) $y = 3\cos 2x$

GRAD : $\frac{dy}{dx} = -6\sin 2x$

$\therefore$ MAX GRADIENT $= 6$

Q28) (A)



(B)

SEPARATION : $f(x) = e^x + 1 - x$

$f'(x) = e^x - 1$

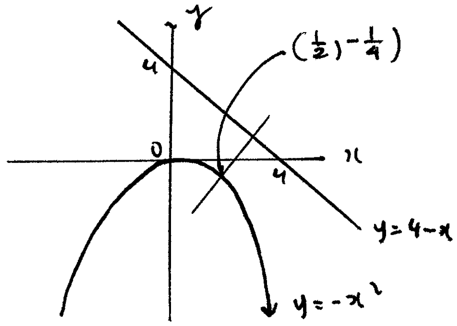
MAX OR MIN WHEN $f'(x) = 0$

$e^x - 1 = 0$

$e^x = 1 \quad \therefore x = 0$

WHEN $x = 0$, $f(0) = e^0 + 1 - 0 = 2$

Q29) (A)



$y = 4 - x \quad m = -1 \quad \cancel{m_1 = +1}$

NORMAL LINE PERP. PROVIDED THE TANGENT LINE IS PARALLEL TO $g(x) = 4 - x$

$m = -1 \quad \therefore f'(x) = -2x = -1$

$x = \frac{1}{2}$

$f(\frac{1}{2}) = -(\frac{1}{2})^2 = -\frac{1}{4}$

$\therefore$ POINT $= (\frac{1}{2}, -\frac{1}{4})$

#12

(Q30) $y = (x-1)^n$

$\frac{dy}{dx} = n(x-1)^{n-1}$

WHEN $x = 2$, $m = \frac{dy}{dx} = n(2-1)^{n-1} = n$

TANGENT $y - y_1 = n(x - x_1)$

$y - 1 = n(x-2)$

WHEN $x = 0$, $y = 0$

$0 - 1 = n(-2)$

$-1 = -2n \quad \therefore \underline{n = \frac{1}{2}}.$

(Q31) $y = ax^2 + bx + c$

$\frac{dy}{dx} = 2ax + b = 0$ FOR ST. PT.

$2ax = -b$

$x = -\frac{b}{2a}$

x	TECHNICALLY SHOULD CHECK THAT THIS IS INDEED A TURNING POINT USING A SIGN DIAGRAM.
$\frac{dy}{dx}$	

(Q32) (A) $y = ax^3 + bx^2 + cx + d$

$\frac{dy}{dx} = 3ax^2 + 2bx + c$

S.t. PTS $x = \frac{-2b \pm \sqrt{4b^2 - 12ac}}{6a}$

(i) NO STAT PT IF ~~.....~~ $\Delta < 0$

$\Delta = (2b)^2 - 4(3a)(c)$

$= 4b^2 - 12ac < 0$

I.E. $\underline{b^2 < 3ac}$

(ii) ONE STAT. POINT : $\Delta = 0$

I.E. $\underline{b^2 = 3ac}$

(iii) TWO STAT POINTS : $\Delta > 0$

I.E. $b^2 > 3ac$.

(Q33) $g(x) = (x-a)^n f(x)$

$g'(x) = (x-a)^n f'(x) + f(x) \cdot n(x-a)^{n-1}$

$= (x-a)^{n-1}\left((x-a)f'(x) + n f(x)\right)$

$g'(a) = \underset{= 0}{(a-a)^{n-1}}\left(0 \cdot f'(x) + n f(a)\right)$

$= 0$ (SINCE $n - 1 \geq 1$)

$\therefore g$ HAS STAT. PT. @ $x = a$

(Q34) $x + y = 9 \qquad x, y \geq 0$

$f(x) = xy^2 = x(9-x)^2$

$= x(81 - 18x + x^2)$

$= x^3 - 18x^2 + 81x$

$f'(x) = 3x^2 - 36x + 81$

$= 3(x^2 - 12x + 27)$

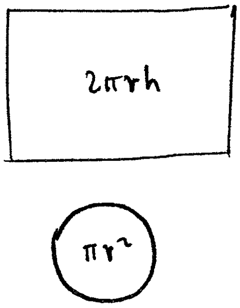
$= 3(x-9)(x-3) = 0$ (ST. PTS)

$\therefore x = 9, 3$ ← ~~..........~~

$f(9) = 0$ ← MIN

$f(3) = 3 \cdot 6^2 = \underline{108}$ ← MAX

(Q35)



$$A = 2\pi r h + \pi r^2 = 6$$

$$h = \frac{6 - \pi r^2}{2\pi r}$$

$$V = \pi r^2 h$$

$$= \cancel{\pi} r \cancel{r} \frac{6 - \pi r^2}{2\cancel{\pi}\cancel{r}}$$

$$= \frac{1}{2} r (6 - \pi r^2)$$

$$= 3r - \frac{\pi}{2} r^3$$

$$\frac{dV}{dr} = 3 - \frac{3\pi}{2} r^2 = 0$$

$$\frac{\cancel{3}\pi}{2} r^2 = \cancel{3}$$

$$r^2 = \frac{2}{\pi}$$

$$\therefore \quad \underline{r = \sqrt{\frac{2}{\pi}}}.$$

(Q36) LET x = # OF ADDED TREES

$\therefore$ TOTAL TREES = $50 + x$

AND OUTPUT PER TREE = $800 - 10x$

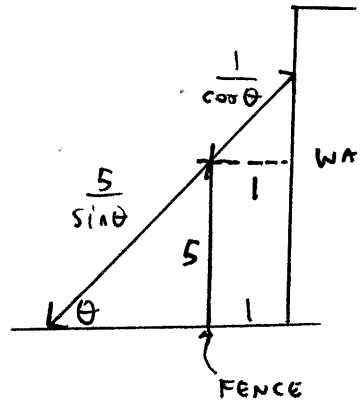
$\therefore$ OUTPUT TOTAL $= (50 + x)(800 - 10x)$

$= 10(50 + x)(80 - x)$

QUADRATIC $\therefore$ TP HALFWAY B/W INTERCEPTS

$$TP = \frac{-50 + 80}{2} = \underline{15}$$ TREES

(Q37)



$$L(\theta) = 5(\sin\theta)^{-1} + (\cos\theta)^{-1}$$

$$L'(\theta) = -5(\sin\theta)^{-2}\cos\theta - 1(\cos\theta)^{-2}(-\sin\theta)$$

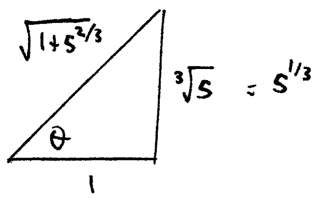
$$= \frac{-5\cos\theta}{\sin^2\theta} + \frac{\sin\theta}{\cos^2\theta}$$

$$= \frac{\sin^3\theta - 5\cos^3\theta}{\sin^2\theta\cos^2\theta} = 0$$

$$\therefore \quad \sin^3\theta = 5\cos^3\theta$$

$$\tan^3\theta = \left(\frac{\sin\theta}{\cos\theta}\right)^3 = 5$$

$$\tan\theta = \sqrt[3]{5}.$$



$$\sin\theta = \frac{5^{1/3}}{\sqrt{1+5^{2/3}}} \qquad \cos\theta = \frac{1}{\sqrt{1+5^{2/3}}}$$

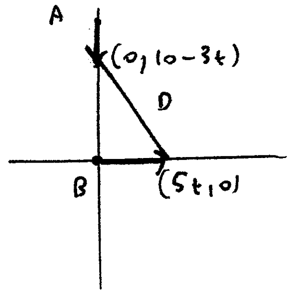
$$\therefore \quad L(\theta) = \frac{5}{5^{1/3}}\sqrt{1+5^{2/3}} + \sqrt{1+5^{2/3}}$$

$$= 5^{2/3}\sqrt{1+5^{2/3}} + \sqrt{1+5^{2/3}}$$

$$= \sqrt{1+5^{2/3}}\,(5^{2/3} + 1)$$

$$= (1 + 5^{2/3})^{3/2}$$

Q38)



BOB x CO-ORD $= 5t$

ALICE y CO-ORD $= 10 - 3t$

$D = \sqrt{(5t)^2 + (10-3t)^2}$

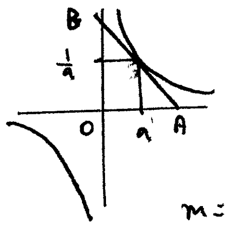
$D^2 = (5t)^2 + (10-3t)^2 = 2(17t^2 - 30t + 50)$

MINIMISE D^2

$\frac{dD^2}{dt} = 2(34t - 30) = 0$ ST. PT.

$t = \frac{30}{34} = \underline{\frac{15}{17}}$ HRS

Q39)



$f(x) = \frac{1}{x} = x^{-1}$

$f'(x) = -x^{-2} = -\frac{1}{x^2}$

$m = f'(a) = -\frac{1}{a^2}$

<u>TANGENT</u> $y - y_1 = m(x - x_1)$

$y - \frac{1}{a} = -\frac{1}{a^2}(x - a)$

$y - \frac{1}{a} = -\frac{x}{a^2} + \frac{1}{a}$

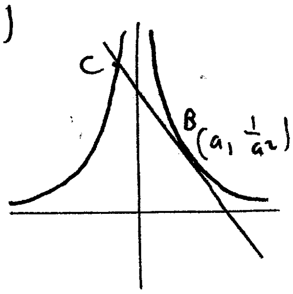
$y = -\frac{x}{a^2} + \frac{2}{a}$

WHEN $x = 0$, $y = \frac{2}{a}$

WHEN $y = 0$, $\frac{x}{a^2} = \frac{2}{a} \Rightarrow x = 2a$

$\therefore$ AREA(ΔAOB) $= \frac{2a \cdot \frac{2}{a}}{2} = \underline{2}$

Q40)



$f(x) = x^{-2} \quad \therefore \quad f'(x) = -2x^{-3}$

$m = f'(a) = -\frac{2}{a^3}$

$y - y_1 = m(x - x_1)$

$y - \frac{1}{a^2} = -\frac{2}{a^3}(x - a)$

$y - \frac{1}{a^2} = -\frac{2}{a^3}x + \frac{2}{a^2}$

$y = -\frac{2}{a^3}x + \frac{3}{a^2}$ ①

$y = \frac{1}{x^2}$ ②

① = ② $\quad \frac{1}{x^2} = -\frac{2}{a^3}x + \frac{3}{a^2}$

$(\times x^2) \quad 1 = -\frac{2}{a^3}x^3 + \frac{3}{a^2}x^2$

$(\times a^3) \quad a^3 = -2x^3 + 3ax^2$

$p(a) = 2x^3 - 3ax^2 + a^3 = 0$

NOTE $p(a) = 2a^3 - 3a^3 + a^3 = 0$

$\therefore (x-a)$ IS A FACTOR

$(x-a)(2x^2 \quad - a^2) = 0$

$(x-a)(2x^2 - ax - a^2) = 0$

$(x-a)(x-a)(a+2x) = 0$

$\therefore x = a, \ x = -\frac{a}{2}.$

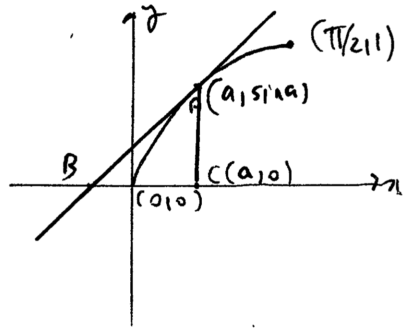
$\downarrow$

$\therefore y = +\frac{1}{(-a/2)^2}$

$= \frac{4}{a^2}$

POINT $= C\left(-\frac{a}{2}, \frac{4}{a^2}\right)$

Q41)



$f(x) = \sin x$

$f'(x) = \cos x$

@ $x = a$ $\quad f'(a) = m = \cos a$

TANGENT : $\quad y - y_1 = m(x - x_1)$

$y - \sin a = \cos a(x - a)$

WHEN $\quad y = 0$

$-\sin a = \cos a\,(x - a)$

$-\tan a = x - a$

$\Rightarrow \quad x = a - \tan a$

$A(\Delta BCA) = \frac{3}{4}$

$\cancel{\frac{(2a - \tan a)\sin a}{2} = \frac{3}{4}}$

BASE OF TRIANGLE $= a - (a - \tan a)$

$= \tan a$

$\therefore \quad A(\Delta BCA) = \frac{\tan a \sin a}{2} = \frac{3}{4}$

$4 \tan a \sin a = 6$

$2 \frac{\sin a}{\cos a} \sin a = 3$

$2 \sin^2 a = 3 \cos a$

$2(1 - \cos^2 a) = 3 \cos a$

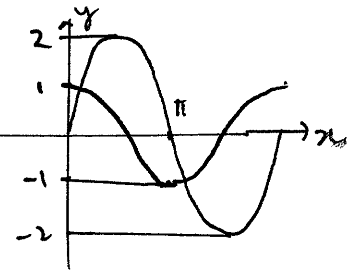
$2\cos^2 a + 3 \cos a - 2 = 0$

$(2\cos a - 1)(\cos a + 2) = 0$

$\cos a = \frac{1}{2}$ or $\cos a = -2$ (NOT POSSIBLE

$\therefore \quad a = \frac{\pi}{3}$

~~Q42)~~



VERT. SEP $\cancel{f(x) = 2\sin x - \cos x}$

$\cancel{f'(x) = 2\cos x + \sin x} = 0$

$\therefore \quad \cancel{\sin x = -2\cos x}$

$\cancel{\tan x = -2}$